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Research paper

Academic Performance in Geometry and Visualization Skills of Pre-Service Teachers

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Abstract

This descriptive-correlational study aimed to enhance the visual imagery and spatial visualization skills in Geometry of pre-service Mathematics teachers, with the end goal of preparing a learning enhancement material. It described the academic performances and visualization skills of 147 pre-service Mathematics teachers, who have taken up Plane and Solid Geometry, stratified from selected colleges and universities in Batangas. The relationship between the levels of their visualization skills and their academic performances in Plane and Solid Geometry was found out. The difficulties encountered by the respondents in solving problems in Geometry, relative to their visualization skills, were also assessed. Results from the findings revealed that the average pre-service Mathematics teacher possessed good academic performance in Plane and Solid Geometry, medium levels of visual imagery and low levels of spatial visualization. The academic performances of pre-service Mathematics teachers in Geometry significantly correlated with their visualization skills. Lastly, most of the pre-service Mathematics teachers often encountered difficulties in solving problems in Geometry relative to their visualization skills. The Visual Enhancers Toolkit was prepared for pre-service Mathematics teachers as a learning enhancement material in Geometry to further develop their visualization skills.

Keywords: *Academic Performance; Geometry; Pre-Service Mathematics Teachers; Spatial Visualization; Visual Imagery.*

1. Introduction

It has been the goal of higher education institutions (HEI's) to produce globally competitive individuals who embody the qualifications of a global professional. Competition grows greater as the needs of the clients of the modern era continue to expand. With this, much is expected from students who are working their hardest to earn a degree. The same is true for pre-service mathematics teachers who will soon be teaching mathematics subjects. They must be equipped with as much knowledge and skills as possible if they desire to qualify as mathematics educators and be pedagogically capable, even more so.

Pedagogically equipped mathematics teachers are able problem solvers. Supported by the Commission on Higher Education (CHED), colleges and universities offering education programs are prompted to make their pre-service teachers meet the standards of an educator. As such, CHED Memorandum Order (CMO) Nos. 74 and 75 s. 2017 implore graduates to become creative problem solvers who can make sound decisions in various classroom situations [7].

Geometry is one of the main branches of mathematics that greatly involves problem solving. In this area, Geometry demands a deeper cognition. True to the framework of MATHTED and SEI-DOST, students are required to explore the characteristics and properties of two and three dimensional geometric shapes and formulate significant geometric relationships [37]. It is also essential for them to use coordinate Geometry to specify locations and describe spatial relationships [37]. Moreover, they must be able to use transformations and symmetry to analyze mathematical situations, use spatial visualization, reasoning and geometric modeling to solve routine and non-routine problems, and use geometric proofs to develop higher-order thinking skills [37].

Geometry thus calls for 2D and 3D manipulation, therefore, visual imagery and spatial visualization (in this study, they were collectively referred to as visualization skills). Students are expected to model properties of shapes and use mathematical communication skills to draw figures given their description. They are also expected to understand properties of geometric figures and apply reasoning skills to make and validate conjectures about transformations and combinations of shapes. Likewise, they are expected to demonstrate various geometric and algebraic connections. As a cognitive process, problem solving in Geometry requires mental activities such as visual imagery and spatial visualization, to name some of the most essential ones, in order to accomplish an intended goal. Being adept with these two, gives an advantage in solving Geometry problems at hand.

Visual imagery and spatial visualization skills are both very essential in solving Geometry problems, and they work together where the former serves as pre-requisite to the latter. Representing and drawing figures are what greatly help to understand and solve a problem and are even keys to a better retention and consideration of a variety of information that may help in deciding on what to do. The National Council of Teachers of Mathematics (NCTM) even reiterates that Geometry learners should be able to use visualization, spatial reasoning, and geometric modeling to solve problems [27].

However, the reality that not all pre-service mathematics teachers are that much adept with such skills and that they take on too much in solving Geometry problems, still lie close at hand. Evidence may be taken from the study of Cordero (2023) stating that the majority of pre-service mathematics teachers performed moderately in Plane and Solid Geometry and should be further improved [8]. Why an individual finds it hard to solve problems in Geometry, fail to arrive at the solution, or misinterpret the meaning of the problem might be resting on their lack of skills to represent and draw on paper the figures that have been the subject of the problem. In the findings of Versoza (2020) among 230 pre-service teachers surveyed, 90% of their mathematical reasoning depends heavily on rules and procedures with no connection to the involved quantities in the problem [42]. This means they fail to relate learned mathematical concepts to its context which is essential in unlocking the answer the problem is seeking.

This lays groundwork on the importance of being skilled in visualizing, manipulating, rotating, and transforming mental images and figures whenever Geometry problems are at hand. Lack of these may lead to anxiety if their mathematical abilities do not catch up to the level of mathematics they are encountering. In the study of Godoy, et al. (2017), visual imagery and spatial visualization were found to be the least of all multiple intelligences where students are adept to [15]. Such circumstances may be traced from how they were towards the subject long before they were set to college mathematics, showing how essential prior knowledge is. For instance, Marasigan (2016) attested to how grade 8 students lacked basic grade 7 mathematics skills, as well as knowledge and understanding of elementary math. This resulted to a difficulty in providing new ideas to the said students [24].

