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Research paper

Some Properties of Connectedness for M-Topological Space

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Abstract

A multiset (mset), or bag, is used to model multi-attribute objects where repetition is significant. The concept of M-topology, or Multiset Topology, serves as a generalization of general topology (point-set topology) within the multiset context. In general topology, fundamental concepts such as homeomorphism, connectedness, and compactness are well-established. In this paper, we introduce the definitions of homeomorphism and some related results specifically for M-topological spaces. Additionally, we explore and establish several results concerning connectedness within the M-topological framework.

Keywords: *Multiset; Multiset Topology; M-connectedness.*

1. Introduction

Set theory plays a crucial role in the formulation of various mathematical structures, and its concepts are fundamental to all branches of mathematics. The basic principle of set theory dictates that elements within a set are distinct, with no repetitions allowed. However, in real-world scenarios, such as in science, we often encounter situations where repetitions of elements are meaningful. In these cases, we need to consider collections where duplicates are allowed. Repetitions in the physical world can be observed in various examples, as found in [1 - 8]. A prime example of this is found in biology, where repeated sequences of nucleotides (such as adenine (A), thymine (T), uracil (U), guanine (G), and cytosine (C)) occur in the strands of DNA and RNA. Additionally, repeated prime factors of natural numbers, multiple roots of equations describing physical phenomena, and many other examples in science and mathematics exhibit the importance of repeated elements.
To address these types of collections, we use the concept of a multiset (or bag). A multiset or a bag is a generalized form of a set, in which each element can appear multiple times, with each occurrence having a certain multiplicity. For example, in the chemical compound propene (C_3H_6), the multiset of atoms could be represented as {C, C, C, H, H, H, H, H, H}, where the three carbon (C) atoms and six hydrogen (H) atoms are repeated. The concept of multisets was first explored by Blizard in his work [1]. A comprehensive survey on multisets and their applications is available in [9].
In recent developments, M-topology was introduced by Girish and John [10] in 2012 as a generalization of general (point-set) topology to incorporate multisets. The M- M-M-topology induced by multiset relations was further explored in their later work [11]. The notion of connectedness in M-topological spaces was studied by Mahalakshmi and Thangavelu [12], while Mahanta and Samanta [13] developed the concept of compactness in M-topological spaces, analyzing various related properties. In 2024, Hoque et. al. in [14] discussed M-connectedness and Compactness via subspace mixed M-topologies.
Other developments include the concept of multiset mixed topological spaces ($T_1(T_2)$ spaces), introduced by Tripathy and Shravan [15], and the exploration of multiset filters by Zakaria et al. [16]. In 2015, Sheikh et al. [17] examined separation axioms in M-topological spaces, establishing important properties related to them. More recently, Ray and Dey [18] extended this concept by introducing separation axioms in mixed multiset topological spaces. Saleh et al. [19] further studied the properties of Ri-axioms in M- M-topological spaces in 2024. A comprehensive survey of M-topology and its applications is available in [20]. Some of the recent advances in the field are found in [21 - 24].
Thus, the study of multisets and M-topology continues to provide valuable insights into mathematical structures that involve repeated elements, offering a rich area for further exploration and application in various scientific and mathematical contexts.
In this paper, we have defined the concept of M-homeomorphism between two M-topological spaces and then discussed some of the properties of M- M-M-M-M-M-M-M-M-M-connectedness for M-topological spaces.
The rest of the paper is organized into four sections. Section 2 presents the basic concepts of multiset theory and multiset topology, which lay the foundation for our study. In Section 3, we introduce the concept of M- M-homeomorphism. Section 4 is dedicated to M-connectedness in M-topological spaces, where we prove and discuss various results, as well as define the notion of a locally connected M-topological space. Finally, Section 5 provides the conclusion.

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2. Preliminaries

In this section, we discuss the basic definitions and concepts of multiset theory and then multiset topology, which are pertinent to our study.

Definition 2.1: *[16]: If X Is a set of elements, then a multiset (mset) or a bag M drawn from the set X is represented by a mapping* $C_M : X \to N$*, where N Is the set of non-negative integers.*

The value $C_M(x)$ indicates the multiplicity of the element x In the multiset. If $X = \{x_1, x_2, \ldots, x_n\}$, then a multiset P drawn from the set X can be expressed as $P = \{p_1/x_1, p_2/x_2, \ldots, p_n/x_n\}$, where p_k represents the multiplicity of the element x_k in P.

