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Research Paper



Tracking of pendulum using particle filter with residual resampling

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Abstract

The phenomenon of simple harmonic motion is more vigilantly explained using a simple pendulum. The angular motion of a pendulum is linear in nature. But the analysis of the motion along the horizontal direction is non-linear. To estimate this, several algorithms like the Kalman filter, Extended Kalman Filter etc. are adopted. Here in this paper, Particle filter is chosen which is a method to form Monte Carlo approximations to the solutions of Bayesian filtering equations. Sequential importance resampling based Particle filters are used where the filtering distributions are multi-nodal or consist of discrete state components since under these circumstances the Bayesian approximations do not always work well.

Keywords: *Bayesian filtering; Extended Kalman Filter; Kalman Filter; Particle filter; Simple pendulum*

1. Introduction

A simple pendulum is a massy object which is suspended from a wire or a string which has a negligible mass. It is also called "Pendulum Bob". The to and fro motion of this simple pendulum produces simple harmonic motion. When the estimation of the motion of the pendulum is done along the angular direction, then it will be linear with respect to time [2]. But if the estimation is along the horizontal direction, then the estimation becomes non-linear. Hence several algorithms are adopted for this purpose. Bayesian estimators are generally used to track target objects with non-linear motion. During earlier studies, the Bayesian estimators like Kalman Filter and Extended Kalman Filter were used. Due to the requirement of improvement in efficiency, Particle filter is adopted in this paper.
The major problem of the sequential importance sampling is that there may occur situations where all the particles have almost zero weights [1]. This problem is called the degeneracy problem in Particle filtering. It can be eliminated or at least reduced by using resampling procedure. In this paper, residual resampling method is used for resampling the particles.

2. Particle Filter

The principle of Particle filter is a sample based approximation of full inference. Particle filters are used to obtain the solutions for the filtering problems which consist of the estimation of internal states in dynamic systems. Earlier, instead of Particle Filter, Kalman Filter and Extended Kalman Filter were used. Kalman Filter used for linear time-variant systems with noise characteristic follows a Gaussian distribution [4]. But when the system is non-linear like a simple pendulum, such a model is no longer efficient. In order to overcome the disadvantage of the Kalman Filter, Kalman and Bucy proposed the Extended Kalman filter. The latter is very useful for non-linear systems because it uses the expansion of the Taylor series using Gaussian approximations to linearize the non-linearities present in the systems. Then the normal Kalman Filter is applied to it. Thus, the Extended Kalman Filter provides better and more efficient results than those which are provided by the Kalman Filter. But when the approximations do not seem to be Gaussian, then Particle filter must be adopted.

The Particle filtering main idea is to generate a set of likely points for the solutions in the state space modeling and representing the probability distribution function (PDF) only in those sampled points by assigning some weights to each sampled point, and then normalize the weights. The set of points in the solution space is generated according to prior pdf. Then the corresponding weight for each point is calculated. The prior pdf is updated according to the resultant weights, and this process is repeated, to yield the solution.

The Particle Filter discretizes the problem into 'particles', where each particle corresponds to a state in the model. The approximation of the probability distribution will be more appropriate when the number of particles is more. Particle filter procedures give a settled technique to create tests from the required appropriation without the requirement of suppositions about the state-space demonstration or the state distributions. Particle Filters are methods to form Monte Carlo approximations to the solutions of the Bayesian Filtering Equations. Monte Carlo approximations or methods replace the closed form computational analysis of the statistical quantities by drawing samples from the distribution and estimating the quantities by the sample averages [1].
In a completely perfect Monte Carlo approximation, N number of independent random samples are drawn.

$$x(i) \sim p(x \mid y1{:}T),\ i = 1,2,\ldots\ldots N \quad (1)$$

The expected estimate is given by [5],

$$E[g(x)|y_{1:T}] \approx \frac{1}{N}\sum \overline{w}^{(i)}\, g(x(i)) \quad (2)$$

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Thus, Monte Carlo methods approximately compute the target distribution as a set of samples that are distributed according to the target density. Using Bayes rule, the non-linear filtering equation is given by,

$$p(x_0, \ldots, x_k \mid y_0, \ldots, y_k) = \frac{p(y0,\ldots,yk \mid x0,\ldots,xk)p(x0,\ldots,xk)}{p(y0,\ldots,yk)} \quad (3)$$