The 2018 Programme for International Student Assessment (PISA) reflects the abovementioned scenario as they found Filipino high school students to have achieved an average of 353 points in mathematical literacy which was way lower than the Organization for Economic Co-operation and Development (OECD) average and was further identified to be below proficiency Level 1 [9]. The average OECD score was 489 which falls under proficiency Level 3 where students can clearly perform described procedures where interpretations are also sound to be a base for building a simple model [9]. They can also interpret and use representations based on different information sources and reason directly from them [9].

The numbers above mean that an average high school student must be able to model a problem by making use of representations as a problem solving key. Yet, given the numbers, the average Filipino high school student could not perform such and other similar tasks. Moreover, it was found that the average Filipino high school student from a public school cannot reach Level 1 proficiency of mathematical literacy. This is in comparison with an average Filipino high school student from a private school that reached Level 1 [9].

It is inevitable that the abovementioned trends might still be at work. It is then probable that the pre-service teachers still have with them their past orientations in mathematics. If they were poorly oriented in mathematics as the trend suggests, then it may be succeeded with poor performance in mathematics, including Geometry as reflected in the common scenario among pre-service mathematics teachers. Most of them may find it hard to illustrate word problems in Geometry which is very essential in accomplishing what is being asked. This may be because before they get into college, they have not been trained enough and oriented to how important some factors are in becoming a successful problem solver (visual imagery and spatial visualization, for example).

Simple modeling and making illustrations start at proficiency Level 2 which progresses up to Level 6, yet the average Filipino high school student can barely reach Level 1 – the same problem shared by 46 out of 78 OECD countries that were significantly below OECD average [22]. This is indeed something that must be reflected on deeply with efforts exerted fully and actions rigorously taken. As a local response to the findings of 2018 PISA, including the findings of 2019 Trends in International Mathematics and Science Study (TIMSS) regarding Philippines' underperformance [23], the Department of Education (DepEd) launched Sulong Edukalidad so as to address the challenge to change the way learners are taught as well as to produce new breed of learners who are globally competitive by globalizing the quality of basic education [10].

From a similar point of view, this study desired to contribute to the endeavor by assessing the level of mathematical literacy starting here in Batangas, in hopes of contributing to its nourishment, thus conducting this study aiming to improve the performance of pre-service mathematics teachers in Geometry. In light to the said aim, a learning enhancement material was prepared to further develop the visual imagery and spatial visualization skills of pre-service mathematics teachers for better performance in said area. The study believes that by enhancing the visual imagery and spatial visualization skills of the pre-service mathematics teachers, it would help them greatly in giving answer to many mathematical concerns and compensate for the skills they lack in Geometry. These pre-service mathematics teachers will soon become secondary or tertiary teachers. The benefits that may be taken from this study may be shared to their future students and bring forth ripples. This work hopes to help Filipinos' mathematical literacy reach greater heights and add to the body of knowledge that strengthens mathematics curriculum both locally and internationally.

2. Methods

2.1. Objectives

This study aimed to relate the visualization skills of pre-service mathematics teachers to their academic performance in Geometry. It specifically aimed to:

1) Describe the academic performance of pre-service mathematics teachers in Plane and Solid Geometry.
2) Determine the level of skills of the respondents in:
a) visual imagery, and;
b) spatial visualization.
3) Find out the relationship between the academic performances of the respondents and their levels of visual imagery and spatial visualization skills;
4) Ascertain the difficulties encountered by the respondents in visual imagery and spatial visualization in relation to solving problems in Plane and Solid Geometry, and;
5) Prepare a learning enhancement material in Geometry.

2.2. Hypothesis

The study was premised on the null hypothesis that there is no significant relationship between the pre-service mathematics teachers' performance in Plane and Solid Geometry and their levels of visual imagery and spatial visualization skills.

2.3. Research design

This study used a descriptive-correlational design of research with questionnaire as the instrument for gathering data. According to Aprecia (2022), descriptive-correlational research describes the variables of the study and the extent of relationships existing between them [1]. As such, the study relied on the response of participants through a written survey or questionnaire. Both quantitative approaches were used in treating the gathered data. Borgstede and Scholz (2021) shared that quantitative approaches are utilized to interpret quantities in order to generalize a given set of data [3].

2.4. Subjects of the study

The respondents consisted of 147 out of 235 pre-service mathematics teachers from selected higher education institutions (HEIs) in Batangas. The sample size was determined at a 95% confidence interval with a 5% margin of error using Raosoft. Stratified random sampling was employed to select samples from the targeted college and universities. The breakdown of the respondents is shown in Table 1.

Table 1: Distribution of Respondents

College/University	Pre-Service Mathematics Teachers	Samples per School
A	140	88
B	4	3
C	14	9
D	53	33
E	24	15
TOTAL	235	147

2.5. Data gathering instruments

A researcher-made questionnaire and two standardized questionnaires were utilized as the main data gathering instruments of the study. The standardized questionnaires used were the Vividness of Visual Imagery Questionnaire (VVIQ) and the Santa Barbara Solids Test (SBST). Each of the questionnaires aimed to quantify the responses concerning visual imagery, spatial visualization, and difficulties encountered in solving problems in Geometry.

