International Journal of Basic and Applied Sciences, 1 (4) (2012) 534-556

www.sciencepubco.com/index.php/IJBAS

Bayesian Estimation of Two-Component Mixture of Gumbel Type II Distribution under Informative Priors

Navid Feroze, and Muhammad Aslam

Department of Mathematics and Statistics, Allama Iqbal Open University, Islamabad, Pakistan
Email: navidferoz@hotmail.com
Department of Statistics, Quaid-i-Azam University, Islamabad, Pakistan
Email: aslamsdqu@yahoo.com

Abstract

In this paper, the Bayesian estimation of the parameters of mixture of two components of Gumbel type II distribution has been considered. A heterogeneous population has been modeled by means of two components mixture of the Gumbel type II distribution under type I censored data. The Bayes estimators of the said parameter have been derived under the assumption of informative priors using different loss functions. A censored mixture data is simulated by probabilistic mixing for the computational purpose. The comparisons among the estimators have been made in terms of corresponding risks.

Keywords: *loss functions, posterior distribution, Bayes risk, censoring*

1 Introduction

The Gumbel type II distribution is used to model the extreme events like extreme earthquake, temperature, floods etc. In hydrology, the Gumbel distribution is used to analyze the variables such as monthly, quarterly and annual maximum values of daily rainfall, wind speeds and river discharge volumes. Gumbel [1] has shown that the maximum value (or first order statistics) in a sample of a random variable following an exponential distribution approaches the Gumbel distribution with increasing sample size. The Gumbel distribution is a special case of generalized extreme value distribution (also identified as the Fisher-Tippett distribution) and the distribution is also known as the log-Weibull distribution and the double exponential distribution (also considered as the Laplace distribution). Kotz and Nadarajah [2] have given some applications of this distribution. Koutsoyiannis and Baloutsos [3] analyzed the annual series of maximum daily rainfall extending through 1860-1995 in Athens using extreme value distribution. The statistical analysis showed that the conventionally employed Extreme Value Type I (EV1 or Gumbel) distribution is inappropriate for the examined record (especially in its upper tail), whereas this distribution would seem as an appropriate model if fewer years of measurements were available. Corsini et al. [4] discussed the maximum likelihood (ML) algorithms and Cramer-Rao (CR) bounds for the location and scale parameters of the Gumbel distribution. Firstly the scale parameter was assumed to be known and the estimator of the location parameter by solving the likelihood equation was obtained and its performance was evaluated. Koutsoyiannis [5] described that the Gumbel distribution has been the prevailing model for quantifying risk associated with extreme rainfall. Several arguments including theoretical reasons and empirical evidence were supposed to support the appropriateness of the Gumbel distribution. Gettinby et al. [6] worked on suitability of the distribution of extreme returns for a UK share index over the years 1975 to 2000. The Gumbel, Frechet, Weibull, Generalised Extreme Value, Generalised Pareto, Log-Normal and Generalised Logistic distributions were considered.

The mixture models have established great interest for the analysts in the recent era. These models include finite and infinite number of components that can analyze different datasets. A finite mixture of probability distribution is suitable to study a population categorized in number of subpopulations. A population of lifetimes of certain electrical elements can be classified into number of subpopulations based on causes of failures. The analysis of mixture models under Bayesian framework has developed a significant interest among the statisticians. Brunner and Lo [7] obtained the limiting posterior distributions based on mixture of priors. Grodzenskii and Domrachev [8] proposed a method for estimating the parameters of a mixture of exponential and Weibull distributions using censored samples. Preliminary estimates, obtained by graphical analysis, were refined by the method of maximum likelihood. The efficiency of the method was confirmed by the results of a statistical modeling. Deniz et al. [9] used the mixture model as prior distribution for Bayesian sensitivity analysis for the credibility theory related to the net premium principle. Saleem and Aslam[10] considered the estimation of parameters of two components mixture of Rayleigh distribution under a Bayesian framework. Different informative and non-informative priors were assumed for estimation. Razali and Salih [11] concentrated on the estimation of the mixing parameter and the parameters of the mixture Weibull distribution using maximum likelihood estimation. In addition, the probability density function, cumulative distribution function, reliability function and failure rate of the mixture Weibull distribution were derived. Saleem et al. [12] dealt with posterior analysis for two-component mixture of power function distribution. Bayes estimators and posterior risks have been derived under uniform, Jeffreys and inverse chi square priors using squared error loss function. Erisoglu et al. [13] proposed a mixture of two different distributions such as Exponential-Gamma, Exponential-Weibull and Gamma-Weibull to model heterogeneous survival data. Various properties of the proposed mixture of two different distributions were discussed. Maximum likelihood estimations of the parameters were obtained by using the EM algorithm. A real life example has also been presented. Majeed and Aslam [14] discussed the Bayesian analysis for mixture of two-component inverted exponential distribution under quadratic loss function. The applicability of the results was discussed under a simulation study. Kazmi et al. [15] derived the expressions for estimators and associated risks for two-component mixture of Maxwel distribution. The precautionary and squared error loss functions were assumed for posterior analysis. Feroze and Aslam [16] estimated the parameter of Gumbel type II distribution under doubly censored data.

2 The Population and the Model

A density function for mixture of two components densities with mixing weights (p,q) is:

$$f(x) = pf_1(x) + qf_2(x) \qquad 0 < p < 1 \tag{1}$$

The following Gumbel type II distribution is considered for both mixture densities:

$$f(x_i;\alpha_i,\beta_i) = \alpha_i\beta_i x_i^{-(\alpha_i+1)} e^{-\beta_i x_i^{-\alpha_i}} \qquad x_i > 0,\ \alpha_i > 0,\ \beta_i > 0 \tag{2}$$

With the cumulative distribution function as:

$$F(x_i;\alpha_i,\beta_i) = 1 - e^{-\beta_i x_i^{-\alpha_i}} \tag{3}$$

The cumulative distribution function for the mixture model is:

$$F(x) = pF_1(x) + qF_2(x) \tag{4}$$

The likelihood function for type I censored data can be obtained as:

$$L(\beta_1,\beta_2,p|\underline{x}) = \prod_{j=1}^{r_1}\left\{pf_1(x_{1j})\right\}\prod_{j=1}^{r_2}\left\{pf_2(x_{2j})\right\}\left[1-F(t)\right]^{n-r} \tag{5}$$

$$L\left(\beta_1,\beta_2,p\mid\underline{x}\right)\propto\beta_1^{r_1}\beta_2^{r_2}p^{r_1}q^{r_2}e^{-\beta_1\sum_{j=1}^{r_1}x_{1j}-\beta_2\sum_{j=1}^{r_2}x_{2j}}\left[pe^{-\beta_1 t^{-\alpha_1}}+qe^{-\beta_2 t^{-\alpha_2}}\right]^{n-r} \tag{6}$$

Expanding the last term by binomial expansion the likelihood function becomes:

$$L\left(\beta_1,\beta_2,p\mid\underline{x}\right)\propto\sum_{k=0}^{n-r}\binom{n-r}{k}\beta_1^{r_1}\beta_2^{r_2}p^{n-k-r_2}q^{r_2+k}e^{-\beta_1 A_{1k}}e^{-\beta_2 A_{2k}} \tag{7}$$

Where $A_{1k}=\sum_{j=1}^{r_1}x_{1j}+\left(n-r-k\right)t^{-\alpha_1}$ and $A_{2k}=\sum_{j=1}^{r_2}x_{2j}+kt^{-\alpha_2}$

3 The Posterior Distributions under Different Informative Priors

In this section, the posterior distributions have been derived under gamma, chi square and exponential priors. The combined priors have been obtained by assuming the independence.