Where,

$$p(y0,\ldots,yk) = p(y0,\ldots,yk|x0,\ldots,xk)p(x0,\ldots,xk)dx0\ldots dxk$$

Since Particle filters are approximations, they will be more accurate when the number of particles is more. Thus, from [6] the above equation becomes

$$p(x_k \mid y_0,\ldots,y_{k-1})$$

$$\text{updating} \quad p(x_k \mid y_0,\ldots,y_k) = \frac{p(yk|xk)p(xk|y0,\ldots,yk-1)}{\int p(yk|xk')p(xk'|y0,\ldots,yk-1)dxk'}$$

$$\text{prediction} \quad p(x_{k+1}|y_0,\ldots,y_k) = \int p(x_{k+1}|x_k)p(x_k|y_0,\ldots,y_k)dx_k \quad (4)$$

There are many methods for Particle Filtering, and each one of them is based on the three operations, particle propagation, weight computation, and resampling. Particle propagation and weight computation fall up to the part of particles and the weight computation, while resampling replaces one set of an arrangement of particles and their weights with another set.

3. Residual Resampling

Resampling means, selecting new positions for particles and assigning new weights to them. It is not necessary that resampling is performed for every step, but performed at regular intervals of time, depending upon the resampling size. The main idea of resampling is to remove particles with lower weights and replace them with particles with higher weights. Resampling is used to prevent inaccurate estimates which have unacceptably large variances. Although the particle generation and weight computation are more expensive steps, resampling is very much important for Particle filter. The resampling step, which is required to truncate off the particles with low normalized weights and then duplicate them with highly normalized weights, is basic because it is a powerful technique to reduce the decline or degeneracy issue in Particle Filters. Resampling is a basic system that is of both hypothetical and practical hugeness for proficient use of the Particle filter.

By using the adaptive resampling technique, the sample size can be selected based on the variance of the particle,

$$n_{effective} = \frac{1}{\sum_{i=1}^{N} (w_k^{(i)})^2} \quad (5)$$

$w_k^{(i)}$-normalized weight of ith particle at kth time step.

The process of resampling is done whenever the effective number of particles is less than the total number of particles. The resampling procedure is done in three steps:

1. First compute the weight of each sample $w_k^{(i)}$, which is the probability of obtaining the ith sample in the distribution $\{ x_k^{(i)} \}$, where, i= 1,2,3,4,5…..N.
2. Remove the samples which are with nearly zero weights and add new samples that are likely to the actual measurement, using resampling.
3. After the resampling procedure, set the weights of new samples and old samples at a constant, say, $w_k^{(i)} = 1/N$.

Since the resampling procedure is not done in every step, there occurs an increase in the variance of the samples. To reduce this variance, several resampling algorithms like systematic resampling, stratified resampling, residual resampling, or multinomial resampling are adopted.

Residual resampling is one of the efficient means of decreasing the variance due to resampling. That is the reason why it is used in this paper. It's also known as **remainder resampling**. In this algorithm, the quantity of the repeated particles is computed first by the removal of the products of the weights of the particles and the quantity of the particles. After this, the quantity of particles that are created is not as much as the required number because of truncation. Along these lines, it is therefore important to resample the remaining deposits to make up for the improved quantity of particles.

Residual resampling method:

From [7], let 'N' represent the number of particles, 'M' represent the number of resampled particles, 'W' represents an array consisting of importance weights of particles, and the output 'R' represent an array of replication factors.

Then,

$$(R) = rr\ (W, N, M) \quad (6)$$

To start with the calculation of the number of replications of the particles due to the truncation of the product and then calculate the remaining number of particles (Mr). The aim is to determine the replica of particles whose weight is greater than 1/N. For N particles, the following steps are performed i.e., N iterations [7].

$$R(m) = \lfloor W(m).M \rfloor \quad (7)$$

$$W_r(m) = W(m).M\text{-}R(m) \quad (8)$$

$$M_r = M_r - R(m) \quad (9)$$

Later the resampling residues are used to produce 'M_r' of the final set of M particles. Later, the quantity of replications of the particles is computed by adding the replication factors that are produced by the above given procedure [7]. The particles are gathered by the residual weights and by utilizing multinomial resampling (another sampling algorithm), where, the likelihood for choosing the particle is directly proportional to the residual weight of that particle [8]. The condition is 'M_r' should be greater than '0' and the following steps are performed N times individually forming a loop.