Definition 2.2: [8] *If M Is a mset drawn from the set? X Then the set denoted by* M^* *and is defined by* $M^* = \{x \in X : C_M(x) > 0\}$ *is called the support set of the mset M.*

Definition 2.3: *(Dressed Epsilon,* $\in^n$*) [8] : If an element x in a mset N occurs for n times i.e.,* $C_N(x) = n$ *Then we write* $x \in^n N$*. If* $x \in^n N$ *Then we can simply write.* $n/x \in N$*.*

Definition 2.4: [5] *The collection of all msets drawn from X Such that no element in a mset occurs more than m Times is known as mset space.* $[X]^m$*. The mset space* $[X]^\infty$ *Is the collection of all msets drawn from X Such that there is no restriction on the number of occurrences of an element in an mset.*

Definition 2.5: [16] *If* N_1 *and* N_2 *There are two msets drawn from the set. X Then we have the following definitions.*

1) $N_1 = N_2$ if $C_{N_1}(x) = C_{N_2}(x), \forall x \in X$.
2) $N_1 \subseteq N_2$ (N_1 is a subset of N_2) if $C_{N_1}(x) \leq C_{N_2}(x), \forall x \in X$.
3) $P = N_1 \cup N_2$ if $C_P(x) = \max\{C_{N_1}(x), C_{N_2}(x)\}, \forall x \in X$.
4) $P = N_1 \cap N_2$ if $C_P(x) = \min\{C_{N_1}(x), C_{N_2}(x)\}, \forall x \in X$.
5) $P = N_1 \ominus N_2$ if $C_P(x) = \max\{C_{N_1}(x) - C_{N_2}(x), 0\}, \forall x \in X$.

The operation $\ominus$ It is called multiset subtraction.

Definition 2.6: *[16] Let M Be a mset drawn from X and* $N \subset M$*. Then the complement* N^c *of N is defined as* $N^c = M \ominus N$*.*

Definition 2.7: *[19] Let M Be a mset and* $k/x \in M$ *Be an element of M, then this single element* k/x *It is called a multipoint or m-point.*

Definition 2.8: *[11] We can define three types of submsets for a given mset as follows.*

1) Whole submset: If M Is an mset then a submset M_1 of M is called a whole subset of M if $C_M(x) = C_{M_1}(x)$, for every $x \in M_1^*$ i.e., the multiplicity of every element in the mset M_1 is the same as its multiplicity in the mset M.
2) Partial whole submset: If M is a mset then a submset N of M is called a partial whole subset if the multiplicity of at least one element in N is the same as its multiplicity in M i.e., $C_M(x) = C_N(x)$, for some $x \in N^*$.
3) Full submset: If M Be a mset then a submset N of M is called full submset, if $N^* = M^*$ with the condition that $C_N(x) \leq C_M(x)$, for all $x \in N^*$.

We now present the concept of a multiset function as defined by Girish and John in [25], and later by Mahanta and Samanta in [13].

Definition 2.9: *[13] If* $M \in [X]^m$ *and* $N \in [Y]^n$ *There are two msets, then a mset function.* $f : M \to N$ *Is a mset relation such that for each* $k/x \in M$*, there exists a unique* $p/y \in N$ *such that* $(k/x, p/y)/kp \in f$ *And we can write the function.* f *as* $f = \{(k/x, p/y)/kp | k/x \in M, f(k/x) = p/y\}$*. We define the range of f as* $R_f = \bigcup_{k/x \in M} \{f(k/x)\} = f(M)$*.*

Definition 2.10: *[13] Let* $M_1 \in [X]^m$ *and* $M_2 \in [Y]^n$ *and let* $f : M_1 \to M_2$ *Be a mset function, if* $N \subseteq M_2$ *then* $f^{-1}(N) = \{k/x \in M_1 | \{f(k/x)\} \subseteq N$*.*

Now, we state three important results as stated by Mahanta and Samanta in [13]. Let $M_1 \in [X]^m$ and $M_2 \in [Y]^n$ and let $f: M_1 \to M_2$ Be a mset function

If $A, B \subseteq M_1$ such that $A \subseteq B$ then $f(A) \subseteq f(B)$.

If $C, D \subseteq M_2$ such that $C \subseteq D$ then $f^{-1}(C) \subseteq f^{-1}(D)$.

If $P, Q \subseteq M_2$ then $f^{-1}(P \cup Q) = f^{-1}(P) \cup f^{-1}(Q)$.