Let $\beta_1:$ Gamma$\left(a_1,b_1\right)$, $\beta_2:$ Gamma$\left(a_2,b_2\right)$ and $p:$ Uniform$\left(0,1\right)$. Under the assumption of independence, the joint prior becomes:

$$h\left(a_1,b_1,p\right)\propto\beta_1^{a_1-1}\beta_2^{a_2-1}e^{-\left(\beta_1 b_1+\beta_2 b_2\right)},\ \beta_i>0 \tag{8}$$

The posterior distribution under the assumption of above prior is:

$$p\left(\beta_1,\beta_2,p\mid\underline{x}\right)=\frac{1}{C_1}\sum_{k=0}^{n-r}\binom{n-r}{k}\beta_1^{r_1+a_1-1}\beta_2^{r_2+a_2-1}p^{n-k-r_2}q^{r_2+k}e^{-\beta_1\left(A_{1k}+b_1\right)}e^{-\beta_2\left(A_{2k}+b_2\right)},\ \beta_i>0 \tag{9}$$

Where $C_1=\sum_{k=0}^{n-r}\binom{n-r}{k}B\left(\theta_{1k},\theta_{2k}\right)\frac{\Gamma\left(r_1+a_1\right)}{\left(A_{1k}+b_1\right)^{r_1+a_1}}\frac{\Gamma\left(r_2+a_2\right)}{\left(A_{2k}+b_2\right)^{r_2+a_2}}$

Again suppose $\beta_1:$ Chi Square$\left(w_1\right)$, $\beta_2:$ Chi Square$\left(w_2\right)$ and $p:$ Uniform$\left(0,1\right)$. Under the assumption of independence, the joint prior becomes:

$$h\left(a_1,b_1,p\right)\propto\beta_1^{\frac{w_1}{2}-1}\beta_2^{\frac{w_2}{2}-1}e^{-\frac{\left(\beta_1+\beta_2\right)}{2}},\ \beta_i>0 \tag{10}$$

The posterior distribution under the assumption of above prior is:

$$p\left(\beta_1,\beta_2,p\left|\underline{x}\right.\right)=\frac{1}{C_2}\sum_{k=0}^{n-r}\binom{n-r}{k}\beta_1^{r_1+w_1/2-1}\beta_2^{r_2+w_1/2-1}p^{n-k-r_2}q^{r_2+k}e^{-\beta_1\left(A_{1k}+1/2\right)}e^{-\beta_2\left(A_{2k}+1/2\right)}$$

$$\beta_i>0 \tag{11}$$

Where $C_2=\sum_{k=0}^{n-r}\binom{n-r}{k}B\left(\theta_{1k},\theta_{2k}\right)\frac{\Gamma\left(r_1+w_1/2\right)}{\left(A_{1k}+1/2\right)^{r_1+w_1/2}}\frac{\Gamma\left(r_2+w_2/2\right)}{\left(A_{2k}+1/2\right)^{r_2+w_2/2}}$

The posterior distribution under the assumption of exponential prior Further consider β_1 : Exponential$\left(d_1\right)$, β_2 : Exponential$\left(d_2\right)$ and p : Uniform$\left(0,1\right)$. Under the assumption of independence, the joint prior becomes:

$$h\left(a_1,b_1,p\right)\propto e^{-\left(\beta_1d_1+\beta_2d_2\right)},\ \beta_i>0 \tag{12}$$

The posterior distribution under the assumption of above prior is:

$$p\left(\beta_1,\beta_2,p\left|\underline{x}\right.\right)=\frac{1}{C_3}\sum_{k=0}^{n-r}\binom{n-r}{k}\beta_1^{r_1}\beta_2^{r_2}p^{n-k-r_2}q^{r_2+k}e^{-\beta_1\left(A_{1k}+d_1\right)}e^{-\beta_2\left(A_{2k}+d_2\right)},\ \beta_i>0 \tag{13}$$

$$C_3=\sum_{k=0}^{n-r}\binom{n-r}{k}B\left(\theta_{1k},\theta_{2k}\right)\frac{\Gamma\left(r_1+1\right)}{\left(A_{1k}+d_1\right)^{r_1+1}}\frac{\Gamma\left(r_2+1\right)}{\left(A_{2k}+d_2\right)^{r_2+1}}$$

4 Bayes Estimators and Corresponding Risks

The Bayes estimators and posterior risks using squared error loss function (SELF), Quadratic loss function (QLF), weighted loss function (WLF) and precautionary loss function (PLF) are presented in the following. Bayes estimator and risk for β1 under SELF using gamma prior are:

$$\beta_{1,SELF}=\frac{1}{C_1}\sum_{k=0}^{n-r}\binom{n-r}{k}B\left(\theta_{1k},\theta_{2k}\right)\frac{\Gamma\left(r_1+a_1+1\right)}{\left(A_{1k}+b_1\right)^{r_1+a_1+1}}\frac{\Gamma\left(r_2+a_2\right)}{\left(A_{2k}+b_2\right)^{r_2+a_2}}$$

$$V\left(\beta_{1,SELF}\right)=\frac{1}{C_1}\sum_{k=0}^{n-r}\binom{n-r}{k}B\left(\theta_{1k},\theta_{2k}\right)\frac{\Gamma\left(r_1+a_1+2\right)}{\left(A_{1k}+b_1\right)^{r_1+a_1+2}}\frac{\Gamma\left(r_2+a_2\right)}{\left(A_{2k}+b_2\right)^{r_2+a_2}}-\left[\frac{1}{C_1}\sum_{k=0}^{n-r}\binom{n-r}{k}B\left(\theta_{1k},\theta_{2k}\right)\frac{\Gamma\left(r_1+a_1+1\right)}{\left(A_{1k}+b_1\right)^{r_1+a_1+1}}\frac{\Gamma\left(r_2+a_2\right)}{\left(A_{2k}+b_2\right)^{r_2+a_2}}\right]^2$$

Bayes estimator and risk for β_2 under SELF using gamma prior are:

$$\beta_{2,SELF}=\frac{1}{C_1}\sum_{k=0}^{n-r}\binom{n-r}{k}B\left(\theta_{1k},\theta_{2k}\right)\frac{\Gamma\left(r_1+a_1\right)}{\left(A_{1k}+b_1\right)^{r_1+a_1}}\frac{\Gamma\left(r_2+a_2+1\right)}{\left(A_{2k}+b_2\right)^{r_2+a_2+1}}$$

$$V\left(\beta_{2,SELF}\right)=\frac{1}{C_1}\sum_{k=0}^{n-r}\binom{n-r}{k}B\left(\theta_{1k},\theta_{2k}\right)\frac{\Gamma\left(r_1+a_1\right)}{\left(A_{1k}+b_1\right)^{r_1+a_1}}\frac{\Gamma\left(r_2+a_2+2\right)}{\left(A_{2k}+b_2\right)^{r_2+a_2+2}}-\left[\frac{1}{C_1}\sum_{k=0}^{n-r}\binom{n-r}{k}B\left(\theta_{1k},\theta_{2k}\right)\frac{\Gamma\left(r_1+a_1\right)}{\left(A_{1k}+b_1\right)^{r_1+a_1}}\frac{\Gamma\left(r_2+a_2+1\right)}{\left(A_{2k}+b_2\right)^{r_2+a_2+1}}\right]^2$$

Bayes estimator and risk for ‘p’ under SELF using gamma prior are:

$$p_{SELF}=\frac{1}{C_1}\sum_{k=0}^{n-r}\binom{n-r}{k}B\left(\theta_{1k}+1,\theta_{2k}\right)\frac{\Gamma\left(r_1+a_1\right)}{\left(A_{1k}+b_1\right)^{r_1+a_1}}\frac{\Gamma\left(r_2+a_2\right)}{\left(A_{2k}+b_2\right)^{r_2+a_2}}$$

$$V\left(p_{SELF}\right)=\frac{1}{C_1}\sum_{k=0}^{n-r}\binom{n-r}{k}B\left(\theta_{1k}+2,\theta_{2k}\right)\frac{\Gamma\left(r_1+a_1\right)}{\left(A_{1k}+b_1\right)^{r_1+a_1}}\frac{\Gamma\left(r_2+a_2\right)}{\left(A_{2k}+b_2\right)^{r_2+a_2}}-\left[\frac{1}{C_1}\sum_{k=0}^{n-r}\binom{n-r}{k}B\left(\theta_{1k}+1,\theta_{2k}\right)\frac{\Gamma\left(r_1+a_1\right)}{\left(A_{1k}+b_1\right)^{r_1+a_1}}\frac{\Gamma\left(r_2+a_2\right)}{\left(A_{2k}+b_2\right)^{r_2+a_2}}\right]^2$$

