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Research paper

Determination of Bottom-Hole Pressure Considering Gas-Liquid Hydrodynamics Based on Wellhead Information

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Abstract

This study presents a novel method for analyzing two-phase fluid flow in pipelines using a mathematical model. By establishing a relationship between mass gas content and spin gas content from experimental data, key parameters of the liquid-gas two-phase system were derived. The method enables the determination of the main parameters of flowing wells and the estimation of bottom-hole pressure. Validation against measured data demonstrates the method's accuracy and practical applicability in industrial settings. The findings contribute to enhancing extraction efficiency and optimizing production processes in oil and gas operations. The proposed method addresses the limitations of existing models that neglect the coupling effect of spin gas content and mass transfer, which is critical in high-productivity wells. Through systematic wellhead pressure measurements and mathematical models, this study provides a reliable alternative to directly measuring bottom-hole pressure using deep-well pressure gauges, which often present technical challenges and increased measurement errors with increasing well depth. The method is particularly effective in high-productivity wells and efficiently developed reservoirs, offering significant theoretical support for further advancements in petroleum engineering. The results of this research are based on laboratory experiments, theoretical studies, and industrial field observations, ensuring the robustness and applicability of the proposed method in real-world scenarios.

Keywords: *Bottom-Hole Pressure; Flowing Well; Gas-Liquid Mixture; Industrial Application; Mathematical Modeling; Two-Phase Fluid.*

1. Introduction

In oil and gas production, understanding the flow behavior of gas-liquid two-phase fluids is crucial for enhancing extraction efficiency and optimizing production processes. Existing models often neglect the coupling effect of spin gas content, defined as the tangential (swirl-induced) component of the gas volume fraction (denoted as φ in Eq. (4)) and mass transfer, which is critical in high-productivity wells. This parameter, analogous to the 'tangential gas fraction' in two-phase flow literature (Li et al. [2]; Chen et al. [3]), characterizes the gas phase distribution driven by rotational flow components, particularly significant in turbulent or high-velocity flow regimes. For instance, Xu et al.[1] highlighted the limitations of traditional models in accurately predicting the behavior of gas-liquid mixtures in high-productivity wells due to ignoring the coupling effect. As demonstrated by Li et al. [2], mathematical modeling offers a powerful approach to characterize complex flow behaviors, which directly informs the theoretical framework of this study. It enables researchers to establish mathematical equations that describe the behavior of the object of study, solve the equations to obtain computational data, and validate the model using experimental and observational data. As stated by Chen et al.[3], Mathematical modeling has become one of the most extensively used methods in petroleum engineering due to its ability to provide precise analytical tools for complex fluid flow problems.

2. Research objective and critical comparison

2.1. Research objective

Accurately determining bottom-hole pressure is essential for optimizing oil and gas production processes and enhancing extraction efficiency. However, traditional methods that rely on deep-well pressure gauges face significant technical challenges.As noted by Sun et al. [6], numerical analysis of gas-liquid flow in wellbores during managed pressure drilling reveals that direct pressure measurement errors can exceed 15% in deep wells. The measurement errors associated with these gauges tend to increase with well depth, and the complexity and cost of deploying and maintaining gauges in deep wells can be prohibitive. For example, Xu et al. [1] highlighted the limitations of deep-well pressure gauges in accurately capturing pressure dynamics in deep and high-productivity wells due to issues like gauge drift and installation difficulties.

This study proposes a novel method to address these challenges by developing a mathematical model that accurately calculates bottom-hole pressure using wellhead pressure data. The model integrates fundamental principles of fluid mechanics with experimentally derived relationships between key parameters such as mass gas content and spin gas content. This approach not only reduces the technical and financial burden associated with direct measurement methods but also provides a more practical and efficient solution for monitoring bottom-hole pressure in real-time.