Vividness of Visual Imagery Questionnaire (VVIQ). The Vividness of Visual Imagery Questionnaire (VVIQ) is a standardized questionnaire developed by David Marks in 1973 to determine individuals' skills in visual imagery by asking them how vivid the images they can generate based on indicated themes [25]. The responses are represented by a five-point scale. Items are grouped into four (4) themes having four (4) items each. Scenarios vary per theme with each having the same response scales and interpretations. The possible responses range from 1 to 5 holding the following verbal interpretations:

Option	Verbal Interpretation
5	Perfectly realistic, as vivid as real seeing
4	Realistic and reasonably vivid
3	Moderately realistic and vivid
2	Dim and vague image
1	No image at all, I only "know" I am thinking of the object

The Vividness of Visual Imagery Questionnaire was utilized upon determining that the questionnaire is open for anyone to use. Moreover, the reliability of the questionnaire was identified to determine whether the study should proceed to using it or not. Nunally (1978), as cited by Campos et al. (2002), recommends its use as it possesses a high internal consistency (Cronbach $\alpha = 0.88$) [4]. The method of Zeng et al. (2022) was adopted in the scoring of the responses in the VVIQ. The responses were summed and categorized as follows [44]:

Score	Verbal Interpretation
VVIQ = 80	Hyperphantasic
62 < VVIQ < 80	High Visualizer
59 < VVIQ ≤ 62	Medium Visualizer
16 < VVIQ ≤ 59	Low Visualizer
VVIQ = 16	Aphantasic

Santa Barbara Solids Test (SBST). SBST is a 30-item, multiple-choice, standardized test developed by Cheryl Cohen and Mary Hegarty. It measures an individual's spatial visualization skill through determining the cross section of solids when cut by a plane (Cohen and Hegarty, 2012) [6].

Figure 1 presents sample solids in the Santa Barbara Solids Test that are cut by a plane. The solids come as single, attached, and nested that were cut vertically, horizontally, or obliquely. The respondents are then required to determine their cross-sections through spatial visualization.

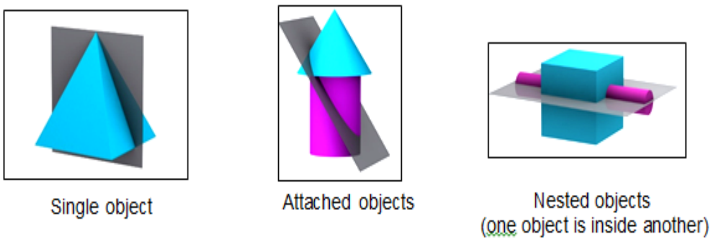

Fig. 1: Solids in Santa Barbara Solids Test.

Source: Cohen, C. A. & Hegarty, M. (2012). Inferring cross sections of 3D objects: A new spatial thinking test. Learning and Individual Differences, 22(6), 868-874.

After examining the solid, respondents select one from the four (4) options which corresponds to the correct cross-section as shown in Figure 2. The test therefore challenges respondents to spatially manipulate three-dimensional objects to gain a two-dimensional image of the cross-section. This gave a very good basis in describing the level of spatial visualization skills of the pre-service mathematics teachers.

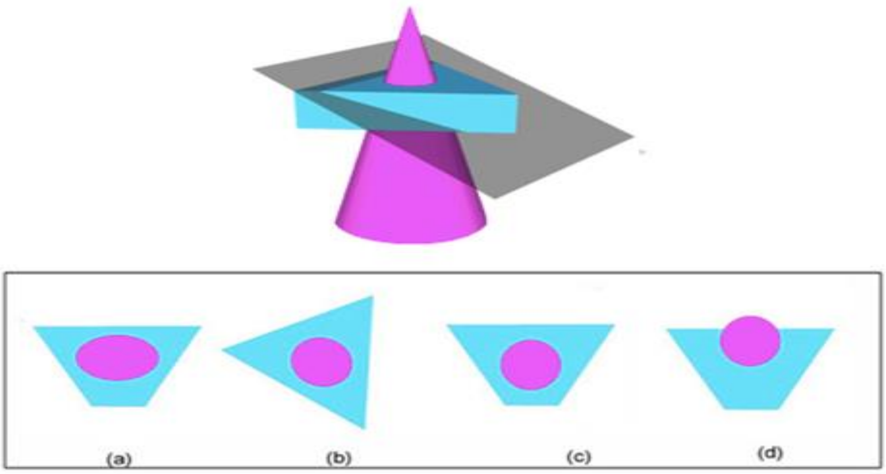

Fig. 2: Sample Item in Santa Barbara Solids Test.

Source: Cohen, C. A. & Hegarty, M. (2012). Inferring cross sections of 3D objects: A new spatial thinking test. Learning and Individual Differences, 22(6), 868-874.

Prior to administration, a letter was written to the authors of the standardized test asking permission if it may be utilized by the study for its data gathering procedure. Permission was granted upon request. The reliability of the test was also identified to determine whether the study should proceed to using it or not. The test claims to possess a Cronbach alpha of α = 0.91 (Cohen and Hegarty, 2012) [6].