Bayes estimator and risk for β_1 under QLF using gamma prior are:

$$\beta_{1,QLF}=\frac{\sum_{k=0}^{n-r}\binom{n-r}{k}B\left(\theta_{1k},\theta_{2k}\right)\frac{\Gamma\left(r_1+a_1-1\right)}{\left(A_{1k}+b_1\right)^{r_1+a_1-1}}\frac{\Gamma\left(r_2+a_2\right)}{\left(A_{2k}+b_2\right)^{r_2+a_2}}}{\sum_{k=0}^{n-r}\binom{n-r}{k}B\left(\theta_{1k},\theta_{2k}\right)\frac{\Gamma\left(r_1+a_1-2\right)}{\left(A_{1k}+b_1\right)^{r_1+a_1-2}}\frac{\Gamma\left(r_2+a_2\right)}{\left(A_{2k}+b_2\right)^{r_2+a_2}}}$$

$$V\left(\beta_{1,QLF}\right)=1-\frac{\frac{1}{C_1}\left[\sum_{k=0}^{n-r}\binom{n-r}{k}B\left(\theta_{1k},\theta_{2k}\right)\frac{\Gamma\left(r_1+a_1-1\right)}{\left(A_{1k}+b_1\right)^{r_1+a_1-1}}\frac{\Gamma\left(r_2+a_2\right)}{\left(A_{2k}+b_2\right)^{r_2+a_2}}\right]^2}{\sum_{k=0}^{n-r}\binom{n-r}{k}B\left(\theta_{1k},\theta_{2k}\right)\frac{\Gamma\left(r_1+a_1-2\right)}{\left(A_{1k}+b_1\right)^{r_1+a_1-2}}\frac{\Gamma\left(r_2+a_2\right)}{\left(A_{2k}+b_2\right)^{r_2+a_2}}}$$

Bayes estimator and risk for β_2 under QLF using gamma prior are:

$$\beta_{2,QLF}=\frac{\sum_{k=0}^{n-r}\binom{n-r}{k}B\left(\theta_{1k},\theta_{2k}\right)\frac{\Gamma\left(r_1+a_1\right)}{\left(A_{1k}+b_1\right)^{r_1+a_1}}\frac{\Gamma\left(r_2+a_2-1\right)}{\left(A_{2k}+b_2\right)^{r_2+a_2-1}}}{\sum_{k=0}^{n-r}\binom{n-r}{k}B\left(\theta_{1k},\theta_{2k}\right)\frac{\Gamma\left(r_1+a_1\right)}{\left(A_{1k}+b_1\right)^{r_1+a_1}}\frac{\Gamma\left(r_2+a_2-2\right)}{\left(A_{2k}+b_2\right)^{r_2+a_2-2}}}$$

$$V\left(\beta_{2,QLF}\right)=1-\frac{\frac{1}{C_1}\left[\sum_{k=0}^{n-r}\binom{n-r}{k}B\left(\theta_{1k},\theta_{2k}\right)\frac{\Gamma\left(r_1+a_1\right)}{\left(A_{1k}+b_1\right)^{r_1+a_1}}\frac{\Gamma\left(r_2+a_2-1\right)}{\left(A_{2k}+b_2\right)^{r_2+a_2-1}}\right]^2}{\sum_{k=0}^{n-r}\binom{n-r}{k}B\left(\theta_{1k},\theta_{2k}\right)\frac{\Gamma\left(r_1+a_1\right)}{\left(A_{1k}+b_1\right)^{r_1+a_1}}\frac{\Gamma\left(r_2+a_2-2\right)}{\left(A_{2k}+b_2\right)^{r_2+a_2-2}}}$$

Bayes estimator and risk for ‘p’ under QLF using gamma prior are:

$$p_{QLF}=\frac{\sum_{k=0}^{n-r}\binom{n-r}{k}B\left(\theta_{1k}-1,\theta_{2k}\right)\frac{\Gamma\left(r_1+a_1\right)}{\left(A_{1k}+b_1\right)^{r_1+a_1}}\frac{\Gamma\left(r_2+a_2\right)}{\left(A_{2k}+b_2\right)^{r_2+a_2}}}{\sum_{k=0}^{n-r}\binom{n-r}{k}B\left(\theta_{1k}-2,\theta_{2k}\right)\frac{\Gamma\left(r_1+a_1\right)}{\left(A_{1k}+b_1\right)^{r_1+a_1}}\frac{\Gamma\left(r_2+a_2\right)}{\left(A_{2k}+b_2\right)^{r_2+a_2}}}$$

$$V\left(p_{QLF}\right)=1-\frac{\frac{1}{C_1}\left[\sum_{k=0}^{n-r}\binom{n-r}{k}B\left(\theta_{1k}-1,\theta_{2k}\right)\frac{\Gamma\left(r_1+a_1\right)}{\left(A_{1k}+b_1\right)^{r_1+a_1}}\frac{\Gamma\left(r_2+a_2\right)}{\left(A_{2k}+b_2\right)^{r_2+a_2}}\right]^2}{\sum_{k=0}^{n-r}\binom{n-r}{k}B\left(\theta_{1k}-2,\theta_{2k}\right)\frac{\Gamma\left(r_1+a_1\right)}{\left(A_{1k}+b_1\right)^{r_1+a_1}}\frac{\Gamma\left(r_2+a_2\right)}{\left(A_{2k}+b_2\right)^{r_2+a_2}}}$$

Bayes estimator and risk for β_1 under WLF using gamma prior are:

$$\beta_{1,WLF}=\left[\frac{1}{C_1}\sum_{k=0}^{n-r}\binom{n-r}{k}B\left(\theta_{1k},\theta_{2k}\right)\frac{\Gamma\left(r_1+a_1-1\right)}{\left(A_{1k}+b_1\right)^{r_1+a_1-1}}\frac{\Gamma\left(r_2+a_2\right)}{\left(A_{2k}+b_2\right)^{r_2+a_2}}\right]^{-1}$$

$$V\left(\beta_{1,WLF}\right)=\frac{1}{C_1}\sum_{k=0}^{n-r}\binom{n-r}{k}B\left(\theta_{1k},\theta_{2k}\right)\frac{\Gamma\left(r_1+a_1+1\right)}{\left(A_{1k}+b_1\right)^{r_1+a_1+1}}\frac{\Gamma\left(r_2+a_2\right)}{\left(A_{2k}+b_2\right)^{r_2+a_2}}-\left[\frac{1}{C_1}\sum_{k=0}^{n-r}\binom{n-r}{k}B\left(\theta_{1k},\theta_{2k}\right)\frac{\Gamma\left(r+a_1-1\right)}{\left(A_{1k}+b_1\right)^{r_1+a_1-1}}\frac{\Gamma\left(r_2+a_2\right)}{\left(A_{2k}+b_2\right)^{r_2+a_2}}\right]^{-1}$$

Bayes estimator and risk for β_2 under WLF using gamma prior are:

$$\beta_{2,WLF}=\left[\frac{1}{C_1}\sum_{k=0}^{n-r}\binom{n-r}{k}B\left(\theta_{1k},\theta_{2k}\right)\frac{\Gamma\left(r_1+a_1\right)}{\left(A_{1k}+b_1\right)^{r_1+a_1}}\frac{\Gamma\left(r_2+a_2-1\right)}{\left(A_{2k}+b_2\right)^{r_2+a_2-1}}\right]^{-1}$$

$$V\left(\beta_{2,WLF}\right)=\frac{1}{C_1}\sum_{k=0}^{n-r}\binom{n-r}{k}B\left(\theta_{1k},\theta_{2k}\right)\frac{\Gamma\left(r_1+a_1\right)}{\left(A_{1k}+b_1\right)^{r_1+a_1}}\frac{\Gamma\left(r_2+a_2+1\right)}{\left(A_{2k}+b_2\right)^{r_2+a_2+1}}-\left[\frac{1}{C_1}\sum_{k=0}^{n-r}\binom{n-r}{k}B\left(\theta_{1k},\theta_{2k}\right)\frac{\Gamma\left(r_1+a_1\right)}{\left(A_{1k}+b_1\right)^{r_1+a_1}}\frac{\Gamma\left(r_2+a_2-1\right)}{\left(A_{2k}+b_2\right)^{r_2+a_2-1}}\right]^{-1}$$

