

Solutions of Volterra Integral Equations of Second Kind Using Various Method

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Abstract

Integral equations are powerful mathematical tools used to model and solve a vast array of real-world problems. Their ability to transform and simplify complex systems makes them essential in theoretical and applied sciences. Volterra integral equations are essential for modeling time-dependent, causal, and memory-influenced systems. They are mathematically tractable and appear in a wide variety of scientific and engineering contexts. Volterra integral equations appear when we convert initial value problem to an integral equation. The solution of volterra integral equation is much easier than the original initial value problem. Many problems of thermodynamics, biology, chemistry, medical sciences, physics, heat flow, neutron diffusion problem, electric circuit problem, transform problem mechanics, engineering can represent mathematically in terms of volterra integral equation of first kind and second kind. We used Successive approximation method; Variational iterative method and Laplace transform Method for solving volterra integral equations of second kind.

Keywords: Volterra Integral Equation; Laplace Transform Method; Inverse Laplace Transform Method; Successive Approximation Method; Variational Iterative Method.

1. Introduction

The theory and application of integral equations is an important subject within applied mathematics, physics, and engineering. In particular, they are widely used in mechanics, geophysics, electricity and magnetism, kinetic theory of gases, hereditary phenomena in biology, quantum mechanics, mathematical economics, and queuing theory [19]. Volterra integral equation can be solved numerically and analytically [14]. Volterra was the first to recognize the importance of the theory and study is very systematically [12]. Volterra integral equations arise in many scientific applications such as the population dynamics spread of epidemics and semi-conductor devices. It is known that many problems of the natural sciences are reduced to the solving of integral equations of variable boundaries, which are called integral equations of volterra type [3]. Vito Volterra (proud Italian) fundamentally investigated these equations and also reduced the mathematical models of many problems of the natural sciences to solve these integral equations. It was also shown that volterra integral equations can be derived from initial value problems. Volterra started working on integral equations in 1884, but his serious study began in 1896. The name integral equation was given by du Bois-Reymond in 1888. However the name Volterra integral equation was first coined by Lalesco in 1908 [3]. Wazwaz proposed Laplace transform method, and Successive approximation method for solving volterra integral equation of first and second kinds [2]. Aggarwal proposed gave the application of Laplace transform for solving population growth and decay problems [12]. Volterra integral equations such as a particularly subset among the various integral equation types because they may have describe the dynamic evolution of systems through as a time [5] [22]. There are several methods to get the approximate solutions of such as Laplace Transform method, Successive approximation method, Variational iterative method (VIM), Homotopy perturbation Method (HPM), Adomain decomposition method can be applied to get approximate solution of ordinary differential equation and partial differential equation [9]. The Variational iterative method (VIM) has no specific requirements such as linearization, small parameters etc. for non-linear operators [20]. The Laplace transform as one of the most important integral transforms, which have many applications in scientific life and in engineering sciences [8].

2. Methodology

Volterra Integral Equation –

A linear integral equation of the form

$$n(x)m(x) = f(x) + \lambda \int_a^x k(x, t)m(t)dt \quad (1)$$



Where the upper limit of the integral is variable, $n(x)$, $f(x)$, $k(x, t)$ are known functions and $m(x)$ is unknown function, is said to be Volterra integral equation of the third kind. If λ is a real or complex parameter and the function $k(x, t)$ is the kernel of the integral equation [5].

First Kind of Volterra Integral Equation-

If we set $n(x) = 0$ in equation (1),

$$f(x) + \lambda \int_a^x k(x, t)m(t)dt = 0 \quad (2)$$

Then it is called Volterra Integral equation of the first kind.

Second kind of Volterra Integral Equation-

Volterra integral equations of the second kind, the unknown function $m(x)$ appears inside and outside the integral sign.

$$m(x) = f(x) + \lambda \int_a^x k(x, t)m(t)dt \quad (3)$$

is called homogeneous Volterra integral equation of the second kind.

a) Laplace Transform Method –

The transform of a function $f(t)$ is defined by

$$L\{F(t)\} = \int_0^{\infty} f(t)e^{-pt} dt = F(p), t \geq 0 \quad (4)$$

Where p is real or complex.

