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Research paper

A Sustainable Retailer Inventory Model for Deteriorating Green Products with Time-Dependent Advertising and Carbon Emission Taxation

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Abstract

This study presents a sustainable inventory model tailored for a retailer managing deteriorating green products, where demand evolves and is influenced by environmental factors. The model incorporates a linearly increasing advertising schedule to reflect the impact of dynamic promotional efforts on eco-conscious consumer behavior. To curb product deterioration, the retailer can invest in preservation technologies that effectively slow the deterioration rate through a factorial reduction mechanism. The cost framework accounts for both time-sensitive reordering efforts and the diminishing returns of financial investment. Additionally, the model factors in carbon emissions from advertising activities, along with penalties for excess emissions and tax credits for adopting green technologies. The retailer aims to maximize overall profit by jointly optimizing selling price, replenishment timing, advertising intensity, and green investments. Analytical insights demonstrate that coordinated decision-making enhances profitability and reduces product obsolescence and environmental impact. The findings highlight the critical role of integrating marketing strategies with sustainability initiatives, offering a practical guide for retailers aiming to achieve economic efficiency and environmental stewardship.

Keywords: *Carbon emission, Deterioration, Dynamic Advertisement, Green product, Preservation technology*

1. Introduction

Growing environmental concerns, stricter sustainability regulations, and shifting consumer preferences have reshaped inventory and supply chain management in recent years. Managing green and perishable products is particularly challenging because these items are time-sensitive, prone to quality loss, and must comply with environmental standards. Retailers are therefore under pressure to balance profitability with sustainability and carbon reduction goals.

Examples of such products include organic food, eco-friendly consumer goods, and biodegradable materials. These items deteriorate over time, leading to spoilage, waste, and reduced customer satisfaction. However, deterioration can be slowed through sustainable technologies such as advanced storage systems, biodegradable packaging, and preservation methods. At the same time, consumer demand for green products is influenced by awareness, which can be shaped by advertising. When advertising efforts vary over time, they can better match the product's life cycle and improve sales response. Yet, advertising activities may also create environmental impacts, such as energy use and emissions.

This research develops an integrated inventory model for a retailer managing perishable green products. The model focuses on three priorities: (i) adjusting advertising intensity over time to stimulate demand, (ii) investing in preservation technologies to reduce spoilage, and (iii) incorporating environmental policies such as carbon taxes and tax credits into decision-making. By considering both profit and sustainability, the model encourages eco-friendly inventory practices.

To the best of our knowledge, no existing study has combined time-dependent advertising, preservation investments that reduce deterioration, and carbon tax-credit mechanisms into a unified framework for perishable green products.

1.1 Motivation

In recent years, retailers of perishable green products such as organic cold-pressed juices have faced the dual challenge of managing product deterioration and stimulating environmentally responsible consumer demand. These products, often sold in eco-friendly packaging and free from preservatives, have a limited shelf life and are highly sensitive to time and storage conditions. Retailers frequently rely on digital advertising campaigns across social media platforms to boost awareness and demand, especially during promotional seasons such as health awareness weeks or fitness-focused months. However, such advertising activities not only incur increasing marginal costs but also contribute to carbon emissions through the energy-intensive nature of digital marketing infrastructures. This creates a need to balance advertising intensity over time while accounting for both inventory spoilage and environmental penalties. The scenario calls for a dynamic inventory model that jointly optimizes order quantity, advertising strategy, and deterioration management under sustainability constraints. This motivates the development of the proposed model, which captures time-varying demand driven by advertising efforts, deterioration affected by product characteristics, and the inclusion of advertising-related emissions costs in a comprehensive profit-maximization framework.

1.2 Novelty of the Study:

- Modeling advertising-driven demand with carbon emission penalties due to promotional activities.
- Simultaneously allowing green investment to reduce the product deterioration rate.
- Including government tax credit policies tied to both green investment and eco-friendly storage infrastructure.
- Structuring all of these components into a profit-maximizing optimization framework for a single retailer.

This model offers a practical and well-rounded approach for retailers striving to balance profitability with environmental goals. By combining analytical insights and numerical evidence, it highlights how aligning marketing efforts, sustainability practices, and compliance with regulations can lead to better performance in managing green products.

2. Literature Review

Over the past decade, researchers have increasingly emphasized embedding sustainability into inventory models, particularly for deteriorating products where demand is shaped by pricing, advertising, environmental policies, and data-driven forecasting. The relevant literature can be grouped into four major streams: advertisement-driven demand, preservation and deterioration, carbon regulation, and data-driven forecasting.

Advertisement-Driven Demand

Several studies have examined advertising as a driver of consumer demand. Hossen [4] analyzed an inventory system under inflation where demand depends on advertising intensity, though his static approach does not capture dynamic consumer responses. Rathore [15] combined advertising and product reliability, while Manna et al. [9] investigated imperfect production systems with defective items and advertisement-sensitive demand. Kausar et al. [6] developed a sustainable production model integrating advertisement- and price-dependent demand with salvage of defectives. Singh and Zaidi [18] extended this by incorporating advertisement and price effects under a hybrid carbon policy and multi-trade credit. Rani et al. [16] also considered advertising alongside eco-packaging and pricing under carbon tax regulation. Unlike these studies, our model introduces a time-dependent advertising strategy that adapts dynamically to demand fluctuations, capturing consumer responsiveness over the product's lifecycle.