The following definition describes various types of functions defined on mset.

Definition 2.11: *[25]*

1) A mset function f is said to be one-to-one (injective) if no two elements in the domain of f Have the same image under f i.e. if k_1/x_1 and k_2/x_2 There are two elements in the domain of f such that $k_1/x_1 \neq k_2/x_2$ then $f(k_1/x_1) \neq f(k_2/x_2)$.
2) A mset function f is said to be onto (surjective) if the range of f It is equal to its co-domain.

3) A mset function f It is said to be bijective if it is both one-to-one (injective) and onto (surjective).
4) A mset function $f: M \to N$ is said to be invertible if the mset relation $f^{-1}: N \to M$ It is also a mset function. The inverse of f Exists if and only if it is bijective.

Theorem 2.1: *[16] Let $f: M \to N$ Be a mset function, $P \subseteq M$ and $Q \subseteq N$. Then*

1) $f^{-1}(N \ominus Q) = f^{-1}(N) \ominus f^{-1}(Q)$.
2) $P \subseteq f^{-1}(f(P))$ Equality holds if f Is injective.
3) $f(f^{-1}(Q)) \subseteq Q$ Equality holds if f Is surjective.

Here, we define Multiset Topology and M-topological space, followed by a discussion of key results, theorems, and concepts related to M-topological spaces that will form the foundation of our study.

Definition 2.12: *(M-topology) [10], [11]: A multiset topology (or M-topology) on a multiset M, where is a multiset drawn from the set X, is a subset $\tau \subseteq P*(M)$ (the power set of the multiset M) than satisfies the following properties.*

1) ϕ and M are in τ.
2) Arbitrary union of sub-collections of elements of τ are in τ.
3) Intersections of finitely many elements of τ are in τ.

The ordered pair (M, τ), where M It is a multiset and τ is an M-topology defined on it is called a multiset topological space or M-Topological Space. If P Be a subset of M such that $P \in \tau$, then P It is called an open mset. A submset Q of M is called a closed mset if $M \ominus Q$ It is an open mset.

Definition 2.13: *[10], [11] If (M, τ) be an M-topological space and $P \in M$ Be a submset. Then, the closure of P is defined as the mset intersection of all the closed msets of M that contain PIt is denoted by $\bar{P}$ or $Cl(P)$. Thus $\bar{P} = \cap\{N \subseteq M: N \text{ is a closed mset and } P \subseteq N\}$ and $C_{\bar{P}}(x) = min\{C_N(x): P \subseteq N\}$.*

Definition 2.14: *[16] Let (M, τ) be an M-topological space, $p/x \in M$ and $P \subseteq M$. Then P is called a neighborhood of p/x If there exists an open mset $N \in \tau$ such that $p/x \in N$ and $C_N(y) \leq C_P(y)$, for all $y \neq x$.*

Definition 2.15: *[12] If (M, τ_1) and (N, τ_2) are two M-topological spaces, then the mset function $f: M \to N$ is called M-continuous if for each open submset P of N the mset $f^{-1}(P)$ is open in M.*

Definition 2.16: *(M-separation)[12] If (M, τ) be an M-topological space. An M-separation of M is defined as $M = P \cup Q$ where P and Q Are disjoint non-empty open sub-msets of M.*

Definition 2.17: *(M-connectedness)[12] An M-topological space (M, τ) is said to be M-connected if there does not exist an M-separation of M. A sub-mset P of M is called M-connected if P is M-connected as sub M-space of M.*

3. M-Homeomorphism

Homeomorphism is an important concept in mathematics. In this section, we define the notion of homeomorphism for two. M-topological spaces and explore some of their consequences. Before formally defining M-homeomorphism, let us first define M-open and M-closed functions.

Definition 3.1: *Let (M, τ_1) and (N, τ_2) are two M-topological spaces and $f: M \to N$ Be a mset function. Then the mset function is called. M-open if for each open mset P of M the mset $f(P)$ is open in N.*

Definition 3.2: *Let (M, τ_1) and (N, τ_2) are two M-topological spaces and $f: M \to N$ Be a mset function. Then the mset function is called. M-closed if for each closed mset R of M the mset $f(R)$ is closed in N.*

Definition 3.3: *(M-homeomorphism). If (M_1, τ_1) and (M_2, τ_2) are two M- topological spaces. An mset function $f: M_1 \to M_2$ is called an M-homeomorphism if*

1) f is a bijective mset function
2) f is M-continuous
3) f^{-1} is M-continuous.