Theorem 1: Laplace Transform for Convolution type linear Volterra Integral Equation of Second Kind -

The Volterra integral equation of second kind is,

$$m(x) = f(x) + \lambda \int_0^x k(x-t)m(t)dt$$

By applying Laplace Transform both sides, we have

$$L[m(x)] = L[f(x)] + \lambda L\left[\int_0^x k(x-t)m(t)dt\right]$$

$$L[m(x)] = L[f(x)] + \lambda L[k(x)]L[m(x)]$$

By Taking Inverse Laplace transform, we get

$$m(x) = L^{-1}\left[\frac{L[f(x)]}{1-\lambda L[k(x)]}\right]$$

Laplace transform of some basic mathematical functions –

Sr. No.	$F(t)$	$L[F(t)] = f(p)$
1	1	$\frac{1}{p}$
2	T	$\frac{1}{p^2}$
3	t^2	$\frac{2!}{p^3}$
4	$t^n, n > -1$	$\frac{\Gamma(n+1)}{p^{(n+1)}}$
5	e^{at}	$\frac{1}{p-a}$
6	$\sin at$	$\frac{a}{p^2 + a^2}$
7	$\cos at$	$\frac{p}{p^2 + a^2}$
8	$\sinh at$	$\frac{a}{p^2 - a^2}$
9	$\cosh at$	$\frac{p}{p^2 - a^2}$

Now, we apply Laplace Transform Method to solve Volterra Integral Equation of second kind.

Solving Volterra integral second kind equation by using Laplace Transform method.

Example I – Solve $m(x) = x^3 + \int_0^x \sin(x-t)m(t)dt$

Solution – Taking Laplace Transform both sides, we have

$$L[m(x)] = L[x^3] + L[\int_0^x \sin(x-t)m(t)dt]$$

$$L[m(x)] = \frac{6}{p^4} + L[\sin x] L[m(x)]$$

$$L[m(x)] = 6\left[\frac{1}{p^4} + \frac{1}{p^5}\right]$$

By Taking inverse Laplace transform both sides, we get

$$m(x) = x^3 + \frac{x^5}{20} \quad (5)$$

This is required solution of volterra integral equation of second kind.

Put $x = 0, 0.1, 0.2, 0.3, \dots, 1$ in equation (5) to obtain the solution of example I.

Put $x = 0$ then $m(0) = 0$

Put $x = 0.1$ then $m(0.1) = 0.001001$

Put $x = 0.2$ then $m(0.2) = 0.008032$ and so on upto

Put $x = 1$ then $m(1) = 0.9275$

Table 1: Solution of $m(x)$ in $[0, 1]$ of Example I.

x	m(x)
0	0
0.1	0.001001
0.2	0.008032
0.3	0.027243
0.4	0.064512
0.5	0.125781
0.6	0.216384
0.7	0.340273
0.8	0.499712
0.9	0.695401
1	0.9275

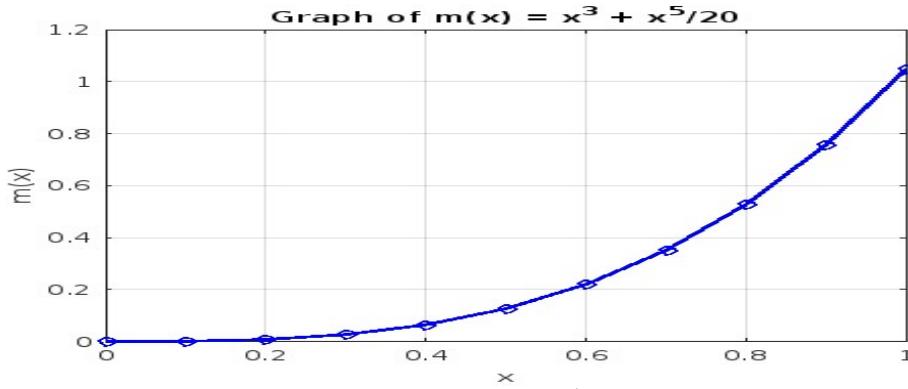


Fig. 1: Graphical solution $m(x) = x^3 + \frac{x^5}{20}$ of Example I.

Above Figure - 1 show the graphical solution of example I.

Example -II Solve $m(x) = x^2 + \int_0^x (x-t)m(t)dt$

Solutions - Applying Laplace Transform both sides, we have

$$L[m(x)] = L[x^2] + L[\int_0^x (x-t)m(t)dt]$$

$$\left[\frac{p^2-1}{p^2}\right] L[m(x)] = \frac{2}{p^3}$$

$$L[m(x)] = 2 \left(\frac{1}{p(p^2-1)}\right)$$

Applying inverse Laplace transform both sides, we get

$$m(x) = 2 \cosh x$$

The series is,

$$m(x) = 2 + x^2 + \frac{x^4}{12} + \frac{x^6}{360} + \dots \quad (6)$$

This is required solution of volterra integral equation of second kind.