Bayes estimator and risk for ‘p’ under WLF using gamma prior are:

$$p_{WLF}=\left[\frac{1}{C_1}\sum_{k=0}^{n-r}\binom{n-r}{k}B\left(\theta_{1k}-1,\theta_{2k}\right)\frac{\Gamma\left(r_1+a_1\right)}{\left(A_{1k}+b_1\right)^{r_1+a_1}}\frac{\Gamma\left(r_2+a_2\right)}{\left(A_{2k}+b_2\right)^{r_2+a_2}}\right]^{-1}$$

$$V\left(p_{WLF}\right)=\frac{1}{C_1}\sum_{k=0}^{n-r}\binom{n-r}{k}B\left(\theta_{1k}+1,\theta_{2k}\right)\frac{\Gamma\left(r_1+a_1\right)}{\left(A_{1k}+b_1\right)^{r_1+a_1}}\frac{\Gamma\left(r_2+a_2\right)}{\left(A_{2k}+b_2\right)^{r_2+a_2}}-\left[\frac{1}{C_1}\sum_{k=0}^{n-r}\binom{n-r}{k}B\left(\theta_{1k}-1,\theta_{2k}\right)\frac{\Gamma\left(r_1+a_1\right)}{\left(A_{1k}+b_1\right)^{r_1+a_1}}\frac{\Gamma\left(r_2+a_2\right)}{\left(A_{2k}+b_2\right)^{r_2+a_2}}\right]^{-1}$$

Bayes estimator and risk for β_1 under PLF using gamma prior are:

$$\beta_{1,PLF}=\left[\frac{1}{C_1}\sum_{k=0}^{n-r}\binom{n-r}{k}B\left(\theta_{1k},\theta_{2k}\right)\frac{\Gamma\left(r_1+a_1+2\right)}{\left(A_{1k}+b_1\right)^{r_1+a_1+2}}\frac{\Gamma\left(r_2+a_2\right)}{\left(A_{2k}+b_2\right)^{r_2+a_2}}\right]^{\frac{1}{2}}$$

$$V\left(\beta_{1,PLF}\right)=2\left[\left[\frac{1}{C_1}\sum_{k=0}^{n-r}\binom{n-r}{k}B\left(\theta_{1k},\theta_{2k}\right)\frac{\Gamma\left(r_1+a_1+2\right)}{\left(A_{1k}+b_1\right)^{r_1+a_1+2}}\frac{\Gamma\left(r_2+a_2\right)}{\left(A_{2k}+b_2\right)^{r_2+a_2}}\right]^{\frac{1}{2}}-\frac{1}{C_1}\sum_{k=0}^{n-r}\binom{n-r}{k}B\left(\theta_{1k},\theta_{2k}\right)\frac{\Gamma\left(r_1+a_1+1\right)}{\left(A_{1k}+b_1\right)^{r_1+a_1+1}}\frac{\Gamma\left(r_2+a_2\right)}{\left(A_{2k}+b_2\right)^{r_2+a_2}}\right]$$

Bayes estimator and risk for β_2 under WLF using gamma prior are:

$$\beta_{2,PLF}=\left[\frac{1}{C_1}\sum_{k=0}^{n-r}\binom{n-r}{k}B\left(\theta_{1k},\theta_{2k}\right)\frac{\Gamma\left(r_1+a_1\right)}{\left(A_{1k}+b_1\right)^{r_1+a_1}}\frac{\Gamma\left(r_2+a_2+2\right)}{\left(A_{2k}+b_2\right)^{r_2+a_2+2}}\right]^{\frac{1}{2}}$$

$$V\left(\beta_{2,PLF}\right)=2\left[\left[\frac{1}{C_1}\sum_{k=0}^{n-r}\binom{n-r}{k}B\left(\theta_{1k},\theta_{2k}\right)\frac{\Gamma\left(r_1+a_1\right)}{\left(A_{1k}+b_1\right)^{r_1+a_1}}\frac{\Gamma\left(r_2+2\right)}{\left(A_{2k}\right)^{r_2+2}}\right]^{\frac{1}{2}}-\frac{1}{C_1}\sum_{k=0}^{n-r}\binom{n-r}{k}B\left(\theta_{1k},\theta_{2k}\right)\frac{\Gamma\left(r_1+a_1\right)}{\left(A_{1k}+b_1\right)^{r_1+a_1}}\frac{\Gamma\left(r_2+a_2+1\right)}{\left(A_{2k}+b_2\right)^{r_2+a_2+1}}\right]$$

Bayes estimator and risk for ‘p’ under WLF using gamma prior are:

$$p_{PLF}=\left[\frac{1}{C_1}\sum_{k=0}^{n-r}\binom{n-r}{k}B\left(\theta_{1k}+2,\theta_{2k}\right)\frac{\Gamma\left(r_1+a_1\right)}{\left(A_{1k}+b_1\right)^{r_1+a_1}}\frac{\Gamma\left(r_2+a_2\right)}{\left(A_{2k}+b_2\right)^{r_2+a_2}}\right]^{\frac{1}{2}}$$

$$V\left(p_{PLF}\right)=2\left[\left[\frac{1}{C_1}\sum_{k=0}^{n-r}\binom{n-r}{k}B\left(\theta_{1k}+2,\theta_{2k}\right)\frac{\Gamma\left(r_1+a_1\right)}{\left(A_{1k}+b_1\right)^{r_1+a_1}}\frac{\Gamma\left(r_2+a_2\right)}{\left(A_{2k}+b_2\right)^{r_2+a_2}}\right]^{\frac{1}{2}}-\frac{1}{C_1}\sum_{k=0}^{n-r}\binom{n-r}{k}B\left(\theta_{1k}+1,\theta_{2k}\right)\frac{\Gamma\left(r_1+a_1\right)}{\left(A_{1k}+b_1\right)^{r_1+a_1}}\frac{\Gamma\left(r_2+a_2\right)}{\left(A_{2k}+b_2\right)^{r_2+a_2}}\right]$$

The expressions for the Bayes estimators and corresponding risks can be derived accordingly.

5 Simulation Study

The results of simulation study have been presented in the following. The parametric space used is: $\left(\beta_1,\beta_2,p\right)\in\left\{\left(4,5,0.35\right),\left(6,8,0.45\right)\right\}$. The purpose of the study is to draw comparisons among the performance of different estimates numerically as it is not possible to compare the expressions derived for the estimates directly. The risks have been given in bold text in the tables. The comparisons of the estimates have been made in terms of posterior risks.

The probabilistic mixing has been used to generate the mixture data. For each observation a random number u has been generated from $U(0,1)$. If $u < p$ the observation has been randomly taken from first subpopulation and if $u > p$ then the observation have been taken from the second subpopulation. The simulation results have been obtained by considering the observations above a fixed censoring time T to be right censored. Under each combination of parametric values, the choice of censoring time has been made so that the censoring rate in the respective sample has been 15%. As one sample cannot completely describe the behavior and properties of the Bayes estimators, the results have been replicated 1000 times and the average of results has been presented in the tables below.

The first property of the estimates, which can be observed, is the convergence of estimates towards the true parametric value by increasing the sample size. The magnitudes of risks tend to decrease by increasing the sample size. All the parameters have been over estimated in each case. This suggests that the corresponding posterior distributions are skewed. These patterns are similar for each prior and under every loss function. It can also be assessed the larger values of the weight parameter ‘p’ imposes a positive impact on the performance of estimates representing the first sub-population and vice versa. While, the greater true parametric values result is lesser efficient estimates irrespective of choice of prior and loss function.

In comparison of priors it can be indicated that the performance of the estimates under chi square prior is the best as the magnitudes of corresponding risks are comparatively lesser. This property holds for each loss function. Whereas, the choice of quadratic loss function leads to the best results under each prior and for all parametric values. It is interesting to note that the amount of risks associated with estimates under precautionary and weighted loss functions are fairly close especially in large samples.

6 Conclusion

The study has been conducted to estimate the parameters of two-component mixture of Gumbel type II distribution under informative priors using different loss functions. The purpose of the study is the find out an appropriate combination of the prior and loss function to estimate the parameters of the said mixture density. The findings of the study suggest that in order to estimate the mentioned parameters, the use of quadratic loss function under chi square prior can be preferred. Some prior elicitation technique may further strengthen this argument.