$$W_r(m) = W_r(m)/M_r \quad (10)$$

$$(R_r) = sr(W_r, N, M_r) \quad (11)$$

$$R(m) = R(m)+R_r(m) \quad (12)$$

Since the final step is obtained from the systematic resampling algorithm, with N number of input and M_r number of output particles, normalization $W_r(m)/M_r$ where m=1, 2, . . ., N, must be implemented before the sequential resampling algorithm [8]. The primary stage relates to a deterministic replication, and thus, the variety of the number of times a particle is resampled is just ascribed to the second stage. The method of Residual resampling has the ability to keep constant sample size. The computational complexity is high for this method. The number of random numbers used can be given by the difference between the number of input particles and the number of replicated particles.

4. Basic Sine Motion of Pendulum

The general differential equation form for a simple pendulum with unit mass and unit length, is given as [5]

$$\frac{d^2(\theta)}{dt^2} = -g\sin(\theta) + \eta(t) \quad (13)$$

where, θ is the angle made by the equilibrium axis with respect to the displacement of the pendulum rod, g is the acceleration due to gravity and $\boldsymbol{\eta}$(t) is the random noise

In terms of state space model, these equations can be represented as,

$$\frac{d}{dt}\begin{pmatrix}x_1\\x_2\end{pmatrix} = \begin{pmatrix}x_2\\-gsin(x_1)\end{pmatrix} + \begin{pmatrix}0\\1\end{pmatrix}\eta(t) \quad (14)$$

where, x_1=θ and x_2=$\frac{d\theta}{dt}$. As already mentioned, the measurement along the horizontal axis gives a non-linear measurement, and it can be given by

$$y_k = \sin(\alpha(y_k)) + \text{noise} \quad (15)$$

The discretization of a continuous nonlinear state with respect to discrete nonlinear measurements can be done as,

$$x_k = f(m_{k-1}, n_{k-1}) \quad (16)$$

$$y_k = q(x_k, y_k) \quad (17)$$

where y represents the measurement vector. The discretization of pendulum model is as follows,

$$\begin{pmatrix}x_{1,k}\\x_{2,k}\end{pmatrix} = \begin{pmatrix}x_1(k-1) + x_2(k-1) * \Delta t\\x_2(k-1) - g * \sin(x_1(k-1)) * \Delta t\end{pmatrix} + n_{k-1} \quad (18)$$

$$y_k = \sin(x_1(k)) + r_k \quad (19)$$

where $n_{k-1} \sim N$ and $r_k \sim N(0,R)$ are noise vectors that are to be considered. Jacobian matrices of f and q for first order system is as follows

$$Q = n^c\left(\frac{\Delta T^3}{3}\ \frac{\Delta T^2}{2}\ ;\frac{\Delta T^2}{2}\ \Delta T\right) \quad (20)$$

where n^c is the spectral density of continuous time process noise. The weights for pendulum state can be computed as

$$w \sim e^{\frac{-1}{2R(y(k)-\sin(x(k,i)))^2}} \quad (21)$$

where, k represents the state; i = 1,2,…N and R represents the Variance Normalize W by

$$W_i = \frac{W_i}{\sum_{j=0}^{N} W_j} \quad (22)$$

Now generate particles based on the weights W and apply resampling at fixed N time intervals.

In the residual resampling approach as given by [3], for i=1,2,.... m,

$$N_s = \lfloor Nw^i \rfloor \quad (23)$$

where, ⌊ ⌋ denotes integer part, $R = \sum_{i=1}^{N}\lfloor Nw^i \rfloor$ and

$$W_s = \frac{(N * W - N_s)}{N - R} \quad (24)$$

5. Experimental Results

The simulation and particle filtering of a simple pendulum is carried out with residual resampling for N=100 times in our experiments. Then, the root mean square error is calculated for all the experimental values on an average, and then the differences between the root mean square errors of the measured and estimated values. Then, it is it identified that the residual resampled estimate values are much closer to the true values, than the un-resampled particle filter estimates.