Put $x = 0, 0.1, 0.2, 0.3, \dots, 1$ in equation (6) to obtain the solution of example II.

Put $x = 0$ then $m(0) = 2$

Put $x = 0.1$ then $m(0.1) = 2.01$

Put $x = 0.2$ then $m(0.2) = 2.09$ and so on upto

Put $x = 1$ then $m(1) = 3.0862$

Table 2: Solution of $m(x)$ in $[0, 1]$ of Example II

X	$m(x) = 2 \cosh x$
0	2
0.1	2.01
0.2	2.09
0.3	2.0907
0.4	2.1621
0.5	2.2553
0.6	2.3709
0.7	2.5103
0.8	2.6749
0.9	2.8662
1	3.0862

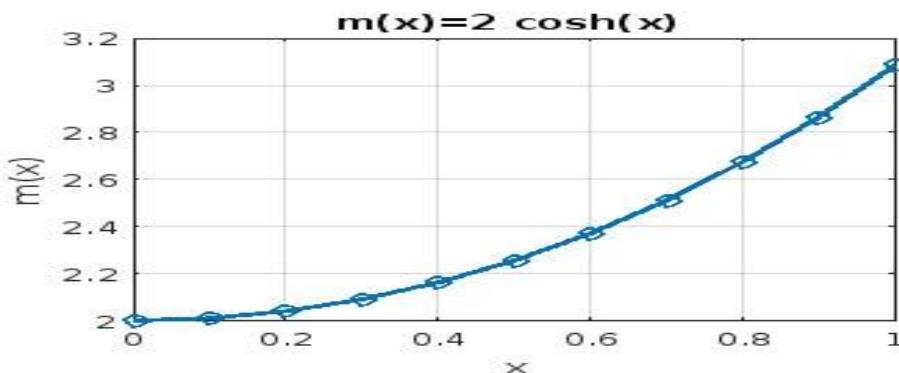


Fig. 2: Graphical solution $m(x) = 2 \cosh x$ of Example II

Above Figure - 2 show the graphical solution of example II.

b) Method of Successive Approximation for solving Volterra Integral equation of the second kind –

Let the volterra integral equation of the second kind be,

$$m(x) = f(x) + \lambda \int_0^x K(x, t) \cdot m(t) dt \quad (7)$$

And also $f(x)$ be continuous is $[0, a]$ and $K(x, t)$ be continuous for $0 \leq x \leq a, 0 \leq t \leq x$.

Given function $m_0(x)$ continuous in $[0, a]$.then, replace by $m(t)$ on R.H.S. of equation (7) by $m_0(t)$, we have

$$m_1(x) = f(x) + \lambda \int_0^x K(x, t) \cdot m_0(t) dt \quad (8)$$

But $m_1(x)$ is not discontinuous in $[0, a]$.

$$m_2(x) = f(x) + \lambda \int_0^x K(x, t) \cdot m_1(t) dt \quad (9)$$

But $m_2(x)$ is not discontinuous in $[0, a]$.

We proceed similarly and arrive at a sequence of functions $m_3(x), m_4(x), \dots, m_n(x), \dots$

$$m_n(x) = f(x) + \lambda \int_0^x K(x, t) \cdot m_{n-1}(t) dt \quad (10)$$

Because of continuity of (x) and $K(x, t)$, the sequences $\{m_n(x)\}$ converges as $n \rightarrow \infty$

$$\text{That is } m(x) = \lim_{n \rightarrow \infty} m_n(x) \quad (11)$$

Thus, the solution $m(x)$ is obtained.

Now, we apply Successive Approximation Method to solve Volterra Integral Equation of second kind.

Solving Volterra integral second kind equation by using Successive approximation method.

Example III Solve $m(x) = 1 - 4 \int_0^x m(t)dt$

Solution – Here $f(x) = 1, \lambda = -4, K(x, t) = 1$

For the zeroth approximation, we can choose $m_0(x) = 1$
By successive approximation method,

$$m_n(x) = 1 - 4 \int_0^x m_{n-1}(t)dt \quad (12)$$

Put $n = 1$ in equation (12), we get

$$m_1(x) = 1 - 4x$$

Put $n = 2$ in equation (12), we get

$$m_2(x) = 1 - 4x + \frac{4^2 x^2}{2!}$$

Similarly,

$$m_3(x) = 1 - 4x + \frac{4^2 x^2}{2!} - \frac{4^3 x^3}{3!}$$

And so on.