Table 1: Bayes estimates and risks under gamma prior

n	$\beta_1 = 4$			
	SELF	QLF	WLF	PLF
50	4.55687 **0.20584**	4.46573 **0.01040**	4.51084 **0.04603**	4.58029 **0.04686**
100	4.52295 **0.09985**	4.47772 **0.00520**	4.50022 **0.02273**	4.53441 **0.02293**
200	4.43625 **0.04767**	4.41406 **0.00260**	4.42513 **0.01112**	4.44183 **0.01117**
300	4.31208 **0.02995**	4.29770 **0.00173**	4.30488 **0.00720**	4.31569 **0.00722**
400	4.21124 **0.02140**	4.20071 **0.00130**	4.20597 **0.00527**	4.21388 **0.00528**
500	4.18306 **0.01688**	4.17469 **0.00104**	4.17887 **0.00419**	4.18515 **0.00419**
n	$\beta_1 = 5$			
	SELF	QLF	WLF	PLF
50	5.69608 **0.25730**	5.58216 **0.01300**	5.63855 **0.05754**	5.72537 **0.05857**
100	5.65368 **0.12481**	5.59715 **0.00650**	5.62527 **0.02841**	5.66801 **0.02866**
200	5.54531 **0.05958**	5.51758 **0.00325**	5.53141 **0.01390**	5.55229 **0.01396**
300	5.39010 **0.03743**	5.37213 **0.00217**	5.38110 **0.00900**	5.39461 **0.00902**
400	5.26405 **0.02674**	5.25089 **0.00163**	5.25746 **0.00659**	5.26735 **0.00660**
500	5.22882 **0.02109**	5.21836 **0.00130**	5.22358 **0.00523**	5.23144 **0.00524**
n	$p = 0.35$			
	SELF	QLF	WLF	PLF
50	0.39434 **0.01781**	0.38646 **0.00090**	0.39036 **0.00398**	0.39637 **0.00405**
100	0.39141 **0.00864**	0.38749 **0.00045**	0.38944 **0.00197**	0.39240 **0.00198**
200	0.38391 **0.00412**	0.38199 **0.00023**	0.38294 **0.00096**	0.38439 **0.00097**
300	0.37316 **0.00259**	0.37192 **0.00015**	0.37254 **0.00062**	0.37347 **0.00062**
400	0.36443 **0.00185**	0.36352 **0.00011**	0.36398 **0.00046**	0.36466 **0.00046**
500	0.36200 **0.00146**	0.36127 **0.00009**	0.36163 **0.00036**	0.36218 **0.00036**

Table 2: Bayes estimates and risks under chi square prior

n	$\beta_1 = 4$			
	SELF	QLF	WLF	PLF
50	4.51084 **0.19964**	4.46573 **0.01030**	4.46573 **0.04511**	4.53379 **0.04591**
100	4.50022 **0.09835**	4.47772 **0.00517**	4.47772 **0.02250**	4.51157 **0.02270**
200	4.42513 **0.04731**	4.41406 **0.00259**	4.41406 **0.01106**	4.43068 **0.01111**
300	4.30488 **0.02980**	4.29770 **0.00173**	4.29770 **0.00717**	4.30848 **0.00720**
400	4.20597 **0.02132**	4.20071 **0.00130**	4.20071 **0.00526**	4.20860 **0.00527**
500	4.17887 **0.01682**	4.17469 **0.00104**	4.17469 **0.00418**	4.18096 **0.00419**
n	$\beta_1 = 5$			
	SELF	QLF	WLF	PLF
50	5.63855 **0.24955**	5.58216 **0.01287**	5.58216 **0.05639**	5.66724 **0.05739**
100	5.62527 **0.12294**	5.59715 **0.00647**	5.59715 **0.02813**	5.63946 **0.02837**
200	5.53141 **0.05914**	5.51758 **0.00324**	5.51758 **0.01383**	5.53835 **0.01389**
300	5.38110 **0.03725**	5.37213 **0.00216**	5.37213 **0.00897**	5.38560 **0.00899**
400	5.25746 **0.02664**	5.25089 **0.00162**	5.25089 **0.00657**	5.26076 **0.00659**
500	5.22358 **0.02103**	5.21836 **0.00130**	5.21836 **0.00522**	5.22620 **0.00523**
n	$p = 0.35$			
	SELF	QLF	WLF	PLF
50	0.39036 **0.01728**	0.38646 **0.00089**	0.38646 **0.00390**	0.39235 **0.00397**
100	0.38944 **0.00851**	0.38749 **0.00045**	0.38749 **0.00195**	0.39042 **0.00196**
200	0.38294 **0.00409**	0.38199 **0.00022**	0.38199 **0.00096**	0.38342 **0.00096**
300	0.37254 **0.00258**	0.37192 **0.00015**	0.37192 **0.00062**	0.37285 **0.00062**
400	0.36398 **0.00184**	0.36352 **0.00011**	0.36352 **0.00045**	0.36421 **0.00046**
500	0.36163 **0.00146**	0.36127 **0.00009**	0.36127 **0.00036**	0.36181 **0.00036**

Table 3: Bayes estimates and risks under exponential prior

n	$\beta_1 = 4$			
	SELF	QLF	WLF	PLF
50	4.49114 **0.20188**	4.40132 **0.01020**	4.44578 **0.04514**	4.51423 **0.04596**
100	4.45771 **0.09793**	4.41313 **0.00510**	4.43531 **0.02229**	4.46901 **0.02249**
200	4.37226 **0.04675**	4.35040 **0.00255**	4.36130 **0.01090**	4.37776 **0.01095**
300	4.24988 **0.02937**	4.23572 **0.00170**	4.24279 **0.00706**	4.25344 **0.00708**
400	4.15050 **0.02098**	4.14013 **0.00128**	4.14531 **0.00517**	4.15310 **0.00518**
500	4.12272 **0.01655**	4.11448 **0.00102**	4.11860 **0.00411**	4.12479 **0.00411**
n	$\beta_1 = 5$			
	SELF	QLF	WLF	PLF
50	5.47700 **0.25136**	5.36746 **0.01270**	5.42168 **0.05621**	5.50516 **0.05722**
100	5.43623 **0.12193**	5.38187 **0.00635**	5.40891 **0.02775**	5.45001 **0.02800**
200	5.33203 **0.05821**	5.30537 **0.00318**	5.31866 **0.01358**	5.33874 **0.01364**
300	5.18279 **0.03657**	5.16551 **0.00212**	5.17413 **0.00879**	5.18713 **0.00882**
400	5.06159 **0.02613**	5.04893 **0.00159**	5.05525 **0.00644**	5.06476 **0.00645**
500	5.02771 **0.02061**	5.01766 **0.00127**	5.02268 **0.00511**	5.03023 **0.00512**
n	$p = 0.35$			
	SELF	QLF	WLF	PLF
50	0.38558 **0.01742**	0.37787 **0.00088**	0.38169 **0.00389**	0.38756 **0.00396**
100	0.38271 **0.00845**	0.37888 **0.00044**	0.38079 **0.00192**	0.38368 **0.00194**
200	0.37537 **0.00403**	0.37350 **0.00022**	0.37443 **0.00094**	0.37585 **0.00094**
300	0.36487 **0.00253**	0.36365 **0.00015**	0.36426 **0.00061**	0.36517 **0.00061**
400	0.35634 **0.00181**	0.35544 **0.00011**	0.35589 **0.00045**	0.35656 **0.00045**
500	0.35395 **0.00143**	0.35324 **0.00009**	0.35360 **0.00035**	0.35413 **0.00035**