Preservation and Deterioration

A second stream of research addresses preservation technologies for reducing spoilage. Barman et al. [2] developed an EPQ model combining preservation and green technology investment during COVID-19. Kumar [7] analyzed preservation investment in an advertisement-sensitive, trade-credit environment. Pervin [14] proposed a sustainable deteriorating inventory model with controllable emissions using green technology. Palanivel et al. [13] integrated preservation, carbon emissions, demand dynamics, and payment latency. Utami et al. [19] examined deterioration and imperfect items under inflation and carbon emission contexts. Mohammed et al. [11] applied the ABC algorithm to optimize inventory for decaying goods under time- and price-dependent demand. Rathore [15] and Manna et al. [9] also indirectly addressed deterioration by combining advertising with imperfect production.

Unlike these works, our framework jointly optimizes preservation investment with dynamic advertising and nonlinear reordering costs, capturing their interactive effects in sustainable retail systems.

Carbon Regulation and Sustainability Policies

Carbon emissions and environmental regulation form another critical stream. Mahato et al. [8] studied deteriorating items under emission policies. Mishra [10] incorporated carbon taxes, cap policies, and advance payments in a supply chain model. Sepehri et al. [17] examined pricing–inventory trade-offs under controllable emissions with permissible payment delays. Gao et al. [3] investigated coordination for used clothing under carbon taxes and labeling. Singh and Zaidi [18] analyzed hybrid carbon policies in inflationary environments. Rani et al. [16] combined carbon regulation with eco-packaging and advertising. Palanivel et al. [13] and Pervin [14] linked carbon emissions with preservation investments. Utami et al. [19] and Mohammed et al. [11] included carbon effects in deteriorating inventory settings.

Unlike these contributions, our model embeds a dynamic carbon tax-credit mechanism that simultaneously penalizes emissions from advertising and rewards green investment, aligning more closely with evolving real-world policies.

Data-Driven Forecasting and Machine Learning

A growing body of literature focuses on predictive methods to enhance inventory decisions. Jia, Schrotenboer, and Chen [5] introduced a predict-then-optimize framework using recurrent neural networks for demand forecasting in multi-retailer replenishment. Bai [1] demonstrated the accuracy of hybrid forecasting (ARIMA + XGBoost) in handling external shocks such as inflation and fuel prices. Namwad, Mishra, and Sangle [12] developed a fuzzy seasonal forecasting model to address uncertainty in deterioration and demand. Zhang, Li, and Wang [20] applied a multi-MLP prediction framework in manufacturing execution systems. Mohammed et al. [11] employed bio-inspired optimization (ABC algorithm) for deteriorating items, and Utami et al. [19] included forecasting under inflationary conditions.

Unlike these forecasting-focused studies, our work does not isolate prediction as a stand-alone tool but embeds sustainability levers—advertising, preservation, and carbon regulation—into a unified optimization framework.

Integrated Approaches

Some studies cross traditional boundaries by integrating multiple drivers. Rathore [15] and Manna et al. [9] combined advertising with production imperfections. Palanivel et al. [13] and Barman et al. [2] integrated preservation, carbon, and demand. Sepehri et al. [17] linked emissions with payment delays. Gao et al. [3] and Rani et al. [16] connected regulation with marketing strategies. Namwad et al. [12] incorporated fuzzy forecasting into deteriorating inventory.

Unlike these fragmented contributions, our model unifies time-dependent advertising, nonlinear reordering costs, and dynamic carbon penalties, offering a more comprehensive approach to economic and environmental sustainability.

In sum, prior work demonstrates significant advances in the four streams of advertising, preservation, regulation, and forecasting. However, most models treat these drivers in isolation, rely on static assumptions, or omit nonlinear decision components. By unifying time-varying advertising, interactive preservation investment, nonlinear reordering costs, and dynamic carbon tax-credit policies, our study provides a more realistic and integrative framework for sustainable retail inventory management.

3. Notations and Assumptions

3.1 Notation

Decision Variables
Q : Order quantity per cycle
T : Cycle length (replenishment time)
$A(t)$: Time-dependent advertisement intensity
G : Investment in green technology per unit
$S \in \{0,1\}$: Storage type (0 = standard, 1 = eco-storage)
Parameters
p : selling price
η : price elasticity
θ : Deterioration rate
c : Unit purchase cost
h : Unit holding cost per unit time
p : Selling price per unit
γ : Green technology cost per unit investment
ρ : Cost of advertisement per unit intensity
δ : Emission per unit advertisement
τ : Carbon tax per unit emission
ζ : Tax credit per unit of green investment or eco-storage
$\phi(S)$: Storage impact on carbon credit; $\phi(1) > \phi(0) = 0$