The n th term is,

$$m_n(x) = 1 - 4x + \frac{4^2 x^2}{2!} - \frac{4^3 x^3}{3!} + \dots + (-1)^n \frac{4^n x^n}{n!} + \dots$$

$$m_n(x) = \sum_{k=0}^n (-1)^k \frac{4^k x^k}{k!}$$

Hence,

$$m(x) = \lim_{n \rightarrow \infty} \left(\sum_{k=0}^n (-1)^k \frac{4^k x^k}{k!} \right)$$

$$m(x) = e^{-4x}$$

The series is,

$$m(x) = 1 - 4x + \frac{4^2 x^2}{2!} - \frac{4^3 x^3}{3!} + \dots \quad (13)$$

This is required solution of volterra integral equation of second kind.

Put $x = 0, 0.1, 0.2, 0.3, \dots, 1$ in equation (13) to obtain the solution of example III.

Put $x = 0$ then $m(0) = 1$

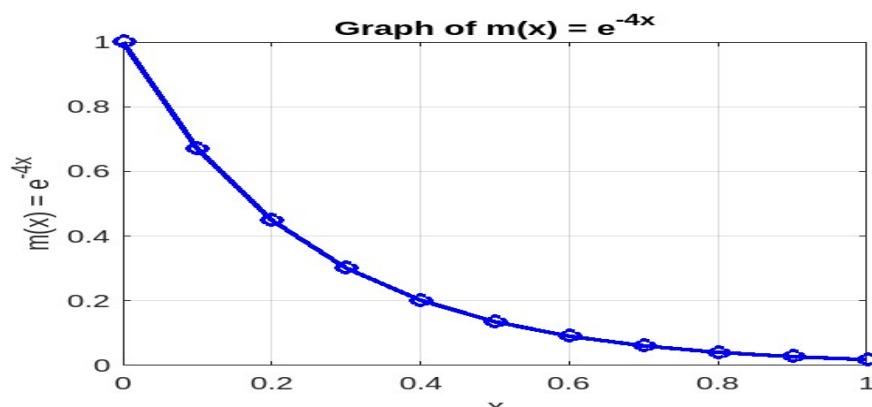
Put $x = 0.1$ then $m(0.1) = 0.6703$

Put $x = 0.2$ then $m(0.2) = 0.4493$ and so on upto

Put $x = 1$ then $m(1) = 0.0183$

Table 3: Solution of $m(x)$ in $[0, 1]$ of Example III.

x	m(x)
0	1
0.1	0.6703
0.2	0.4493
0.3	0.3012
0.4	0.2019
0.5	0.1353
0.6	0.0907
0.7	0.0608
0.8	0.0408
0.9	0.0273
1	0.0183

Fig. 3: Graphical solution $m(x) = e^{-4x}$ of Example III

Above Figure - 3 show the graphical solution of example III.

Example IV- Solve $m(x) = x - \int_0^x (x-t) m(t) dt$ (14)

Solution – Here $f(x) = x$, $\lambda = -1$, $K(x, t) = (x-t)$

For the zeroth approximation, we can choose $m_0(x) = 0$
By successive approximation method,

$$m_n(x) = x - \int_0^x (x-t) m_{n-1}(t) dt \quad (15)$$

Put $n = 1$ in equation (15), we get

$$m_1(x) = x$$

Put $n = 2$ in equation (15), we get

$$m_2(x) = x + \frac{x^3}{3!}$$

Put $n = 3$ in equation (15), we get

$$m_3(x) = x + \frac{x^3}{3!} + \frac{x^5}{5!}$$

And so on.

The n th term is,

$$m_n(x) = 1 + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \cdots + \frac{x^{2n+1}}{(2n+1)!} + \cdots$$

$$m_n(x) = \sum_{k=0}^n \frac{x^{2k+1}}{(2k+1)!}$$

Hence,

$$m(x) = \lim_{n \rightarrow \infty} \left(\sum_{k=0}^n \frac{x^{2k+1}}{(2k+1)!} \right)$$

$$m(x) = \sinh x \quad (16)$$

The series is,

$$m(x) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$$

This is required solution of volterra integral equation of second kind.

Put $x = 0, 0.1, 0.2, 0.3, \dots, 1$ in equation (16) to obtain the solution of example IV.