The scores of the respondents in the SBST were directly recorded and normed using Median Split method, supported by the authors of the test. Median split is a widely used method of expressing continuous scores into categories. In the study of Downing et al. (2005), they used median split to categorize the spatial visualization abilities (SVA) of the participants. Participants' scores above the median were labeled as High SVA, while those below the median, Low SVA [12]. In a similar manner, the median of all possible SBST scores (from 0 to 30) was determined (Median = 15). All gathered SBST scores were categorized using the Median as reference and were interpreted as follows:

Score	Verbal Interpretation
Above 15	High
15 and below	Low

Researcher-made Questionnaire. Part of the questionnaire also included the difficulties encountered by the respondents in solving problems in Geometry related to visual imagery and spatial visualization. This questionnaire also aimed to determine the academic performance of the respondents in Geometry through their grades in Plane and Solid Geometry.

Prior to the actual administration of the questionnaire, a pilot test was conducted. Cronbach Alpha was utilized which resulted to an internal consistency of 0.817. After the administration and retrieval of questionnaires, the results were organized and treated statistically. The grades of the respondents in Plane and Solid Geometry ranged from 3.00 to 1.00. They were categorized and interpreted as follows:

Grade	Verbal Interpretation
1.00	Excellent
1.25	Superior
1.50	Very Good
1.75	Good
2.00	Meritorious
2.25	Very Satisfactory
2.50	Satisfactory
2.75	Fairly Satisfactory
3.00	Passing

Meanwhile, the indicators for the difficulties encountered by respondents in solving problems in Geometry were itemized and answered in a range of 1 to 4, with 1 being the lowest and 4 being the highest. The data were interpreted in terms of the criteria based on the following scale continuum:

Option	Scale Range	Verbal Interpretation
4	3.50 – 4.00	Very Often
3	2.50 – 3.49	Often
2	1.50 – 2.49	Rarely
1	1.00 – 1.49	Very Rare

3. Results and Discussion

3.1. Academic performance of students in plane and solid geometry

The grades obtained by the pre-service mathematics teachers in Plane and Solid Geometry were used in this study to assess their performance. Table 2 presents the distribution of academic performances of pre-service mathematics teachers in Plane and Solid Geometry, expressed in terms of their grades. These results were utilized to characterize their performance in Geometry.

Table 2: Academic Performances of the Respondents in Plane and Solid Geometry

Level	f	%
Excellent (1.00)	1	0.7
Superior (1.25)	16	10.9
Very Good (1.50)	22	15
Good (1.75)	44	29.9
Meritorious (2.00)	32	21.8
Very Satisfactory (2.25)	19	12.9
Satisfactory (2.50)	8	5.4
Fairly Satisfactory (2.75)	4	2.7
Passing (3.00)	1	0.7
Total	147	100

Mean = 1.85 (Good); SD = 0.382; Min = 3.00 (Passing); Max = 1.00 (Excellent).

Among 147 pre-service mathematics teachers surveyed, the majority (44, 29.9%) achieved a good grade (1.75) in Plane and Solid Geometry. This was followed by 32 respondents (21.8%) whose grades were deemed meritorious (2.00). Additionally, 22 respondents (15%) obtained a very good grade of 1.50 while 19 of them (12.9%) received a very satisfactory grade 2.25. Some pre-service mathematics teachers attained a superior grade of 1.25 which comprised 10.9% of the respondents surveyed (16 respondents). Furthermore, 8 respondents (5.4%) reported having a satisfactory grade of 2.50 in the subject, while 4 confirmed to have a fairly satisfactory grade of 2.75 comprising 2.7% of those surveyed. The remaining two pre-service mathematics teachers (1, 0.7%) obtained excellent and passing grades of 1.00 and 3.00, respectively. Notably, these grades represent the highest and lowest scores, respectively (Min = 3.00, Max = 1.00).

The numbers indicate that the academic performances of the surveyed pre-service mathematics teachers in Plane and Solid Geometry tend to cluster around a Good grade (closer to 1.75 interpreted as Good), with an average of 1.85 (SD = 0.382), as evidenced by their descriptive results. These findings suggest that, on average, the respondents demonstrate a good level of performance in Geometry based on their grades in Plane and Solid Geometry. However, as Patkin and Sarafty (2012) have argued, there is potential for promoting geometric thinking. Despite the grades being considered good, there remains ample opportunity for improvement [32].

3.2. Level of visualization skills of the respondents

Visual Imagery. The levels of visual imagery were assessed based on the quality of images formed. Individuals are classified as aphantasic if they have zero capacity to form mental images, while, hyperphantasics possess the ability to visualize and simulate scenarios in the mind akin to real life. Although aphantasics and hyperphantasics are rare, the latter can be found with a higher percentage of possibility (Zeng et al., 2022) [44]. Table 3 shows the distribution of the pre-service Mathematics teachers according to their level of visual imagery.

Table 3: Visual Imagery Skills of the Respondents

Level	f	%
Hyperphantasic (VVIQ = 80)	0	0
High Visualizer (62 < VVIQ < 80)	71	48.3
Medium Visualizer (59 < VVIQ ≤ 62)	15	10.2
Low Visualizer (16 < VVIQ ≤ 59)	61	41.5
Aphantasic (VVIQ = 16)	0	0
Total	147	100

Mean = 59.61 (Medium Visualizer); SD = 10.484; Min = 29 (Low Visualizer); Max = 76 (High Visualizer).