3.2 Assumptions

1. The retailer handles a single green perishable product (e.g., organic juices, biodegradable goods) that deteriorates over time. While multi-product settings exist in reality, the single-product assumption provides analytical tractability and serves as a foundation for future multi-product extensions.
2. The effective deterioration rate (θ) is constant but can be reduced through preservation technologies. The deterioration reduction is modeled exponentially as $\theta = \theta_0 \ e^{-rk}$, where θ_0 Is the natural deterioration rate and $r > 0$ Is the preservation efficiency parameter. Exponential reduction is widely used in perishable inventory studies to capture diminishing returns from preservation investment. Although in practice, improvements may sometimes follow more complex nonlinear patterns, the exponential form realistically reflects how technologies (e.g., refrigeration, biodegradable packaging) provide significant early gains that taper with higher investment.
3. Consumer demand depends on the selling price (p) and the time-dependent advertising effort $A(t)$, modeled as:

$$D(t) = \alpha \, p^{-\eta} + \beta \, A(t),$$

Where is the base demand, η is the price elasticity, and β is the advertising sensitivity coefficient. The advertising effort increases linearly over time:

$$A(t) = a_0 + a_1 \ t,$$

where a_0 and a_1 They are positive constants. Linear advertising captures realistic scenarios where campaigns intensify over a selling season (e.g., festive promotions). Although real-world advertising may show nonlinear effects (e.g., saturation), the linear assumption balances realism and analytical tractability, and future work may extend to nonlinear forms.

4. The ordering cost is both time- and effort-sensitive and includes: a fixed cost (b_0), a time-dependent labor cost, and an inverse effort-based monetary cost. The ordering cost is given by:

$$OC(t) = b_0 + b_1(m_1 + m_2 \, t) + \frac{b_2}{(m_3 - m_4 \, t)},$$

Where all parameters ensure positivity for feasible values of time t. This reflects the practical observation that longer cycles increase labor cost, while operational efficiency reduces certain costs.

5. The cost of preservation investment is modeled as a quadratic function:

$$GI = c_1 \ G + c_2 \ G^2,$$

Where c_1 and c_2 are the linear and nonlinear cost coefficients, respectively. The quadratic form mirrors real-world conditions where small investments (e.g., basic cooling) are inexpensive, but advanced technologies (e.g., automated temperature control) require higher marginal spending.

6. Carbon emissions generated by advertising are directly proportional to advertising intensity: $E(t) = \delta \ A(t)$, where δ Is the emission coefficient. A carbon tax is levied at the rate τ per unit of emission. The retailer receives a green tax credit at a rate. ξ, which is proportional to the preservation investment G. These assumptions reflect current policy frameworks where governments penalize emissions while incentivizing sustainability. Although actual policies may vary, this formulation captures the essential trade-off faced by retailers.

7. These costs are proportional to the average inventory level and are affected by both the deterioration rate and time-dependent demand. This reflects the practical reality that holding larger inventories of perishables amplifies both deterioration losses and associated costs.

4. Model Formulation

This study develops a dynamic inventory model for a retailer managing a green and perishable product over a finite planning horizon. $[0, T]$. The primary objective is to determine the optimal time-varying advertising effort that maximizes the retailer's total profit while accounting for deterioration, environmental impact, and order management complexities. Demand is modeled as a function of both price and time-dependent advertising effort, capturing consumer responsiveness and marketing influence. The inventory level depletes over time due to both deterioration and sales, governed by a first-order differential equation. Advertising intensity is treated as a continuous control variable, affecting both demand and carbon emissions. The model incorporates a nonlinear ordering cost that accounts for both time investment (such as delay, cancellation, and reordering efforts) and inverse monetary effort. Specifically, the total ordering cost is expressed as a function of time: a fixed component plus a linearly increasing time-based cost and an inversely proportional monetary cost.

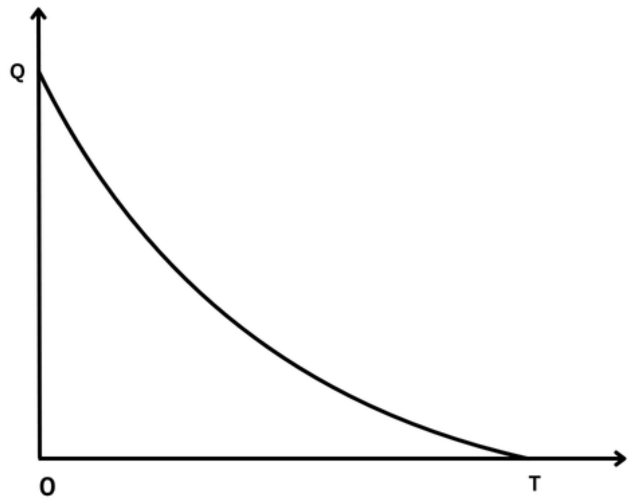

Fig 1: Inventory Level

The inventory level $I(t)$ Satisfies the following differential equation:

$$\frac{dI(t)}{dt} + \theta I(t) = -\left(\alpha p^{-\eta} + \beta A(t)\right)$$

Using the boundary conditions, $I(T) = 0$, we get

$$I(T) = \frac{(\alpha p^{-\eta} + \beta a_0)}{\theta} + \frac{\beta a_1(\theta T - 1)}{2}\left[e^{\theta(T-t)} - 1\right]$$