Put $x = 0$ then $m(0) = 0$

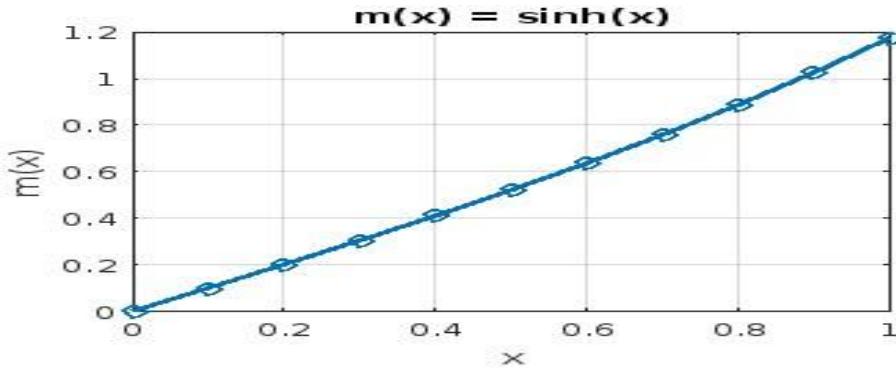
Put $x = 0.1$ then $m(0.1) = 0.1002$

Put $x = 0.2$ then $m(0.2) = 0.2013$ and so on upto

Put $x = 1$ then $m(1) = 1.1752$

Table 4: Solution of $m(x)$ in $[0, 1]$ of Example IV.

x	$m(x) = \sinh(x)$
0	0
0.1	0.1002
0.2	0.2013
0.3	0.3045
0.4	0.4108
0.5	0.5211
0.6	0.6367
0.7	0.7586
0.8	0.8881
0.9	1.0265
1	1.1752

**Fig. 4:** : Graphical solution $m(x) = \sinh(x)$ of Example IV

Above Figure - 4 show the graphical solution of example IV.

c) Variation Iteration Method (VIM) –

The Variational iteration method is employed to solve the time dependent reaction diffusion equation which has special importance in engineering and sciences and constitutes a good model for many systems in various fields [3]. The Variation Iteration Method was developed in 1999, by J.H. He. This method is, now, widely used by many researchers to study linear and non-linear problems. The method introduced a reliable and efficient process for a wide variety of scientific and engineering applications. It is based on Lagrange multiplier and it has the merits of simplicity and easy execution [19]. The main idea of variational iteration method is to approximate the solution of differential equation by using an iteration formula in the form of a correctional functional which involves Lagrange's multiplier. By applying variational theory, Lagrange's multiplier can be determined. The iteration is initiated by a simple function such as linear function. To illustrates the main concept of this method [13].

Consider the following,

$$Tm(t) = g(t) \quad (17)$$

Where T is a differential operator that acts on sufficiently smooth function u defined on such an interval $I \subseteq R$. The given function g is also defined in I . Initially, split T into its linear and non-linear part, namely,

$$Lm(t) + Nm(t) = f(t)$$

Where L and N are linear and non-linear operator respectively.

A correctional functional for equation (17) is then defined iteratively as,

$$m_{n+1}(t) = m_n(t) + \int_0^x \lambda(r) (Lm(r) + Nm(r) - f(r)) dr$$

Where $\lambda(r)$ is Lagrange multipliers, u_n is the n th approximate solution and m_n is restricted variations so that, $m_n = 0$. This means that $m_{n+1}(t)$ can be considered as an approximated solution for equation,

$$m_{n+1}(t) = m_n(t) + \int_0^x \lambda(r) (Lm_n(r) + Nm_n(r) - f(r)) dr$$

Now, we apply Variational Iteration Method to solve Volterra Integral Equation of second kind.

Solving Volterra integral second kind equation by using Variational iteration method

$$\text{Example V Solve } m(x) = 5 + 2x^2 - \int_0^x (x-t)m(t)dt \quad (18)$$

Solution – differentiate with respect to x both sides of equation (18), we get

$$m'(x) = 4x - \frac{d}{dx} \left[\int_0^x (x-t)m(t)dt \right]$$

By using Leibnitz rule,

$$\begin{aligned}
 m'(x) &= 4x - \int_0^x m(t)dt \\
 m'(x) - 4x + \int_0^x m(t)dt &= 0
 \end{aligned} \tag{19}$$

The correctional functional formula is,

$$m_{n+1}(x) = m_n(x) + \int_0^x \lambda(r)(m'_n(r) - 4r + \int_0^r m(t)dt)dr$$

We find that $\lambda(r) = -1$

Therefore,

$$m_{n+1}(x) = m_n(x) - \int_0^x (m'_n(r) - 4r + \int_0^r m_n(t)dt)dr \tag{20}$$

We can choose initial conditions $m_0(x) = m(0) = 5$ & $m'_0(x) = 0$

Put $n = 0$ in equation (20), we get

$$m_1(t) = 5 - \frac{x^2}{2}$$

Put $n = 1$ in equation (20), we get

$$m_2(x) = 5 - \frac{x^2}{2!} + \frac{x^4}{4!}$$

Similarly,

$$m_3(x) = 5 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$$

& so on.