Table 4: Bayes estimates and risks under gamma prior

n	$\beta_1 = 4$			
	SELF	QLF	WLF	PLF
50	4.53496 **0.20485**	4.44426 **0.01035**	4.48915 **0.04581**	4.55827 **0.04663**
100	4.50120 **0.09937**	4.45619 **0.00518**	4.47858 **0.02262**	4.51261 **0.02282**
200	4.41492 **0.04744**	4.39284 **0.00259**	4.40385 **0.01106**	4.42047 **0.01111**
300	4.29135 **0.02980**	4.27704 **0.00173**	4.28418 **0.00716**	4.29494 **0.00719**
400	4.19099 **0.02129**	4.18052 **0.00129**	4.18575 **0.00525**	4.19362 **0.00526**
500	4.16294 **0.01679**	4.15462 **0.00104**	4.15878 **0.00417**	4.16503 **0.00417**
n	$\beta_1 = 5$			
	SELF	QLF	WLF	PLF
50	5.78371 **0.26126**	5.66804 **0.01320**	5.72529 **0.05842**	5.81345 **0.05947**
100	5.74066 **0.12673**	5.68326 **0.00660**	5.71181 **0.02885**	5.75521 **0.02910**
200	5.63062 **0.06050**	5.60247 **0.00330**	5.61651 **0.01411**	5.63771 **0.01417**
300	5.47302 **0.03801**	5.45478 **0.00220**	5.46389 **0.00914**	5.47760 **0.00916**
400	5.34504 **0.02716**	5.33167 **0.00165**	5.33835 **0.00669**	5.34839 **0.00670**
500	5.30926 **0.02142**	5.29864 **0.00132**	5.30395 **0.00531**	5.31192 **0.00532**
n	$p = 0.45$			
	SELF	QLF	WLF	PLF
50	0.50388 **0.02276**	0.49381 **0.00115**	0.49879 **0.00509**	0.50647 **0.00518**
100	0.50013 **0.01104**	0.49513 **0.00058**	0.49762 **0.00251**	0.50140 **0.00254**
200	0.49055 **0.00527**	0.48809 **0.00029**	0.48932 **0.00123**	0.49116 **0.00123**
300	0.47682 **0.00331**	0.47523 **0.00019**	0.47602 **0.00080**	0.47722 **0.00080**
400	0.46567 **0.00237**	0.46450 **0.00014**	0.46508 **0.00058**	0.46596 **0.00058**
500	0.46255 **0.00187**	0.46162 **0.00012**	0.46209 **0.00046**	0.46278 **0.00046**

Table 5: Bayes estimates and risks under chi square prior

n	$\beta_1 = 4$			
	SELF	QLF	WLF	PLF
50	4.46746 **0.19772**	4.42279 **0.01020**	4.42279 **0.04467**	4.49020 **0.04547**
100	4.45695 **0.09740**	4.43466 **0.00512**	4.43466 **0.02228**	4.46819 **0.02248**
200	4.38258 **0.04685**	4.37162 **0.00257**	4.37162 **0.01096**	4.38808 **0.01100**
300	4.26349 **0.02951**	4.25638 **0.00171**	4.25638 **0.00711**	4.26705 **0.00713**
400	4.16553 **0.02111**	4.16032 **0.00129**	4.16032 **0.00521**	4.16814 **0.00522**
500	4.13869 **0.01666**	4.13455 **0.00103**	4.13455 **0.00414**	4.14076 **0.00415**
n	$\beta_1 = 5$			
	SELF	QLF	WLF	PLF
50	5.68192 **0.25147**	5.62510 **0.01297**	5.62510 **0.05682**	5.71084 **0.05783**
100	5.66854 **0.12388**	5.64020 **0.00652**	5.64020 **0.02834**	5.68284 **0.02859**
200	5.57396 **0.05959**	5.56002 **0.00327**	5.56002 **0.01393**	5.58096 **0.01400**
300	5.42249 **0.03753**	5.41346 **0.00218**	5.41346 **0.00904**	5.42702 **0.00906**
400	5.29790 **0.02685**	5.29128 **0.00164**	5.29128 **0.00662**	5.30122 **0.00664**
500	5.26377 **0.02119**	5.25850 **0.00131**	5.25850 **0.00526**	5.26640 **0.00527**
n	$p = 0.45$			
	SELF	QLF	WLF	PLF
50	0.49879 **0.02208**	0.49381 **0.00114**	0.49381 **0.00499**	0.50133 **0.00508**
100	0.49762 **0.01088**	0.49513 **0.00057**	0.49513 **0.00249**	0.49888 **0.00251**
200	0.48932 **0.00523**	0.48809 **0.00029**	0.48809 **0.00122**	0.48993 **0.00123**
300	0.47602 **0.00329**	0.47523 **0.00019**	0.47523 **0.00079**	0.47642 **0.00080**
400	0.46508 **0.00236**	0.46450 **0.00014**	0.46450 **0.00058**	0.46537 **0.00058**
500	0.46209 **0.00186**	0.46162 **0.00011**	0.46162 **0.00046**	0.46232 **0.00046**

Table 6: Bayes estimates and risks under exponential prior

n	$\beta_1 = 4$			
	SELF	QLF	WLF	PLF
50	4.45609 **0.19990**	4.36697 **0.01010**	4.41108 **0.04470**	4.47900 **0.04551**
100	4.42292 **0.09697**	4.37869 **0.00505**	4.40069 **0.02207**	4.43413 **0.02227**
200	4.33814 **0.04629**	4.31645 **0.00253**	4.32726 **0.01080**	4.34360 **0.01085**
300	4.21672 **0.02908**	4.20266 **0.00168**	4.20968 **0.00699**	4.22025 **0.00701**
400	4.11811 **0.02078**	4.10781 **0.00126**	4.11295 **0.00512**	4.12069 **0.00513**
500	4.09055 **0.01639**	4.08236 **0.00101**	4.08645 **0.00407**	4.09260 **0.00407**
n	$\beta_1 = 5$			
	SELF	QLF	WLF	PLF
50	5.65227 **0.25334**	5.53922 **0.01280**	5.59517 **0.05665**	5.68133 **0.05767**
100	5.61019 **0.12289**	5.55409 **0.00640**	5.58200 **0.02797**	5.62441 **0.02822**
200	5.50265 **0.05867**	5.47514 **0.00320**	5.48886 **0.01368**	5.50958 **0.01374**
300	5.34864 **0.03686**	5.33081 **0.00213**	5.33971 **0.00886**	5.35311 **0.00889**
400	5.22356 **0.02633**	5.21050 **0.00160**	5.21702 **0.00649**	5.22683 **0.00650**
500	5.18860 **0.02077**	5.17822 **0.00128**	5.18340 **0.00515**	5.19120 **0.00516**
n	$p = 0.45$			
	SELF	QLF	WLF	PLF
50	0.49162 **0.02237**	0.48178 **0.00113**	0.48665 **0.00500**	0.49414 **0.00509**
100	0.48796 **0.01085**	0.48308 **0.00057**	0.48550 **0.00247**	0.48919 **0.00249**
200	0.47860 **0.00518**	0.47621 **0.00028**	0.47740 **0.00121**	0.47921 **0.00121**
300	0.46521 **0.00325**	0.46366 **0.00019**	0.46443 **0.00078**	0.46560 **0.00078**
400	0.45433 **0.00232**	0.45319 **0.00014**	0.45376 **0.00057**	0.45461 **0.00057**
500	0.45129 **0.00183**	0.45038 **0.00011**	0.45084 **0.00045**	0.45151 **0.00046**

Table 7: Bayes estimates and risks under gamma prior

n	$\beta_1 = 6$			
	SELF	QLF	WLF	PLF
50	6.78973 **0.27171**	6.65394 **0.01373**	6.72115 **0.06076**	6.82464 **0.06185**
100	6.73919 **0.13180**	6.67180 **0.00686**	6.70532 **0.03000**	6.75627 **0.03027**
200	6.61001 **0.06292**	6.57696 **0.00343**	6.59344 **0.01468**	6.61833 **0.01474**
300	6.42500 **0.03953**	6.40358 **0.00229**	6.41427 **0.00950**	6.43038 **0.00953**
400	6.27475 **0.02824**	6.25906 **0.00172**	6.26690 **0.00696**	6.27868 **0.00697**
500	6.23275 **0.02228**	6.22029 **0.00137**	6.22651 **0.00553**	6.23588 **0.00554**
n	$\beta_1 = 8$			
	SELF	QLF	WLF	PLF
50	8.94285 **0.33963**	8.76399 **0.01716**	8.85252 **0.07595**	8.98883 **0.07731**
100	8.87628 **0.16475**	8.78752 **0.00858**	8.83168 **0.03750**	8.89878 **0.03783**
200	8.70613 **0.07865**	8.66260 **0.00429**	8.68431 **0.01835**	8.71709 **0.01843**
300	8.46245 **0.04941**	8.43425 **0.00286**	8.44833 **0.01188**	8.46954 **0.01191**
400	8.26456 **0.03530**	8.24390 **0.00215**	8.25422 **0.00870**	8.26974 **0.00872**
500	8.20925 **0.02784**	8.19283 **0.00172**	8.20103 **0.00691**	8.21336 **0.00692**
n	p = 0.35			
	SELF	QLF	WLF	PLF
50	0.40223 **0.02351**	0.39419 **0.00119**	0.39817 **0.00526**	0.40430 **0.00535**
100	0.39924 **0.01141**	0.39524 **0.00059**	0.39723 **0.00260**	0.40025 **0.00262**
200	0.39158 **0.00544**	0.38963 **0.00030**	0.39060 **0.00127**	0.39208 **0.00128**
300	0.38062 **0.00342**	0.37936 **0.00020**	0.37999 **0.00082**	0.38094 **0.00082**
400	0.37172 **0.00244**	0.37079 **0.00015**	0.37126 **0.00060**	0.37196 **0.00060**
500	0.36924 **0.00193**	0.36850 **0.00012**	0.36887 **0.00048**	0.36942 **0.00048**