Using the boundary condition $I(0) = Q$, we get:

$$Q = \frac{(\alpha p^{-\eta} + \beta a_0)}{\theta} + \frac{\beta a_1(\theta T - 1)}{2}\left[e^{\theta T} - 1\right]$$

Sales Revenue

$$\begin{aligned} SR &= \int_0^T p \cdot D(t)\, dt \\ &= p^{1-\eta}\alpha T + \beta\left[a_0 T + a_1\frac{T^2}{2}\right] \end{aligned}$$

Ordering cost

$$\begin{aligned} OC &= \int_0^T b_0 + b_1(m_1 + m_2 t) + \frac{b_2}{m_3 - m_4 t}\, dt \\ &= b_0 T + b_1 m_1 T + b_1 m_2\frac{T^2}{2} + \frac{b_2 m_4}{(m_3 - m_4 T)^2} - \frac{b_2 m_4}{m_3^2} \end{aligned}$$

Purchase Cost

$$C_1 = cQ$$

Holding Cost

$$\begin{aligned} HC = &\ h\int_0^T I(t)\,dt \\ = &\ h\left\{\frac{(\alpha p^{-\eta}+\beta a_0)T}{\theta}+\frac{\beta a_1(\theta T-1)}{2}\left[-\frac{1}{\theta}+\frac{e^{\theta T}}{\theta}-T\right]\right\} \end{aligned}$$

Deterioration cost

$$\begin{aligned} DC = &\ d_c\int_0^T I(t)\,dt \\ = &\ d_c\left\{\frac{(\alpha p^{-\eta}+\beta a_0)T}{\theta}+\frac{\beta a_1(\theta T-1)}{2}\left[-\frac{1}{\theta}+\frac{e^{\theta T}}{\theta}-T\right]\right\} \end{aligned}$$

Preservation Technology Investment Cost

$$PTI = PQ$$

Advertising Cost

$$\begin{aligned} AC = &\ \rho\int_0^T A(t)\,dt \\ = &\ \rho\left[a_0T+\frac{a_1T^2}{2}\right] \end{aligned}$$

Green Investment Cost

$$GI = c_1G + c_2G^2$$

Carbon Emission Tax
Total emission over the cycle:

$$\begin{aligned} Ctax = &\ \tau\cdot\delta\int_0^T A(t)\,dt \\ = &\ \tau\cdot\delta\left[a_0T+\frac{a_1T^2}{2}\right] \end{aligned}$$

Tax Credit

$$TC = \zeta\cdot\big(G+\phi(S)\big)$$

The total profit function is:

$$\begin{aligned} TP = &\ SR - OC - PC - HC - DC - PTI - AC - GI - Ctax + TC \\ = &\ p^{1-\eta}\alpha T + \beta\left[a_0T + a_1\frac{T^2}{2}\right] - b_0T + b_1m_1T + b_1m_2\frac{T^2}{2} + \frac{b_2m_4}{(m_3-m_4T)^2} - \frac{b_2m_4}{m_3^2} - cQ \\ &-h\left\{\frac{(\alpha p^{-\eta}+\beta a_0)T}{\theta}+\frac{\beta a_1(\theta T-1)}{2}\left[-\frac{1}{\theta}+\frac{e^{\theta T}}{\theta}-T\right]\right\} - PQ - \rho\left[a_0T+\frac{a_1T^2}{2}\right] \\ &-d_c\left\{\frac{(\alpha p^{-\eta}+\beta a_0)T}{\theta}+\frac{\beta a_1(\theta T-1)}{2}\left[-\frac{1}{\theta}+\frac{e^{\theta T}}{\theta}-T\right]\right\} - c_1G + c_2G^2 \\ &-\tau\cdot\delta\left[a_0T+\frac{a_1T^2}{2}\right] + \eta\cdot\big(G+\phi(S)\big) \end{aligned}$$

4.1 Special Case: Constant Advertising

If $A(t) = A$, a constant, then:

$$D_1 = \alpha p^{-\eta} + \beta A$$

The inventory differential equation is

$$\frac{dI(t)}{dt} + \theta I(t) = -D$$

With boundary conditions, when $t = T$ then $I(t) = 0$

$$I(t) = \frac{D_1}{\theta}\left[e^{\theta(T-t)} - 1\right]$$

With boundary conditions, when $t = 0$, then $I(t) = Q$

$$Q_1 = \frac{D_1}{\theta}\left[e^{\theta(T)} - 1\right]$$

Holding cost

$$\begin{aligned} HC_1 = &\ h\int_0^T I\,(t)dt \\ = &\ h\left(\frac{D_1}{\theta}\left[\frac{e^{\theta T} - T\theta - 1}{\theta}\right]\right) \end{aligned}$$

Deteriorating cost

$$\begin{aligned} DC_1 = &\ d\int_0^T I\,(t)dt \\ = &\ d\left(\frac{D_1}{\theta}\left[\frac{e^{\theta T} - T\theta - 1}{\theta}\right]\right) \end{aligned}$$