The n th term is,

$$m_n(x) = 5 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^5}{6!} + \dots + (-1)^n \frac{x^{2n}}{2n!}$$

Hence,

$$m(x) = \lim_{n \rightarrow \infty} \left(4 + \sum_{k=0}^n (-1)^k \frac{x^{2k}}{2k!} \right)$$

$$m(x) = 4 + \cos x$$

The series is,

$$m(x) = 5 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^5}{6!} + \dots \tag{21}$$

This is required solution of volterra integral equation of second kind.

Put $x = 0, 0.1, 0.2, 0.3, \dots, 1$ in equation (21) to obtain the solution of example V.

Put $x = 0$ then $m(0) = 5$

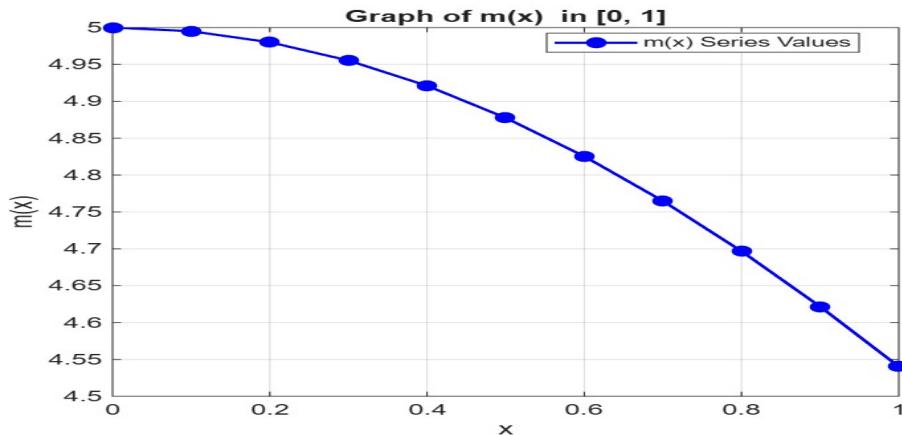
Put $x = 0.1$ then $m(0.1) = 4.995$

Put $x = 0.2$ then $m(0.2) = 4.98$ and so on upto

Put $x = 1$ then $m(1) = 4.5403$

Table 5: Solution of $m(x)$ in $[0, 1]$ of Example V.

X	m(x)
0	5
0.1	4.995
0.2	4.98
0.3	4.95534
0.4	4.9211
0.5	4.87758
0.6	4.8253
0.7	4.7684
0.8	4.9553
0.9	4.6216
1	4.5403

Fig. 5: Graphical solution $m(x) = 4 + \cos x$ of Example V

Above Figure - 5 show the graphical solution of example V.

Example VI Solve $m(x) = 1 + 2x + 4 \int_0^x (x-t)m(t)dt$ (22)

Solution - Differentiate with respect to x both sides of equation (22), we get

$$m'(x) = 2 + 4 \frac{d}{dx} \left(\int_0^x (x-t)m(t)dt \right)$$

By using Leibnitz rule,

$$m'(x) = 2 + 4 \int_0^x m(t)dt$$

$$m'(x) - 2 - 4 \int_0^x m(t)dt = 0 \quad (23)$$

The correctional functional is,

$$m_{n+1}(x) = m_n(x) + \int_0^x \lambda(r) (m'_n(r) - 2 - 4 \int_0^r m_n(t)dt) dr$$

We find that $\lambda(r) = -1$

$$m_{n+1}(x) = m_n(x) - \int_0^x (m'_n(r) - 2 - 4 \int_0^r m_n(t)dt) dr \quad (24)$$

We have to choose initial condition $m_0(x) = m(0) = 1$ & $m'_0(x) = 0$

Put $n = 0$ in equation (24), we get

$$m_1(x) = 1 + 2x + \frac{2^2 x^2}{2!}$$

Put $n = 1$ in equation (24), we get

$$m_2(x) = 1 + 2x + \frac{2^2 x^2}{2!} + \frac{2^3 x^3}{3!} + \frac{2^4 x^4}{4!}$$

And so on.

The n th term is,

$$m_n(x) = 1 + 2x + \frac{2^2 x^2}{2!} + \frac{2^3 x^3}{3!} + \frac{2^4 x^4}{4!} + \dots + \frac{2^n x^n}{n!}$$

Hence,

$$m(x) = \lim_{n \rightarrow \infty} \left(\sum_{k=0}^n \frac{2^k x^k}{k!} \right)$$

$$m(x) = e^{2x}$$

The series is,

$$m(x) = 1 + 2x + \frac{2^2 x^2}{2!} + \frac{2^3 x^3}{3!} + \dots \quad (25)$$

This is required solution of volterra integral equation of second kind.