Table 8: Bayes estimates and risks under chi square prior

n	$\beta_1 = 6$			
	SELF	QLF	WLF	PLF
50	6.72115 **0.26353**	6.65394 **0.01359**	6.65394 **0.05954**	6.75535 **0.06060**
100	6.70532 **0.12982**	6.67180 **0.00683**	6.67180 **0.02970**	6.72224 **0.02996**
200	6.59344 **0.06245**	6.57696 **0.00342**	6.57696 **0.01460**	6.60172 **0.01467**
300	6.41427 **0.03933**	6.40358 **0.00228**	6.40358 **0.00947**	6.41963 **0.00950**
400	6.26690 **0.02814**	6.25906 **0.00171**	6.25906 **0.00694**	6.27082 **0.00696**
500	6.22651 **0.02221**	6.22029 **0.00137**	6.22029 **0.00552**	6.22963 **0.00553**
n	$\beta_1 = 8$			
	SELF	QLF	WLF	PLF
50	8.85252 **0.32941**	8.76399 **0.01699**	8.76399 **0.07443**	8.89757 **0.07576**
100	8.83168 **0.16228**	8.78752 **0.00854**	8.78752 **0.03713**	8.85395 **0.03745**
200	8.68431 **0.07806**	8.66260 **0.00428**	8.66260 **0.01825**	8.69522 **0.01833**
300	8.44833 **0.04917**	8.43425 **0.00286**	8.43425 **0.01184**	8.45539 **0.01187**
400	8.25422 **0.03517**	8.24390 **0.00214**	8.24390 **0.00867**	8.25939 **0.00869**
500	8.20103 **0.02776**	8.19283 **0.00171**	8.19283 **0.00690**	8.20514 **0.00691**
n	$p = 0.35$			
	SELF	QLF	WLF	PLF
50	0.39817 **0.02281**	0.39419 **0.00118**	0.39419 **0.00515**	0.40019 **0.00524**
100	0.39723 **0.01123**	0.39524 **0.00059**	0.39524 **0.00257**	0.39823 **0.00259**
200	0.39060 **0.00540**	0.38963 **0.00030**	0.38963 **0.00126**	0.39109 **0.00127**
300	0.37999 **0.00340**	0.37936 **0.00020**	0.37936 **0.00082**	0.38031 **0.00082**
400	0.37126 **0.00243**	0.37079 **0.00015**	0.37079 **0.00060**	0.37149 **0.00060**
500	0.36887 **0.00192**	0.36850 **0.00012**	0.36850 **0.00048**	0.36905 **0.00048**

Table 9: Bayes estimates and risks under exponential prior

n	$\beta_1 = 6$			
	SELF	QLF	WLF	PLF
50	6.69180 **0.26648**	6.55797 **0.01346**	6.62421 **0.05959**	6.72621 **0.06066**
100	6.64199 **0.12927**	6.57557 **0.00673**	6.60861 **0.02942**	6.65883 **0.02969**
200	6.51467 **0.06171**	6.48210 **0.00337**	6.49834 **0.01439**	6.52287 **0.01446**
300	6.33233 **0.03877**	6.31122 **0.00224**	6.32176 **0.00932**	6.33763 **0.00935**
400	6.18425 **0.02770**	6.16879 **0.00168**	6.17651 **0.00682**	6.18813 **0.00684**
500	6.14286 **0.02185**	6.13057 **0.00135**	6.13671 **0.00542**	6.14594 **0.00543**
n	$\beta_1 = 8$			
	SELF	QLF	WLF	PLF
50	8.59889 **0.33180**	8.42692 **0.01676**	8.51204 **0.07420**	8.64310 **0.07553**
100	8.53489 **0.16095**	8.44954 **0.00838**	8.49200 **0.03664**	8.55652 **0.03696**
200	8.37128 **0.07683**	8.32942 **0.00419**	8.35030 **0.01792**	8.38182 **0.01800**
300	8.13697 **0.04827**	8.10985 **0.00279**	8.12339 **0.01160**	8.14379 **0.01164**
400	7.94669 **0.03449**	7.92683 **0.00210**	7.93675 **0.00850**	7.95168 **0.00851**
500	7.89351 **0.02720**	7.87772 **0.00168**	7.88560 **0.00675**	7.89746 **0.00676**
n	$p = 0.35$			
	SELF	QLF	WLF	PLF
50	0.39329 **0.02299**	0.38543 **0.00116**	0.38932 **0.00514**	0.39531 **0.00523**
100	0.39036 **0.01115**	0.38646 **0.00058**	0.38840 **0.00254**	0.39135 **0.00256**
200	0.38288 **0.00532**	0.38097 **0.00029**	0.38192 **0.00124**	0.38336 **0.00125**
300	0.37217 **0.00334**	0.37092 **0.00019**	0.37154 **0.00080**	0.37248 **0.00081**
400	0.36346 **0.00239**	0.36255 **0.00015**	0.36301 **0.00059**	0.36369 **0.00059**
500	0.36103 **0.00188**	0.36031 **0.00012**	0.36067 **0.00047**	0.36121 **0.00047**

Table 10: Bayes estimates and risks under gamma prior

n	$\beta_1 = 6$			
	SELF	QLF	WLF	PLF
50	6.75709 **0.27040**	6.62195 **0.01366**	6.68883 **0.06047**	6.79183 **0.06155**
100	6.70679 **0.13117**	6.63972 **0.00683**	6.67309 **0.02986**	6.72379 **0.03012**
200	6.57823 **0.06262**	6.54534 **0.00342**	6.56174 **0.01461**	6.58651 **0.01467**
300	6.39411 **0.03934**	6.37279 **0.00228**	6.38343 **0.00946**	6.39946 **0.00948**
400	6.24458 **0.02811**	6.22897 **0.00171**	6.23677 **0.00692**	6.24850 **0.00694**
500	6.20279 **0.02217**	6.19038 **0.00137**	6.19658 **0.00550**	6.20590 **0.00551**
n	$\beta_1 = 8$			
	SELF	QLF	WLF	PLF
50	9.08043 **0.34486**	8.89882 **0.01742**	8.98871 **0.07712**	9.12712 **0.07850**
100	9.01284 **0.16729**	8.92271 **0.00871**	8.96755 **0.03808**	9.03569 **0.03842**
200	8.84007 **0.07986**	8.79587 **0.00436**	8.81792 **0.01863**	8.85120 **0.01871**
300	8.59265 **0.05017**	8.56400 **0.00290**	8.57830 **0.01206**	8.59984 **0.01210**
400	8.39171 **0.03585**	8.37073 **0.00218**	8.38120 **0.00883**	8.39697 **0.00885**
500	8.33554 **0.02827**	8.31887 **0.00174**	8.32720 **0.00702**	8.33972 **0.00703**
n	p = 0.45			
	SELF	QLF	WLF	PLF
50	0.51396 **0.03004**	0.50368 **0.00152**	0.50877 **0.00672**	0.51660 **0.00684**
100	0.51014 **0.01457**	0.50503 **0.00076**	0.50757 **0.00332**	0.51143 **0.00335**
200	0.50036 **0.00696**	0.49786 **0.00038**	0.49910 **0.00162**	0.50099 **0.00163**
300	0.48635 **0.00437**	0.48473 **0.00025**	0.48554 **0.00105**	0.48676 **0.00105**
400	0.47498 **0.00312**	0.47379 **0.00019**	0.47438 **0.00077**	0.47528 **0.00077**
500	0.47180 **0.00246**	0.47086 **0.00015**	0.47133 **0.00061**	0.47204 **0.00061**