Advertising Cost

$$AC_1 = A_d A$$

Sales Revenue

$$SR_1 = pQ_1$$

Total profit

$$\begin{aligned} TP_1 = &\ SR_1 - OC - PC - HC_1 - DC_1 - PTI - AC_1 - GI + TC \\ = &\ pQ_1 - b_0T + b_1m_1T + b_1m_2\frac{T^2}{2} + \frac{b_2m_4}{(m_3 - m_4T)^2} - \frac{b_2m_4}{m_3^2} - cQ_1 \\ &-h\left(\frac{D_1}{\theta}\left[\frac{e^{\theta T} - T\theta - 1}{\theta}\right]\right) - d\left(\frac{D_1}{\theta}\left[\frac{e^{\theta T} - T\theta - 1}{\theta}\right]\right) - PQ_1 - A_dA \\ &-c_1G + c_2G^2 - \eta\cdot\big(G + \phi(S)\big) \end{aligned}$$

5. Concavity Analysis

Concavity Analysis of Total Profit Function $TP(p, A, T)$
The retailer's total profit function $TP(p, A, T)$ depends on three decision variables: selling price p, advertisement effort A, and replenishment cycle time T.
To examine the concavity of $TP(p, A, T)$We analyze the second-order partial derivatives concerning the decision variables. The Hessian matrix is given by:

$$H = \begin{bmatrix} \frac{\partial^2 TP}{\partial p^2} & \frac{\partial^2 TP}{\partial p\,\partial A} & \frac{\partial^2 TP}{\partial p\,\partial T} \\ \frac{\partial^2 TP}{\partial A\,\partial p} & \frac{\partial^2 TP}{\partial A^2} & \frac{\partial^2 TP}{\partial A\,\partial T} \\ \frac{\partial^2 TP}{\partial T\,\partial p} & \frac{\partial^2 TP}{\partial T\,\partial A} & \frac{\partial^2 TP}{\partial T^2} \end{bmatrix}$$

The term involving p is $p^{1-\eta}\alpha T$, so:

$$\frac{\partial TP}{\partial p} = (1 - \eta)p^{-\eta}\alpha T, \quad \frac{\partial^2 TP}{\partial p^2} = -\eta(1 - \eta)p^{-\eta-1}\alpha T < 0 \quad \text{for } \eta > 0$$

Thus, the profit function is concave in p when $\eta > 0$.
Assuming $\beta(A) = \beta_0 + \beta_1 A$, the dependence on A Is linear:

$$\frac{\partial TP}{\partial A} = \beta_1\left[a_0T + \frac{a_1T^2}{2}\right] - 1, \quad \frac{\partial^2 TP}{\partial A^2} = 0$$

Hence, the function is linear in A. If diminishing returns to advertising are introduced (e.g., $\beta(A) = \log(1 + A)$), concavity in A Can be established.
The function includes several terms in T, such as:

- Revenue terms: linear and quadratic in T

- Cost terms: quadratic and rational expressions in T
- Exponential terms in the holding and deterioration cost

Due to the presence of increasing and convex cost terms (especially exponential and rational terms), the second derivative for T Is negative for realistic parameter values:

$$\frac{\partial^2 TP}{\partial T^2} < 0$$

Thus, the function is concave in T Over feasible ranges.
Mixed Partials

$$\frac{\partial^2 TP}{\partial p\, \partial A} = 0$$
$$\frac{\partial^2 TP}{\partial p\, \partial T} = (1-\eta)p^{-\eta}\alpha > 0$$
$$\frac{\partial^2 TP}{\partial A\, \partial T} = \beta_1 a_0 + \beta_1 a_1 T > 0$$

Mixed partials are positive but do not disrupt concavity significantly if their magnitude is relatively small.
Under standard assumptions:

- $TP(p, A, T)$ is concave in p when $\eta > 0$
- Concave in T for realistic cost parameters
- Linear in A; may be concave with a non-linear advertisement response

Therefore, the total profit function is jointly concave in p and T, and potentially concave overall if the advertisement response is modeled appropriately. This concavity ensures that the optimal solution can be obtained using convex optimization techniques.

6. Solution Procedure

The system defined by the state equation, adjoint equation, and optimality condition forms a two-point boundary value problem. The solution can be obtained using a forward-backward sweep method:

- Initialize $\lambda(t)$ (e.g., set $\lambda(t) = 0$ for all t),
- Solve the state equation forward in time from $t = 0$ to T,
- Solve the adjoint equation backward from $t = T$ to 0 using the transversality condition,
- Update $A(t)$ using the optimal control expression,
- Repeat the forward-backward iterations until convergence is achieved.

This approach yields the optimal advertising intensity. $A^*(t)$, inventory trajectory $I(t)$, and the corresponding total profit Π over the cycle $[0, T]$.