Put $x = 0, 0.1, 0.2, 0.3, \dots, 1$ in equation (25) to obtain the solution of example VI.

Put $x = 0$ then $m(0) = 1$

Put $x = 0.1$ then $m(0.1) = 1.2214$

Put $x = 0.2$ then $m(0.2) = 1.4918$ and so on upto

Put $x = 1$ then $m(1) = 7.3891$

Table 6: Solution of $m(x)$ in $[0, 1]$ of Example VI.

X	m(x)
0	1
0.1	1.2214
0.2	1.4918
0.3	1.8221
0.4	2.2255
0.5	2.7183
0.6	3.3201
0.7	4.0552
0.8	4.953
0.9	6.0496
1	7.3891

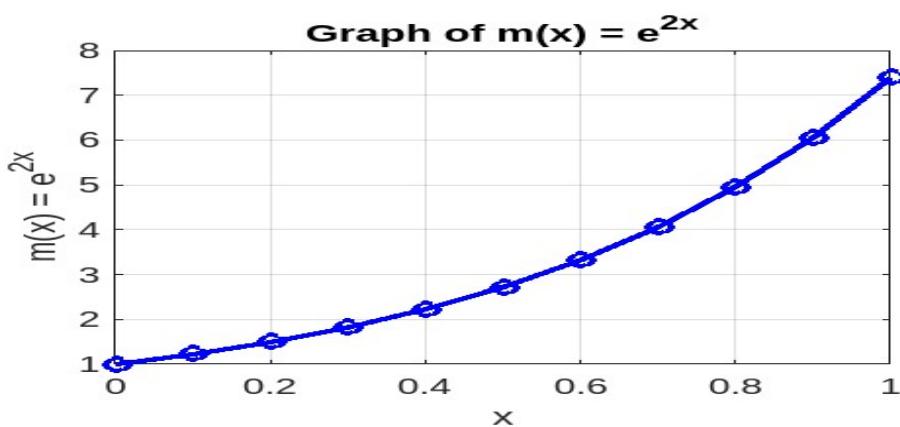


Fig. 6: : Graphical solution $m(x) = e^{2x}$ of Example VI

Above Figure - 6 show the graphical solution of example VI.

3. Conclusion

In this paper, we solved successfully the linear volterra integral equation of second kind by Laplace transform Method, Successive Approximation Method and Variational Iteration Method. The research finding indicate that the Laplace transform, Successive Approximation Method and Variational Iterative Method is very useful integral transform for solving a linear Volterra Integral equation of the second kind. Through comparative analysis, it can be concluded that all three methods are reliable and complementary. The Laplace Transform Method is best suited for problems with well-behaved kernels and known transform pairs, while the Successive Approximation and VIM are more flexible for complex or nonlinear cases. In the future, the proposed scheme can be applied for nonlinear Volterra integral equations and their systems.

Future scope

- 1) Extension to Nonlinear and Complex Kernel Systems
- 2) Hybrid and Modified Methods
- 3) Comparative Computational Analysis
- 4) Application to Fractional and Stochastic Volterra Equations
- 5) Algorithm Automation and Symbolic Computation
- 6) Incorporation of Artificial Intelligence and Machine Learning
- 7) Real-World and Multidisciplinary Applications

References

- [1] Abdul Majid Wazwaz, "Linear and Non-linear integral equations Method and Applications" Higher Education Press, Beijing and Springer – Verlag Berlin Heidelberg 2011.
- [2] D.C. Sharma and M.C. Goyal, "Integral Equations" PHI Learning Private limited Delhi 110092 2017.
- [3] Dinkar Patil, Apeksha Deshmukh, Mansi Patil, "Application of General Integral Transform for solving linear volterra integral equation of second kind," International Journal of Advances in engineering and Management, Volume 5, Issue 1, Pages 801-807, Jan 2023, www.ijaem.net.
- [4] D.N.Warade, Sheetal R.Gamkar, P.H.Munjankar and Arun R.Kamble, "Study of Integral Equations used in various fields of Science and Engineering," IJRBAT, Volume (II), Issue (XI), Pages 59-63, May 2023, www.ijrbat.in. <https://doi.org/10.29369/ijrbat.2023.02.1.0010>.