Out of the 147 pre-service mathematics teachers surveyed, 71 were identified as high visualizers (62 < VVIQ < 80), comprising the majority (48.3%) of the respondents. Low visualizers (16 < VVIQ ≤ 59) constituted the second highest group, with a percentage of 41.5%, closely following the high visualizers, totalling 61 respondents. Medium visualizers (59 < VVIQ ≤ 62) comprised the smallest portion of the distribution, with 15 respondents (10.2%). Interestingly, no respondents were classified as aphantasic (VVIQ = 16) or hyperphantasic (VVIQ = 80). The lowest VVIQ score recorded was 29 (Low Visualizer), while the highest was 76 (High Visualizer).

These findings suggest that the levels of visual imagery among 147 pre-service mathematics teachers surveyed were distributed, with the average person being classified as a medium visualizer (Mean = 59.61[Medium Visualizer], SD = 10.484, Min = 29 [Low Visualizer], Max = 76 [High Visualizer]). Although the majority of the respondents were identified as high visualizers (62 < VVIQ < 80), a significant proportion also fell into the low visualizers category (16 < VVIQ < 59), indicating a potential need for enhancement. This aligns with the perspectives of Burmark (as cited by Schaffhauser, 2012) and Goldstein (2011) who advocate for enhancing visual imagery as a form of literacy [36] [16], providing a basis for intervention aimed at improving the visual imagery of pre-service mathematics teachers in Batangas.

Spatial Visualization. Another skill deemed contributory to performance in Geometry is spatial visualization. Omar et al. (2019) defined it as the ability to apprehend, encode, and mentally manipulate three-dimensional spatial forms [29]. It is believed that proficiency in this skill may lead to better performance in Geometry. Table 4 presents the distribution of the respondents according to their levels of spatial visualization skills.

Table 4: Spatial Visualization Skills of the Respondents

Level	f	%
High (above 15)	33	22.4
Low (15 and below)	114	77.6
Total	147	100

Mean = 10.83 (Low); SD = 5.239; Min = 0 (Low); Max = 28 (High).

Among 147 pre-service mathematics teachers surveyed, majority were found to have a low level of spatial visualization (114, 77.6%) with scores below the median of 15. This encompassed a significant portion of the respondents. Conversely, 33 respondents (22.4%) were found to have high level of spatial visualization.

These findings indicate that the levels of spatial visualization skills among 147 pre-service mathematics teachers surveyed were widely distributed. On average, respondents possess low level of spatial visualization skill (Mean = 10.83 [Low], SD = 5.239, Min = 0 [Low], Max = 28 [High]). These analytical results provide insights into the need to enhance spatial visualization skills. Such enhancement has been demonstrated as feasible by Kurtulus and Uygan (2010) who utilize Sketch Up-based Geometry activities to improve spatial visualization [20].

Many pre-service mathematics teachers were found to struggle with visual imagery and spatial visualization, which may have affected their performance in Geometry. These struggles may stem from a lack of attention and intervention focusing on these skills. A number of studies attest to this problem stating that there is limited recognition given to spatial visualization as proven by the limited number of studies conducted, exploring the potentials of the said skill to improve in geometry-related areas (Putri et al., 2025) [33]. Therefore, it is deemed appropriate to develop a learning enhancement material aimed at helping them improve their performance in Geometry, particularly concerning these skills.

3.3. Relationships between the academic performances of the respondents and their level of visualization skills

The relationship between pre-service mathematics teachers' visualization skills and their academic performances in Plane and Solid Geometry was examined and presented in Table 5. The results showed that 16.1% of the variances ($R^2 = 0.161$) in the academic performances of the respondents in Plane and Solid Geometry can be explained by visual imagery and spatial visualization. Additionally, there was evidence of statistical significance indicating that the visualization skills of the respondents are associated with their level of performance in Plane and Solid Geometry (F stat = 13.827; $p < 0.01$).

Table 5: Relationship between Visualization Skills of Pre-service Mathematics Teachers and their Academic Performance in Plane and Solid Geometry

Visualization Skills	Unstandardized Coefficients		Standardized Coefficients	t
	B	Std. Error	Beta	
(Constant)	2.628	.172		15.311
Visual Imagery	-.010	.003	-.263	-3.398*
Spatial Visualization	-.019	.006	-.263	-3.393*

Dependent Variable: Academic Performance in Plane and Solid Geometry.
R^2: 0.161; F stat: 13.827**.
*$p < 0.05$; **$p < 0.01$.

The relationship between visual imagery and performance in Plane and Solid Geometry also showed evidence of statistical significance (t = -3.398; $p < 0.05$). This indicates that as the level of skill in visual imagery increases, the level of performance in Plane and Solid Geometry improves as well (B = -0.010). In the schools targeted, the lower numbers correspond to higher grades, which contributed to a negative B coefficient (B = -0.010). This technically means that as one value increases, the other decreases, and vice-versa.

Due to the nature of the grading system among the targeted colleges and universities, the study interpreted this as meaning that as the level of visual imagery increased, the grades also improved. These findings suggest that if the pre-service mathematics teachers work on enhancing this skill, they can expect better performance in Geometry. This aligns with the findings of Fastame (2021) [14], which showed that individuals with high visual imagery abilities performed better in Geometry compared to those with lower abilities.