7. Numerical Example

To illustrate the applicability of the proposed model, we consider a numerical example using parameter values that reflect a typical operating environment of a retailer managing deteriorating green products. The base market demand is set at $\alpha = 200$ Units/day, while the advertising sensitivity is $\beta = 5$ Units per unit of advertising intensity per day. The product deteriorates at a rate of $\theta = 0.1$ Per day, and the selling price per unit is $p = 100$ Currency units. The holding cost is $h = 0.3$ Currency units per unit per day, and the deterioration cost per unit is $d = 0.1$ Currency units. Ordering incurs a fixed cost of $b_0 = 30$ currency units, a time-based cost with parameters $b_1 = 20$ currency units per unit time, and a monetary effort component defined by $b_2 = 30$ currency·time units, with efficiency parameters $m_1 = 10, m_2 = 12,\ m_3 = 14$, and $m_4 = 15$. Advertising intensity is time-dependent, starting at $a_0 = 0.2$ ad units and increasing at a rate of $a_1 = 0.3$ Ad units/day. Demand is also influenced by price elasticity, represented by $\eta = 0.23$. The unit purchasing cost is c = 0.5 currency units, and advertising incurs a cost of $\rho = 0.3$ Currency units per ad unit per day. Environmental factors include a carbon tax rate of $\tau = 0.2$ currency units per unit of emission and an emission rate of $\delta = 0.1$ Units per ad unit. To mitigate environmental impact, the retailer can invest in preservation technologies, with a green investment amount of $G = 5$ currency units and a corresponding tax credit rate of $\gamma = 0.5$. $P = 0.5, A = 10$; $c_1 = 0.1,\ c_2 = 0.2$. These values are used to evaluate the model's performance under realistic retail and sustainability conditions. The optimal solution is obtained as follows: The replenishment cycle length is $\boldsymbol{T = 10}$ **days,** $\boldsymbol{Q = 705}$, Total profit $\boldsymbol{TP = 51611}$ currency units, $\boldsymbol{Q_1 = 172}$**,** Total profit $\boldsymbol{TP_1 = 2458}$ Currency units.

8. Sensitivity Analysis and Managerial Insights

Table 1: Sensitivity Analysis

Parameter	Percentage	$\boldsymbol{Q}$	$\boldsymbol{TP}$	$\boldsymbol{Q_1}$	$\boldsymbol{TP_1}$
α	-30	847	50626.49	883	70940.8
	-20	1192	50949.44	937	60290.51
	-10	1097	70833.84	1067	73813.7
	10	946	55693.52	780	51316.83
	20	802	75857.44	784	71812.13
	30	872	55569.7	934	69217.73
β	-30	995	79594.3	737	51192.77
	-20	908	53431.74	745	47977.71
	-10	933	52780.5	873	54867.93
	10	815	50662.27	957	66373.68

Parameter	Percentage	Q	TP	Q_1	TP_1
	20	1064	58918.84	815	64530.68
	30	921	68499.61	782	50518.88
θ	-30	843	54375.93	1046	51514.5
	-20	883	74671.48	1061	64495.93
	-10	864	61115.52	1020	50859.18
	10	1062	54697.08	800	64165.59
	20	968	73349.25	1065	72613.84
	30	846	59858.61	990	74424.67
h	-30	1060	51684.5	933	52317.41
	-20	1186	68066.38	721	66471.05
	-10	880	51125.5	810	45915.25
	10	1098	59407.71	1080	67916
	20	868	71295.53	835	59326.39
	30	843	70387.58	865	45423.87
d	-30	1148	70092.11	765	50176.5
	-20	907	63244.15	977	54656.78
	-10	949	75105.98	768	74170.27
	10	1114	60284.57	800	57140.67
	20	1044	67267.29	784	54373.88
	30	1139	69265.61	893	57487.56
T	-30	1085	64425.37	1009	46955.12
	-20	927	62701.89	913	60755.32
	-10	884	61752.71	961	74188.72
	10	1181	73353.53	896	56659.8
	20	974	68900.55	1077	66000.17
	30	1062	55690.46	847	64338.63
η	-30	822	63598.18	930	64924.68
	-20	1058	79458.84	735	47116.52
	-10	1080	68053.12	835	49158.59
	10	914	78817.17	810	56044
	20	1181	55630.88	920	56235.29
	30	817	77257.44	895	47472.93
ρ	-30	916	61290.15	1026	65819.08
	-20	894	50582.45	1035	51770.43
	-10	1128	56850.65	924	62051.42
	10	1045	67718.88	987	68038.22
	20	1026	77855.72	899	54168.16
	30	1002	53883.13	821	56759.42
τ	-30	1185	53629.54	944	59271.36
	-20	1093	63035.19	766	52414
	-10	1037	55644.45	1023	74144.07
	10	993	78955.08	849	54787.02
	20	955	74078.5	885	53901.79
	30	873	66436.75	926	73962.05
G	-30	845	54634.71	752	45957.38
	-20	942	64272.11	702	56058.78
	-10	1135	60033.1	842	50848.61
	10	1029	63314.74	737	53162.52
	20	837	58343.8	714	69948.51
	30	1176	62271.2	1076	71903.98
P	-30	1018	58153.88	1082	62055.85
	-20	1184	74009.24	985	69608.77
	-10	1146	76446.08	945	70169.69
	10	1149	66510.4	902	60359.58
	20	1136	73262.32	831	65169.77
	30	1060	79826.41	848	60277.6
$CapA$	-30	1168	54230.34	768	47526.34
	-20	1028	68414.69	798	53894.65
	-10	1115	57908.99	1009	57918.94
	10	1106	50280.91	1028	65369.11
	20	1130	66315.33	789	72298.77
	30	1123	72623.28	726	54319.46
p	-30	949	50355.4	1090	48803.18
	-20	1040	59869.92	843	52997.96
	-10	1032	62864.56	1041	73706.46
	10	1042	57339.15	725	49501.04
	20	1065	62032.78	882	73224.66
	30	1052	75881.22	1015	51414.2