A property of the M-topological space M Which is invariant under M- A homeomorphism is called a topological property in the M-topological context or simply M-topological property. The M- A homeomorphism helps us in the classification of M-topological spaces based on their intrinsic topological structure. It also helps us in establishing topological equivalence between two M-topological spaces. The study of M- Homeomorphism is important to extend classical invariants like connectedness, separation axioms, and continuity to multiset settings.

Definition 3.4: *(M-embedding). Let (M_1, τ_1) and (M_2, τ_2) are two M-topological spaces and $f: M_1 \to M_2$ be M-continuous. If the mset function $g: M_1 \to f(M_1)$ be an M-homeomorphism, then the mset function $f: M_1 \to M_2$ is called an M-embedding of M_1 in M_2*

Theorem 3.1: *Let (M_1, τ_1) and (M_2, τ_2) are two M-topological spaces and $f: M_1 \to M_2$ Be a bijective mset function. Then the following are equivalent.*

1) f is an M-homeomorphism
2) f is M-continuous and M-open.

Proof: (i) $\Leftrightarrow$ (ii) Let f is M-homeomorphism, then f is M-continuous. Now, f^{-1} Is continuous implies for each open set U in M_1, the mset $(f^{-1})^{-1}(U) = f(U)$ is open in M_2. Thus f is M-open. Conversely, let f be M-continuous and M-open. We have to show that f^{-1} is M-continuous. As f is M-open so for each open msets A in M_1 the mset f (A) open in M_2. Now, the mset $(f^{-1})^{-1}(A) = f(A)$ is open in M_2. Thus f^{-1} is M-continuous.

4. Some Results on m-Connectedness

In this section, we provide proof of some of the results on M-connectedness for M-a topological space that Mahalakshmi and Thangavelu have stated without proof in [12]. Then we discuss the notion of local connectedness in M- a topological space, and also prove that the property of M-connectedness is preserved under M-homeomorphism.

Theorem 4.1: *An M-topological space (X, τ) is M-connected if and only if the only submsets of X Those are both open and closed in. X are the empty mset and X Itself.*

Proof: Let X be M-connected. Let there exist non-empty submsets M_1 of X other than X such that M_1 is both open and closed in X, then the msets M_1 and $X \ominus M_1$ are both open, disjoint, and non-empty with $M_1 \cup (X \ominus M_1) = X$. Thus M_1 and $X \ominus M_1$ forms an M-separation of X. Which is a contradiction to the fact that X is M-connected. Thus if X is M-connected, then the only subsets of X Those are both open and closed in. X are the empty mset and X Itself.
We will use a contrapositive argument to prove the converse part. Let M_1 and M_2 form a M- separation of X, then M_1 is non-empty and different from X, and also M_1 is both open and closed in X.
Hence the result.

Theorem 4.2: *Let M-topological space (X, τ) be M-connected, and if $M_1 \cup M_2$ is an M-separation of X and if N is an M-connected subspace of X then N lies entirely within M_1 or M_2.*

Proof: As M_1 and M_2 Are open msets in X So the msets $M_1 \cap N$ and $M_2 \cap N$ are open in N . Now, the msets $M_1 \cap N$ and $M_2 \cap N$ They are disjoint, and their union is N Itself. If $M_1 \cap N$ and $M_2 \cap N$ If both are nonempty, then they will form an M-separation to N. But this is not true as N is M-connected, so one of these two msets must be empty. Hence N must lie entirely in M_1 or in M_2.

Theorem 4.3: *Let (X, τ) be an M-topological space. The union of a collection of M-connected subspaces of X Having an m-point in common is M- connected.*

Proof: Let $\{M_\alpha\}$ Be a collection of M-connected subspaces of M-topological space X. Let k/x Be an m-point common to each of M_α i.e., $k/x \in \cap M_\alpha$. We have to prove that the space. $Y = \cup M_\alpha$ is M-connected. Consider $Y = U \cup V$ as an M-separation of Y. The m-point k/x is in one of the msets Uor V, suppose $k/x \in U$. As each M_α is M-connected it must lie entirely (by Theorem 4.2) in either U or in V. But it cannot lie in. V Because it contains the m-point k/x of U . Hence $M_\alpha \subset U$ for each α, so that $\cup M_\alpha \subset U$. This shows that the mset V Is empty. Which is a contradiction. Hence the theorem.