[5] Dr. Subhamay Dutta, "The Method of Successive Approximation (Neumann's Series) of Volterra Integral Equation of the second kind, " Pure and Applied Mathematical Journal, Volume 5, Issue 6, Pages 211-219. <https://doi.org/10.11648/j.pamj.20160506.16>

[6] E. Rama, K. Somaiah, K. Sambaiah, "A study of Variational iteration method for solving various types of problems," Malaya Journal of Matematik, Volume 9, Issue 1, Pages 701-708, 2021. <https://doi.org/10.26637/MJM0901/0123>.

[7] Fawzia Al-saar and Kirtiwant P. Ghadle, "Combined L.T, with analytical methods for solving Volterra integral equations with a convolution kernel," JKSIM, Volume 22, No-2, Pages 125-136, (2018)

[8] Hanan abushahma, Zieneb Elshegmani "Using the Laplace Transform to Solve the Volterra Integral Equation" Scientific Journal of Faculty of Education, Misurata University-Libya, Vol. 9, No. 22, Jun. 2023

[9] M.Rahman "Integral equations and their Applications" WIT Press

[10] Mohammed S. Mechee Adil M. Al Ramahi, Raad M. Kadum, "Application of Variational Iteration Method for Solving A Class of Volterra Integral Equations," Journal of Babylon university Pure and Applied Sciences, Volume 24, Issue 9, 2016

[11] Narhari Patil, Avinash Khambayat, "Differential Transform Method for system of Linear Differential Equations," Research Journal of Mathematical and Statistical Sciences, Volume 2, No-3, PP 4-6, March 2014.

[12] Poonam Jagtap, Avinash Khambayat, "Coparision of Volterra Integral Equation by Successive Approximation and Adomain Decomposition Methods," Journal of Emerging Technologies and Innovative Research (JETIR), Volume 11, Issue 8 , August 2024

[13] Raman Chauhan, Sudhanshu Aggarwal, "Laplace Transform for convolution type linear volterra Integral equation of 2nd kind," Journal of Advanced Research in Applied Mathematics and Statistics, Volume 4, Issue 3 & 4, Page No. 1-7, (2019)

[14] Sudhanshu Aggarwal, Aakansha Vyas, "Laplace Transform for the solution of Non-Linear Volterra Integral equation of second kind", Journal of Advanced Research in Applied Mathematics and Statistics, Volume 8, Issue 3& 4, Page no 18-25 <https://doi.org/10.24321/2455.7021.202304>

[15] Sahar Muhar Jaabar, Ahmed Hadi Hussain, "A Move Recent Review of the Integral Equtions and their applications", Journal of Physics: conference serie, 1818 (2021) 012170. <https://doi.org/10.1088/1742-6596/1818/1/012170>.

[16] Sudhanshu Aggarwal, Nidhi Sharma, Raman Chauhan, "Application of Kamal Transform for solving Linear Volterra Integral Equations of First Kind", International Journal of Research in Advent Technology, Volume 6, Issue 8, August 2018 www.ijrat.org E-ISSN 2321-9637

[17] Sudhanshu Aggrawal, Sanjay Kumar, "Laplace Transform for system of 2nd kind linear volterra integro -Differential equation", JETIR, Volume 8, Issue 6, June 2021

[18] S. Shakeri, R.Saadati , S.M. Vaezpour, J. Vahidi, " Variational Iteration Method for Solving Integral Equations", Journal of Applied Sciences, Volume 9, Issue 4, Pages 799-800, 2009. <https://doi.org/10.3923/jas.2009.799.800>.

[19] Teshome Bayleyegn Matebie "The Method of Successive Approximations (Neumann's Series) of Volterra Integral Equation of the Second Kind" Pure and Applied Mathematics Journal 2016; 5(6): 211-219. <https://doi.org/10.11648/j.pamj.20160506.16>.

[20] Yuvraj Pardeshi, "Analytical Solution of Partial Integro Differential Equations Using Laplace Differential Transform method and Comparision with DLT and DET", Asian Journal of applied Science Technology, Volume 6, Issue 2, Pages 127-137, April-June 2022. <https://doi.org/10.38177/ajast.2022.6214>.

[21] Zaid M. Odibat, "A study on the convergence of Variational iteration method", Mathematical and computer Modelling, Volume 51, pages 1181-1192, 2010. <https://doi.org/10.1016/j.mcm.2009.12.034>.

[22] Zana Mohammed Hassan, Sadeq Taha Abdulazeez, Sarbast Kamal Rasheed, "Investigating Solutions of Volterra Integral Equations Using the Successive Approximations", Mathematical Statistician and Engineering Applications, Volume 72, Issue 2, Pages 75-82, 2023.