It is thereby identified that the root mean square error has decreased and therefore, particle filter with residual resampling has increased the scope of estimating the actual motion of the pendulum.

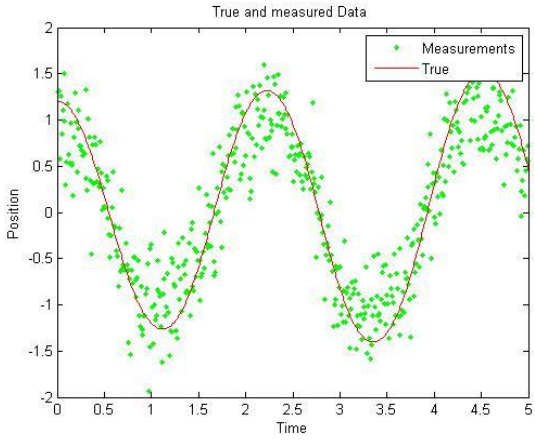


Figure1: The simulation of a simple pendulum: Red line and green dots indicate the true and measured values of the pendulum position before resampling.

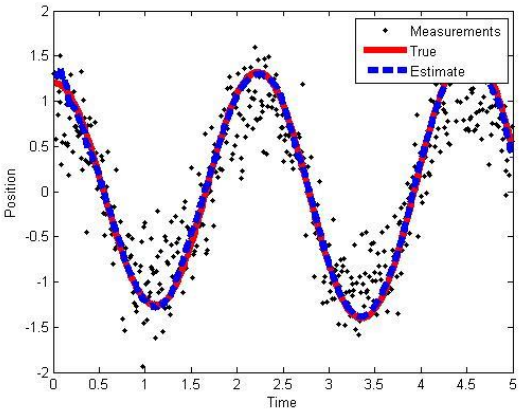


Figure 2: Residual Resampling Estimate: The red line and green dots indicate the true and measured values of the pendulum position after resampling.

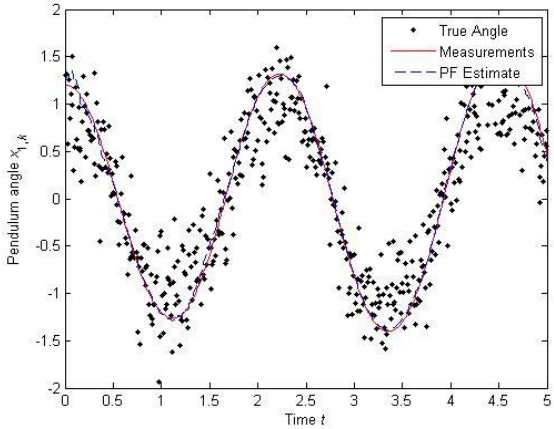


Figure 3: Particle Filter estimate: the dots represent the true angle, continuous line represents the measurements, and the dotted line represents the Particle Filter estimate.

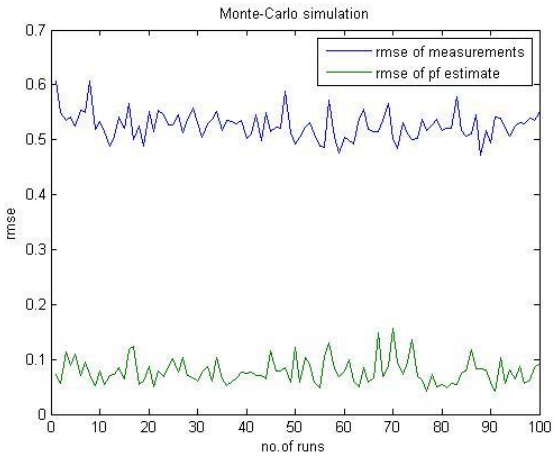


Figure 4: Monte-Carlo simulation giving RMSE values of measurements and estimates for hundred samples along the horizontal axis and the angle of pendulum is along the vertical axis.

6. Conclusion

In the pendulum model, there are many non-linearities, and therefore, Particle filter is used for the optimal estimation of the pendulum position. The average root mean square error of measurement values is 0.526065 and the average of estimated root mean square error 0.078238. The RMS of measured values is greater than that of the RMS of estimated values. This implies that the Particle filter gives an optimal estimation of the system when compared to the previous non-linear filters.

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