Theorem 4.4: *Let the M-topological space (M, τ) be an M-connected space. Let N be a M-connected subspace of M. If $N \subseteq B \subseteq Cl(N)$, then B is also M-connected.*

Proof: Let N be M-connected and let $N \subseteq B \subseteq Cl(N)$. Suppose $B = M_1 \cup M_2$ be an M-separation of B. Then by Theorem 4.2, the mset N must lie entirely in M_1 or in M_2; we consider $N \subset M_1$. Then $Cl(N) \subset Cl(M_1)$; since $Cl(M_1)$ and M_2 are disjoint, B cannot have some point common in M_2. This is a contradiction to the fact that M_2 is a non-empty subset of B. Hence, B cannot have a separation. This completes the proof.

Theorem 4.5: *If the mset function $f: X \to M$ is M-continuous and if X Is M-connected then $f(X) \subseteq M$ is also M-connected.*

Proof: Let the space X be M-connected and the mset function f: X → M be M-continuous. We have to prove that the space f (X) is M-connected. As f: X → M is M-continuous the function g: X → f (X) is surjective and M-continuous. Suppose that $f(X) = U \cup V$ be an M-separation of f (X). Then the msets $g^{-1}(U)$ and $g^{-1}(V)$ are disjoint open msets in X such that $X = g^{-1}(U) \cup g^{-1}(V)$. The msets $g^{-1}(U)$ and $g^{-1}(V)$ are non-empty as the mset function g is surjective. Thus $g^{-1}(U)$ and $g^{-1}(V)$ forms an M-separation of X. This is a contradiction that X is M-connected. Thus, we cannot have an M-separation of f (X), hence f (X) is M-connected.

The following theorem shows that the property of M-connectedness for M-topological space is preserved under M-homeomorphism.

Theorem 4.6: *Let (M_1, τ_1) and (M_2, τ_2) are two M-topological spaces. If M_1 is M-connected and $f: M_1 \to M_2$ be an M-homeomorphism then $f(M_1)$ is M-connected. More precisely, M-connectedness is a topological invariant under M-homeomorphism.*

Proof: Let the space M_1 be M-connected and the mset function f: $M_1 \to M_2$ be M-homeomorphism. We have to prove that the space $f(M_1)$ is M-connected. As f: $M_1 \to M_2$ is M-homeomorphism, so the function g: $M_1 \to f(M_1)$ is surjective and M-continuous. Suppose that $f(M_1) = U \cup V$ be an M-separation of $f(M_1)$. Then the msets $g^{-1}(U)$ and $g^{-1}(V)$ are disjoint open msets in M_1 such that $M_1 = g^{-1}(U) \cup g^{-1}(V)$. The msets $g^{-1}(U)$ and $g^{-1}(V)$ are non-empty as the mset function g is surjective. Thus $g^{-1}(U)$ and $g^{-1}(V)$ forms an

M-separation of M_1. This is a contradiction that M_1 is M-connected. Thus, we cannot have an M-separation of $f(M_1)$, hence $f(M_1)$ is M-connected.

Now, we define the notion of local connectedness in an M-topological space.

If (M, τ) be an M-topological space, then we can define a mset relation R on M such that $k/x R p/y$ if there is a connected M-subspace of M containing both m-points k/x and p/y. This mset relation is an equivalence mset relation. The reflexivity and symmetry are very obvious for this relation. Now, consider N to be a connected M-subspace of M containing the m-points k/x and p/y, and V be a connected M-subspace of M containing the m-points p/y and q/z. Then we have a subspace $N \cup V$ containing both k/x and q/z. The subspace is M-connected as p/y is common to them. Thus, the relation is transitive. The each m-equivalence class of this equivalence relation is called an M-connected component.

Definition 4.1: *(locally connected) Let (M, τ) be an M-topological space. Then the M-space M is said to be locally connected at an m-point k/x if, for every neighborhood U of k/x, there is a connected neighborhood V of k/x that contains in U. If M is locally connected at each of its points, then we say that the M-topological space M is locally connected.*

5. Conclusion

In this paper, we have introduced the concepts of M-open and M-closed functions, M-homeomorphism, and local connectedness in M-topological spaces. Furthermore, we have discussed additional properties related to connectedness in M-topological spaces that have previously been unexplored. In future, we can investigate whether properties such as M-compactness and Hausdorffness are preserved under M-homeomorphism. Additionally, we can explore the concept of M-connectedness and related results in product M-topological spaces.

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