The relationship of spatial visualization skill and performance in Plane and Solid Geometry also showed evidence of statistical significance (t = -3.393; $p < 0.05$). This similarly indicates that as the level of skill in spatial visualization increases, the level of performance in Plane and Solid Geometry improves (B = -0.019). Again, since lower numbers correspond to higher grades, this contributed to a negative B coefficient (B = -0.019), meaning that as one value increases, the other decreases, and vice-versa.

In the findings, the interpretation that the grades improved as the level of spatial visualization increased, was also used. This was considering the nature of the governing grading system among the targeted schools. This suggests that if the pre-service mathematics teachers would want to improve their performance in Plane and Solid Geometry, it would be of great help if their spatial visualization skill would be enhanced. In a similar view, van Garderen (2006) stated that students with high spatial visualization have better access to using schematic imagery which is a more refined way of solving mathematical word problems such as in Geometry [41].

The levels of performances of the pre-service mathematics teachers in Plane and Solid Geometry were found to be significantly predicted by their levels of skills in visual imagery and spatial visualization. This established the stand of the study regarding the role the indicated variables play towards better performance in Geometry. It called for an enhancement material focusing on the featured variables that could possibly help them improve their skills in solving various problems in Geometry. It was then determined that the study must consider the results of these relationships in the process of preparing the material.

3.4. Difficulties encountered in solving problems in Geometry related to visual imagery and spatial visualization

It was found that visual imagery and spatial visualization skills are correlated with the level at which a pre-service mathematics teacher can perform in Geometry. Therefore, these skills must be enhanced if they are found to be low. The varying levels among those surveyed indicated that they either succeeded or struggled with the use of their visualization skills in different ways. There is more to explore regarding these skills, especially the challenges encountered by the respondents, as the majority represented those who struggled. Table 6 presents the difficulties encountered by the pre-service mathematics teachers in solving Geometry problems.

Table 6: Difficulties Encountered in Solving Problems in Geometry

Items	WM	SD	VI
1. When solving, there is difficulty in solid mensuration whenever the problem involves hidden lines.	2.99	0.707	O
2. When solving, there is difficulty in relating parts of a solid and derive concepts through spatial manipulation.	2.94	0.654	O
3. When solving, there is difficulty determining which learned geometric concepts are applicable to the problem.	2.91	0.682	O
4. When solving, Geometry proving is difficult whenever image construction is required.	2.89	0.713	O
5. When solving, there is difficulty associating definitions, properties, theorems, and postulates that are necessary in Geometry proving.	2.89	0.713	O
6. When solving, there is difficulty in generating appropriate solutions relative to a given illustration for the problem.	2.88	0.614	O
7. When solving, there is difficulty in visualizing the cross section of a solid essential to solving the problem.	2.88	0.661	O
8. When solving, there is difficulty in translating the word problem to an illustration.	2.86	0.669	O
9. When solving, the lack of understanding of related geometric terms shows.	2.82	0.712	O
10. When solving, the illustration does not fit the figure described in the problem, leading to an incorrect answer.	2.78	0.763	O
Overall	2.88	0.689	O

Legend: O – Often.

It was found among 147 pre-service mathematics teachers surveyed that most of the pre-service mathematics teachers confirmed to often find difficulty in solid mensuration whenever the problem involved hidden lines (WM = 2.99, SD = 0.707). Also, they were found to often encounter difficulties in solving Geometry problems whenever they relate parts of a solid and derive concepts through spatial manipulation which they often encountered (WM = 2.94, SD = 0.654).

These perspectives regarding the difficulty level of Solid Geometry is understandable since it is introduced in the last part of Plane and Solid Geometry, thus requiring the accumulation of concepts in Plane Geometry introduce in the first part of the course. To be able to see the hidden lines of a solid, relate its parts, and derive concepts essential to the problem, one may try rotating the figure mentally to unlock a solution. This problem then entails good spatial manipulation skill. Metzger (2019) put it that lack of exposure to challenges requiring spatial reasoning causes learners to doubt their own Mathematical aptitude. As a mathematical area, Geometry can be made more visual with activities that emphasize spatial reasoning [26].

At a weighted mean of 2.91 relative to the previous ones, the respondents were ascertained to often find difficulty in determining which learned geometric concepts are applicable to the problem (SD = 0.682). It was found to be easier for someone to accomplish this if one has mastery of the said concepts and what they are. Geometric concepts come with definitions and properties that must be made familiar with anyone solving a problem in Geometry. Doing so provides mental images that help understand the language of the problem.

Having this said, it will also be helpful if one has a good understanding of the context of the problem which can sharpen the mental images that may be formed. This difficulty encountered by respondents is calling for good visualization skills partnered with good linguistic skills. This perspective found rest on the study of Erdogan (2017) which implied that being good with these skills would require one to be accustomed to them and the words associated with them. Words in Geometry that pre-service mathematics teachers are familiar with dictate how well they could translate word problems into representations [13].

Meanwhile, many respondents verified that they often find Geometry proving difficult whenever image construction is required (WM = 2.89, SD = 0.713). In the same way, this may be mended by enhancing one's visual imagery. Image construction here entails doing as described in the problem similar to following instructions to every word. This may be applied as well to how a number of the respondents often find difficulty associating definitions, properties, theorems, and postulates that are necessary in Geometry proving (WM = 2.89, SD = 0.713). This shows how important it is to have a relatively big word bank for geometric concepts. In connection, Bombio & Del Rosario (2022) recommended that educators may expose their students to a wide variety of reading materials in enhancing geometric proof skills [2].