Table 11: Bayes estimates and risks under gamma prior

n	$\beta_1 = 6$			
	SELF	QLF	WLF	PLF
50	6.65652 **0.26100**	6.58996 **0.01346**	6.58996 **0.05897**	6.69040 **0.06002**
100	6.64085 **0.12857**	6.60765 **0.00676**	6.60765 **0.02942**	6.65760 **0.02968**
200	6.53004 **0.06185**	6.51372 **0.00339**	6.51372 **0.01446**	6.53824 **0.01453**
300	6.35259 **0.03896**	6.34201 **0.00226**	6.34201 **0.00938**	6.35790 **0.00941**
400	6.20664 **0.02787**	6.19888 **0.00170**	6.19888 **0.00687**	6.21052 **0.00689**
500	6.16664 **0.02200**	6.16048 **0.00136**	6.16048 **0.00546**	6.16973 **0.00547**
n	$\beta_1 = 8$			
	SELF	QLF	WLF	PLF
50	8.92061 **0.33195**	8.83141 **0.01712**	8.83141 **0.07500**	8.96601 **0.07634**
100	8.89961 **0.16352**	8.85511 **0.00860**	8.85511 **0.03741**	8.92206 **0.03774**
200	8.75111 **0.07866**	8.72924 **0.00431**	8.72924 **0.01839**	8.76210 **0.01847**
300	8.51331 **0.04954**	8.49912 **0.00288**	8.49912 **0.01193**	8.52043 **0.01196**
400	8.31771 **0.03544**	8.30731 **0.00216**	8.30731 **0.00874**	8.32292 **0.00876**
500	8.26411 **0.02797**	8.25585 **0.00173**	8.25585 **0.00695**	8.26825 **0.00696**
n	$p = 0.45$			
	SELF	QLF	WLF	PLF
50	0.50877 **0.02914**	0.50368 **0.00150**	0.50368 **0.00658**	0.51136 **0.00670**
100	0.50757 **0.01436**	0.50503 **0.00076**	0.50503 **0.00328**	0.50885 **0.00331**
200	0.49910 **0.00691**	0.49786 **0.00038**	0.49786 **0.00161**	0.49973 **0.00162**
300	0.48554 **0.00435**	0.48473 **0.00025**	0.48473 **0.00105**	0.48595 **0.00105**
400	0.47438 **0.00311**	0.47379 **0.00019**	0.47379 **0.00077**	0.47468 **0.00077**
500	0.47133 **0.00246**	0.47086 **0.00015**	0.47086 **0.00061**	0.47156 **0.00061**

Table 12: Bayes estimates and risks under gamma prior

n	$\beta_1 = 6$			
	SELF	QLF	WLF	PLF
50	6.63957 **0.26387**	6.50678 **0.01333**	6.57251 **0.05901**	6.67371 **0.06007**
100	6.59015 **0.12800**	6.52425 **0.00667**	6.55703 **0.02914**	6.60685 **0.02939**
200	6.46382 **0.06110**	6.43150 **0.00333**	6.44762 **0.01425**	6.47196 **0.01432**
300	6.28291 **0.03839**	6.26196 **0.00222**	6.27242 **0.00923**	6.28817 **0.00926**
400	6.13598 **0.02743**	6.12064 **0.00167**	6.12830 **0.00676**	6.13983 **0.00677**
500	6.09491 **0.02163**	6.08272 **0.00133**	6.08881 **0.00537**	6.09797 **0.00538**
n	$\beta_1 = 8$			
	SELF	QLF	WLF	PLF
50	8.87406 **0.33441**	8.69658 **0.01690**	8.78442 **0.07478**	8.91968 **0.07613**
100	8.80800 **0.16222**	8.71992 **0.00845**	8.76374 **0.03692**	8.83033 **0.03725**
200	8.63916 **0.07744**	8.59597 **0.00422**	8.61751 **0.01806**	8.65004 **0.01814**
300	8.39736 **0.04865**	8.36937 **0.00282**	8.38334 **0.01170**	8.40439 **0.01173**
400	8.20099 **0.03476**	8.18048 **0.00211**	8.19072 **0.00856**	8.20613 **0.00858**
500	8.14610 **0.02742**	8.12981 **0.00169**	8.13794 **0.00680**	8.15018 **0.00681**
n	$p = 0.45$			
	SELF	QLF	WLF	PLF
50	0.50145 **0.02952**	0.49142 **0.00149**	0.49638 **0.00660**	0.50403 **0.00672**
100	0.49772 **0.01432**	0.49274 **0.00075**	0.49521 **0.00326**	0.49898 **0.00329**
200	0.48817 **0.00684**	0.48573 **0.00037**	0.48695 **0.00159**	0.48879 **0.00160**
300	0.47451 **0.00430**	0.47293 **0.00025**	0.47372 **0.00103**	0.47491 **0.00104**
400	0.46341 **0.00307**	0.46226 **0.00019**	0.46283 **0.00076**	0.46371 **0.00076**
500	0.46031 **0.00242**	0.45939 **0.00015**	0.45985 **0.00060**	0.46054 **0.00060**

References

[1] E. J. Gumbel, Statistics of extremes, Dover Publications Edition 2004, ISBN, 0, 486, 43604 7, originally published Colombia University Press, 1958.

[2] S. Kotz, S. Nadarajah, Extreme value distributions, Theory and Applications. Imperial College Press, (2000).

[3] D. Koutsoyiannis, G. Baloutsos, Analysis of a long record of annual maximum rainfall in Athens, Greece, and design rainfall inferences, Natural Hazards 22 (2000) 29-48.

[4] G. Corsini, F. Gini, M. V. Gerco, et al., Cramer-Rao bounds and estimation of the parameters of the Gumbel distribution, IEEE, 31(2002) 1202-1204.

[5] D. Koutsoyiannis, On the appropriateness of the Gumbel distribution in modelling extreme rainfall, Proceedings of the ESF LESC Exploratory Workshop held at Bologna, Italy, (2003)

[6] G. D. Gettinby, C. D. Sinclair, D. M. Power., et al., An analysis of the distribution of extreme share returns in the UK from 1975 to 2000, Journal of Business Finance & Accounting, 31(2004) 607-646.

[7] L. J. Brunner, A. Y. Lo, Limiting posterior distributions under mixture of conjugate priors, Statistica Sinica 6(1996) 187-197.

[8] S. Y. Grodzenskii, V. G. Domrachev, Estimation of the parameters of a mixture of exponential and weibull distributions with progressive censoring. Measurement Techniques, 45(2002) 1115-1118.

[9] G. Deniz, T. J. Polo, J. M. Sanchhaz, A note on mixture prior distribution with application in actuarial statistics, Estudios De Econimia Aplicada 22(2004) 1-15.

[10] M. Saleem, M. Aslam, Bayesian analysis of the two component mixture of the rayleigh dist. With the uniform and the Jeffreys priors, J. of Applied Statistical Science 16(2008) 105-113.

[11] A. M. Razali, A. A. Salih, Combining two weibull distributions using a mixing parameter", European Journal of Scientific Research 31(2009) 296-305.

[12] M. Saleem, M. Aslam, P. Economou, On the Bayesian analysis of the mixture of power function distribution using the complete and the censored sample, Journal of Applied Statistics 37(2010) 25-40.

[13] U. Erisoglu, M. Erisoglu, H. Erol, A mixture model of two different distributions Approach to the analysis of heterogeneous survival data, World Academy of Science, Engineering and Technology 78(2011) 41-45.

[14] M. Y. Majeed, M. Aslam, Bayesian analysis of the two component mixture of inverted exponential distribution under quadratic loss functions, International Journal of Physical Sciences 7(2012) 1424-1434.

[15] S. M. A. Kazmi, M. Aslam, S. Ali, On the Bayesian estimation for two component mixture of Maxwell distribution, assuming type I censored data, International Journal of Applied Science and Technology (IJAST) 2(2012) 197-218.

[16] N. Feroze, M. Aslam, Bayesian analysis of Gumbel type II distribution under doubly censored samples using different loss functions, Caspian Journal of Applied Sciences Research 1(10) (2012) 1-10.