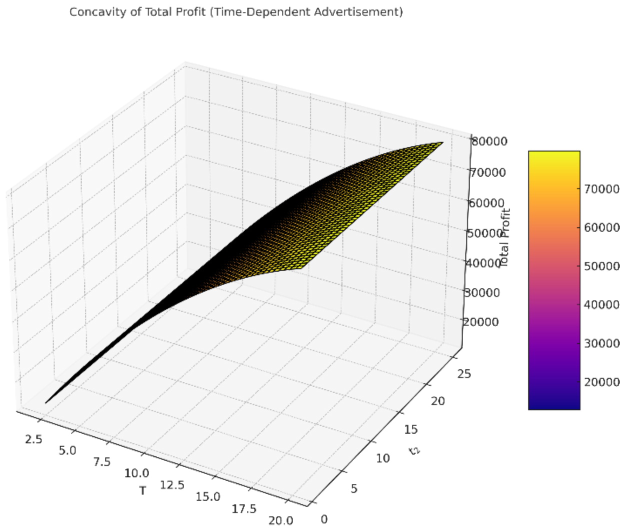

Fig 2: Concavity of the Total Profit

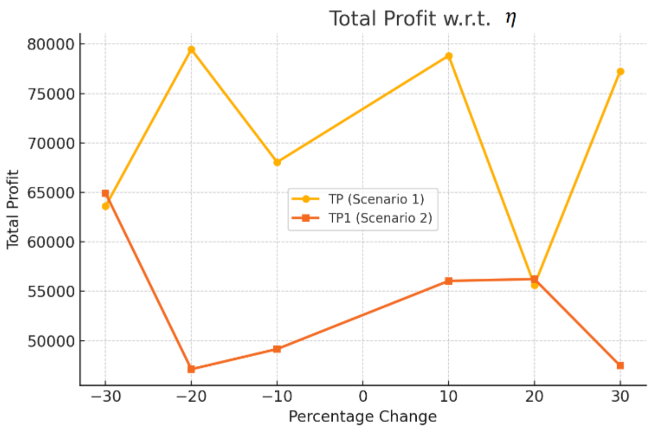

Fig 3: Percentage changes in Total Profit w.r.t η

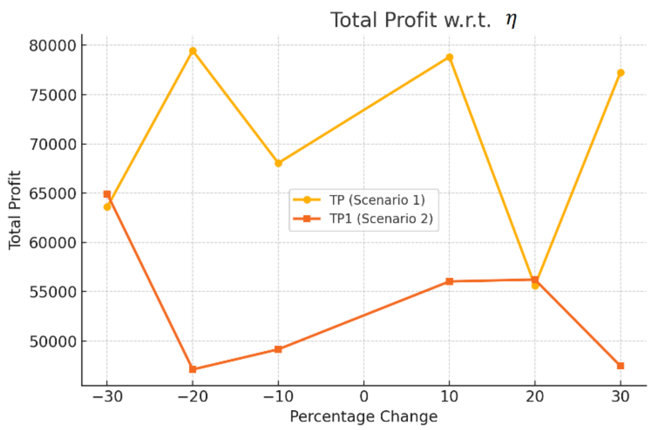

Fig 4: Percentage changes in Total Profit w.r.t T

8.1 Sensitivity Analysis

This section evaluates the sensitivity of the model outputs (Q, TP, Q_1, TP_1) to changes in key parameters. TP denotes the total profit under time-dependent advertisement, and TP_1 represents the total profit under constant advertisement. Table 2 summarizes the variations, and Figure 2 illustrates the convexity behavior concerning selected parameters. Figures 2–4 collectively illustrate the structural properties and sensitivity of the proposed model. Figure 2 confirms the concavity of the profit function with respect to order cycle length (T) and advertising effort (t_1), ensuring the existence of a unique optimal solution and demonstrating that excessive investment in either decision variable yields diminishing returns. Figure 3 shows the sensitivity of total profit to changes in demand elasticity (η), where profits are higher under time-dependent advertising (TP1) than under constant advertising (TP), confirming the superior performance of dynamic strategies. Figure 4 further demonstrates how total profit varies with cycle length (T), indicating that dynamic advertising stabilizes profitability across different planning horizons. Together, these figures validate the robustness of the model and provide actionable insights: retailers can improve profitability and sustainability by adopting time-dependent advertising while carefully balancing demand sensitivity and cycle length decisions.

The sensitivity analysis highlights several important trends. Total Profit (TP) under time-dependent advertising is more responsive to parameter changes compared to constant advertising (TP_1). Specifically, TP increases with demand parameter (α), reaching a maximum of 75,857.44 at +20% variation, whereas TP_1 peaks earlier at 73,813.70 at -10%, confirming that dynamic advertising is especially profitable in high-demand conditions. Interestingly, TP achieves its maximum (79,594.30) at -30% for η, suggesting that moderate responsiveness optimizes profitability, while excessive price sensitivity reduces demand and margins.