Many of the respondents were also found to often encounter difficulty in generating appropriate solutions relative to a given illustration for the problem (WM = 2.88, SD = 0.614). These illustrations contain problem-related components that can be incorporated with one another or can be used to derive relationships that can help lead to the correct answer. Failure to understand this may have led the pre-service mathematics teachers to fail to relate an illustration to the context of a problem leading to their difficulty in generating appropriate solutions. If their Geometry word bank serves them well, together with a good linguistic competence, they can match the components of an illustration to what the problem is looking for. With this in mind, the analytical framework developed by Dejoras & Vistro-Uy (2022) regarding definitions from a number of mathematical terms can be a great help [11].

Meanwhile, it was made apparent that a big portion of the respondents often find difficulty in a similar level in visualizing the cross section of a solid essential to solving the problem (WM = 2.88). Though still close to one another, the responses were found to be more distributed (SD = 0.661) compared to the previous. A lot of word problems in Solid Geometry require information that can be taken from dissecting a solid such as area or dimensions of its cross section when sliced in any particular manner. This calls for a good spatial visualization skill. Being good with this can save pre-service mathematics teachers from this difficulty by freely manipulating the solid mentally. With this, the study may find itself supporting Olkun et al. (2017), Khine (2017), Görselleştirme (2017), and Patkin & Plaskin (2019) regarding the need to highlight spatial visualization in the mathematics curriculum [28] [18] [17] [31].

Majority of the respondents were also found to often have difficulty in translating the word problem to an illustration (WM = 2.86, SD = 0.669). Some respondents even testified that they get intimidated whenever the problem is very long to read. To be able to turn a Geometry-related term to an illustration, one must know what it is. This includes what it looks like, its features, and its properties. This greatly helps in understanding the language of the problem thus giving a clearer image of what the problem is pertaining to. Such difficulty may also be mended with a good visual imagery skill together with good linguistic competence. The respondents of Patac et al. (2022) experienced a similar dilemma. As such, they failed to explicitly symbolize theorems that are conveyed in words. Therefore, they stressed that mastering the required representation registers can assist students in posing problems [30].

Similarly, most of the pre-service mathematics teachers surveyed confirmed that whenever they solve problems in Geometry, the lack of understanding of related geometric terms often shows (WM = 2.82, SD = 0.712). This greatly implied that they lacked familiarity of the terms which may have led to a lot of problems. Moreover, whenever the respondents try to solve problems that require geometric construction, the illustration does not fit the figure described, leading to an incorrect answer (WM = 2.78, SD = 0.763). These findings also spoke about the importance of one's registry of geometric terms and concepts. Geometric construction requires following descriptions of geometric figures word per word.

Lack of familiarity of geometric terms and failure to understand fully how the figure was described can lead to varying interpretations of the problem which can give incorrect answers. Some pre-service mathematics teachers even felt anxious whenever they miss small details crucial to the problem. This can be mended by taking the suggestion of Vukovic & Lesaux (2013) about making mathematical concepts and representations relatable to all types of learners [43]. Casilla (2021) supported this by suggesting the use of native tongue so as to provide better and more specific examples for various concepts [5].

Most of the pre-service mathematics teachers encountered problems in solving problems in Geometry related to their visual imagery and spatial visualization skills (Overall WM = 2.88). This was reinforced by how near the responses are to each other (Overall SD = 0.689) which indicated that they often encountered the abovementioned problems. This showed the need to enhance the said skills. It also appeared that majority of the respondents found more difficulty in Solid Geometry compared to Plane Geometry. It would seem the former required more visualization skills than the latter, contributing to the recorded perspectives of the respondents.

3.5. Prepared learning enhancement material

The Visual Enhancers Toolkit, a learning enhancement material, was founded on Gardner's Theory of Multiple Intelligences, Paivio's Dual Coding Theory, Kosslyn's Quasi-pictorial Theory, and theory of Enactivism. Gardner's theory supported what pre-service mathematics teachers can gain in their performance in Geometry if their visual imagery and spatial visualization skills are enhanced. In constructing the material, it was ensured pre-service mathematics teachers are familiar with defined terms essential to the topic. It was made sure that the images used to represent essential concepts are vividly exemplified to ensure better retention of the problem solving process, according to Paivio's and Kosslyn's claims [34] [21]. To reinforce the skills, the exercises featured examples and problems that lead pre-service mathematics teachers to refer to their surroundings, based on the theory of Enactivism [39].

Gardner's theory of multiple intelligences, specific to visual-spatial intelligence, concerns imagination, conceptualization, and problem solving (Kurniati et al., 2017) [19]. Individuals strong in this suit are likely to excel in Geometry which is a visual area in mathematics. As one of the multiple intelligences, everyone is safely assumed to possess it, giving the researchers the assurance that the subjects of the study who are weak in the said aspect could be enhanced.

Considering that Geometry is an area consisting of abstract concepts and ideas often presented as written words, Paivio's dual coding theory perfectly fits the study since these words are to be transformed into figures [34]. In a shared work of Sadoski et al. (2012), dual coding is defined as the association of words and images (codes) through representations which affect concept retention [35]. In theory, words evoke images that initiate recall of stored concepts. These images depend on what has been learned thus far (Thomas, 2014) [40]. Geometry problems are required to be detailed and specific to avoid misleading and misrepresentation since proper and correct geometric construction is crucial to get the correct answer. This showcases how important it is to have a bank of geometric concepts and figures (images). The study of Mir et al. (2023) on examining the impact of Dual Coding to enhance students' retention was found to be an inspiration as it showed that incorporating the theory to learning interventions positively influences retention and vocabulary. Therefore, if a learning enhancement material in Geometry is to be prepared, it must ensure that it helps struggling pre-service mathematics teachers to add more concepts and images in their memory needed in solving geometric problems. Words must be properly selected and at the same time, description and instructions must be specific and detailed assuring problem solvers that they have a fair chance of answering the problem. Dual coding theory is also a constructivist theory [34] leading the learning enhancement material to be learner-centered.

Hand-in-hand with Paivio's dual coding theory, Stephen Kosslyn's quasi-pictorial theory suggests some guide on what must be done if the targets for enhancement are visual imagery and spatial visualization. Kosslyn lays how deep representations play a crucial role in solving visually and spatially connected problems such as in geometry (Thomas, 2014) [40]. If Paivio points how words are associated with image generation which hints how word problems must be constructed, Kosslyn's points give way to what can be done to prime problem solvers in visualizing effectively leading to their improved approach in solving word problems in Geometry. Exercises and activities enhancing visual imagery and spatial visualization are therefore encouraged which can be done by using descriptions and depictions which make information explicit and accessible [21]. This suggests that the more vivid the descriptions, the better the retention of the images in the long-term memory which may later be extracted once needed.

It is also important to consider how the environment may be associated in creating activities and exercises targeting cognitive skills such as visual imagery and spatial visualization, as suggested by enactivism [38]. Sima (2011) noted how greatly the environment influences problem solvers to better visual imagery and spatial visualization [38]. It is an entirely large warehouse of images that could be replicated as one solves a problem. The NCTM regards how crucial representation and modeling are in Geometry [27]. The environment provides the vision numbers of experiences that could be stored in the individual's schemata that can be used in solving various problems in Geometry. The theory therefore suggests that relating activities to real-world scenarios and actual experiences help enrich visualization. Sutiarso (2023) noted that using REACT (Relating, Experiencing, Applying, Cooperating, Transferring) learning strategy to develop mathematical representation abilities [39]. It can be coupled by social interaction and reflective processes while using a material that is exciting and easy to understand [39].

Considering the aforementioned insights and findings of this study, the Visual Enhancers Toolkit was materialized and ensured to enhance the visualization skills of pre-service mathematics teachers so as to improve their performance in Geometry. It was divided into components arranged in the following manner: Visual Exercises, Problem Solving, and Points of Reflection.

Visual Exercises presents activities that encourage learners to use their visualization skills repeatedly to make the sensation more familiar to them. Moving images (rotating, showing all views) are presented through QR codes for self-checking so that students can compare the images and simulations they form mentally to the ones provided in the material. Meanwhile, Problem Solving provides problems with suggested strategies that fully utilize visualization skills. Key concepts go together with corresponding illustrations to deliver vivid images essential to problem solving in Geometry. Lastly, Points of Reflection provides key points to allow learners to concretize all processes undergone in all phases of the material.

The material shows sample topics in the course Plane and Solid Geometry such as Triangle Congruence and Surfaces and Solids, where visual imagery and spatial visualization are very much needed. Each topic presents sample problems with tips on how students can properly utilize visuals – illustrated or non-illustrated. All these were considered in view of helping pre-service mathematics teachers to better see the ideas presented by various Geometry problems rather than just knowing routine procedures and computation processes. This material also presents the engaging part of Geometry problem solving where implicit relationships among given information are revealed and made clearer through pre-made or self-constructed illustrations. Utilizing Visual Enhancers Toolkit, therefore, may suggest a new approach to teach Geometry-related subjects through reinforcement of visualization skills.

4. Conclusion

In light to the foregoing findings, it was concluded that the pre-service mathematics teachers have a good level of academic performance in Plane and Solid Geometry. They are also medium visualizers with low spatial visualization skills. Moreover, higher level of visualization skills significantly corresponds to higher academic performance in Plane and Solid Geometry. The pre-service mathematics teachers also find solving Geometry problems difficult in terms of interpreting geometric figures and visualizing three-dimensional objects. Lastly, the Visuals Enhancers Toolkit was prepared as a learning enhancement material to help enhance the visualization skills of pre-service mathematics teachers.

5. Recommendations

The Visual Enhancers Toolkit may be subjected to evaluation and have it utilized by pre-service teachers to strengthen their visualization skills so as to enhance their academic performance in Geometry. The mathematics teacher education curriculum may also highlight students' visualization skills in Geometry classes. This may be complemented with the utilization of the toolkit once validated.
Further, though it was found that some of the variations in academic performance in geometry are due to visual imagery and spatial visualization skills, others may still be due to other factors which may be explored in future studies. A similar study may also be conducted focusing on non-Euclidean geometry or other fields of mathematics where visualization skills can be utilized.

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