Both TP and TP_1 exhibit convexity with respect to cycle length (T), peaking around $T \approx 3.5$, where replenishment is balanced against holding and deterioration costs. Extending the cycle beyond this point leads to diminishing profits due to higher spoilage costs—an especially critical consideration for perishable goods.

The relationship between selling price (p) and profit is concave, with both TP and TP_1 peaking in the p = 35–40 range. Beyond this threshold, consumer demand falls sharply due to price sensitivity, reducing overall profitability.

Moderate elasticity values ($\eta \approx 0.5$) optimize both TP and TP_1, while higher elasticity reduces profit since even small price increases trigger significant demand drops. Similarly, higher green investment (G) improves TP_1, reaching 71,903.98 at +30%, underscoring the role of sustainable practices in enhancing profits under constant advertising. In contrast, rising preservation costs (P) diminish profitability for both scenarios, with TP_1 suffering a sharper decline (from 76,446.08 at -10% to 60,277.60 at +30%). This emphasizes the importance of adopting efficient preservation technologies.

In summary, the analysis shows that time-dependent advertising consistently delivers higher profits across parameter variations, while effective pricing, optimal cycle times, green investments, and cost-efficient preservation are critical levers for sustaining profitability in perishable goods markets.

8.2 Managerial Insights

The results provide several actionable insights for retailers managing perishable green products, with organic juice retailers serving as a practical illustration.

- First, time-dependent advertising (TP) consistently outperforms constant advertising (TP_1), demonstrating that dynamically adjusting promotional intensity over time is more effective for boosting sales and profitability. For example, an organic juice retailer could increase advertising intensity during weekends or health-awareness campaigns when consumer demand is more responsive, rather than maintaining a uniform advertising budget.
- Second, profitability improves with a higher base demand (α) and moderate price elasticity (η), but only when the selling price (p) is optimally set. The convexity analysis shows that profits peak when the price is maintained within a threshold range (e.g., $p \approx 35$–40). For organic juice, this suggests that pricing should balance affordability with consumer willingness to pay a premium for eco-friendly products.
- Third, the results highlight an optimal replenishment cycle of approximately $T \approx 3.5$. Extending beyond this leads to higher holding and spoilage costs. In the context of organic juice, which has a short shelf life, retailers should schedule replenishments every few days to avoid deterioration and waste.
- Fourth, green investment significantly enhances profitability, especially under constant advertising. For juice retailers, this could translate into investing in eco-friendly refrigeration or biodegradable packaging, which simultaneously preserves quality and strengthens the brand's green image.
- Finally, while preservation costs (P) can reduce profits if too high, adopting cost-effective preservation technologies—such as energy-efficient cooling or natural preservatives—can mitigate this trade-off. The sensitivity results also emphasize that key variables like α, β, p, and T should be closely monitored in practice to optimize both profitability and sustainability outcomes.
- Overall, these findings suggest that sustainable retail strategies, such as dynamic advertising and targeted preservation investments, are not only theoretically beneficial but also practically implementable in industries dealing with highly perishable green products.

9. Conclusion

This research proposes a novel and integrated inventory model for retailers handling perishable green products, introducing a dynamic framework that jointly considers time-dependent advertising, preservation investment, and environmental factors. A distinctive contribution of the model lies in its factorial reduction of deterioration through green investments and its inclusion of carbon emission penalties and green tax incentives elements rarely combined in existing inventory literature. The model uncovers several important insights. First, it demonstrates that dynamically increasing advertising intensity outperforms fixed promotional strategies in terms of profitability. Second, preservation investment effectively curbs spoilage, aligning economic performance with sustainability. Importantly, the profit function exhibits a well-structured concave form, ensuring the existence and stability of optimal decision solutions across key variables. The analytical and numerical results confirm that coordinated decisions in pricing, advertising, and green investment significantly enhance overall performance, providing a clear path for retailers to align profitability with environmental responsibility. This work bridges critical gaps in sustainable inventory modeling and serves as a decision-making tool for real-world retail applications.

To strengthen applicability, several specific directions for future research are identified. First, extending the model to multi-product green retail systems could capture advertising budget allocation and preservation trade-offs across interdependent products. Second, incorporating stochastic demand and carbon taxation uncertainty would reflect real-world consumer behavior and policy volatility. Third, future research may ask: How would the model perform if demand were forecasted in real time using machine learning techniques? Fourth, another promising direction is to explore whether blockchain technology can enhance the tracking and accountability of green investments across the supply chain. Finally, examining dual-channel retailing (online–offline integration) and nonlinear preservation technologies would provide deeper insights into practical implementation. These targeted research avenues build directly on the current model's limitations and outline concrete pathways for advancing sustainable retail strategy research.

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Conflict of Interest

The authors declare that there are no known financial or non-financial conflicts of interest that could have influenced the work reported in this manuscript.

Author Contributions

- Ruba Priyadharshini: Conceptualization, model formulation, analysis, and manuscript writing.
- Dr. R. Uthayakumar**:** Model validation, critical review, supervision, and final approval of the manuscript.

Ethics Approval

This article does not contain any studies with human participants or animals performed by the authors. Hence, ethics approval was not applicable.

Data Availability

All data generated or analyzed during this study are included in this manuscript. Additional details can be made available by the corresponding author upon reasonable request.

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