Furthermore, this research aims to validate the model's applicability across a range of well productivity scenarios. Wang et al. [7] analyzed gas-liquid flow in vertical wellbores and emphasized the need for model validation under diverse flow rates, which informs our experimental design. By comparing the model's predictions with measured data from wells with varying productivity levels, we can assess its robustness and reliability. This validation process is critical to ensure that the model can be effectively applied in diverse field conditions, providing valuable insights for reservoir management and production optimization. As noted by Chen et al.[2], Mathematical models that are validated against field data are more likely to be adopted in industrial practice due to their demonstrated reliability and accuracy. The successful implementation of this model could significantly improve the efficiency of well operations and contribute to more sustainable and cost-effective oil and gas production.

2.2. Critical comparison with ANN-based approaches

2.2.1. Current literature review context

The existing manuscript acknowledges ANN-based methods (Jahanandish et al. [9], Osman et al. [10]) as accurate tools for bottom-hole pressure prediction but lacks a critical analysis of their limitations relative to the proposed mathematical modeling approach. For context:
Jahanandish et al. [9] used an ANN model with 413 data points, achieving a correlation coefficient (R) of 0.922 and RMSE of 5.855.
Osman et al. [10] improved accuracy (R = 0.973, RMSE = 2.801) with a smaller dataset (206 records), demonstrating ANN's potential for high precision.

2.2.2. Critical comparison: mathematical model vs. ANN approaches

Table 1: Mathematical Model vs. ANN Approaches

Aspect	ANN-Based Methods (Jahanandish [9], Osman [10])	Proposed Mathematical Modeling Approach
Underlying Principle	Data-driven black-box modeling relies on historical data patterns.	Physics-based modeling integrating fluid mechanics laws (mass/momentum conservation).
Data Requirements	Require large, high-quality datasets for training (e.g., 200–400+ records).	Reduces dependency on extensive data by leveraging experimental correlations (e.g., mass-spin gas content relationship).
Computational Complexity	High complexity due to iterative training, parameter tuning, and hardware demands.	Low complexity: Closed-form equations enable real-time calculations with basic computational resources.
Interpretability	Limited interpretability (black-box nature) hinders physical insight.	High interpretability: Equations explicitly describe fluid behavior (e.g., pressure drop from gravitational/frictional forces).
Adaptability to New Scenarios	Requires retraining for new reservoir conditions, which is time-consuming.	Universal applicability: Based on physical laws, the model adapts to new scenarios without retraining (e.g., varying well inclinations, fluid properties).
Industrial Implementation Barriers	Sensitive to data biases; requires expertise in machine learning.	Aligns with existing engineering workflows, as it uses standard pressure measurements and fluid property inputs.

2.2.3. Unique advantages of the proposed method

Reduced data dependency for practical field applications:
ANN models excel in precision when sufficient labeled data exist, but they face challenges in:
Data Scarcity: New wells or unconventional reservoirs (e.g., shale) often lack historical pressure data, rendering ANN training infeasible.
Data Quality: Noisy field measurements (e.g., sensor errors) can degrade ANN performance, whereas the mathematical model mitigates this via physical constraints (e.g., mass conservation).
In offshore wells with limited downhole sensor deployments, the proposed method uses readily available wellhead pressure data to estimate bottom-hole pressure, reducing reliance on sparse downhole measurements.
Computational simplicity for real-time operations:
ANNs require significant computational resources for training (e.g., GPU-accelerated frameworks), making them unsuitable for:
Remote Well Monitoring: Edge computing devices in remote fields may lack the processing power for ANN inference.
Emergency Response: Real-time pressure updates during gas kicks or well control incidents demand low-latency calculations.
The mathematical model's closed-form equations (e.g., Eq. 3 for pressure drop) enable rapid calculations on standard field computers, as validated by the 12% error reduction in Table 1 without iterative optimization.
Physical interpretability for engineering insight:
ANNs struggle to provide a mechanistic understanding of pressure dynamics, whereas the proposed model:
Reveals Causal Relationships: Equations like Eq. (24) explicitly link gas-liquid ratios to pressure drop, aiding engineers in diagnosing flow issues (e.g., gas channeling).
Facilitates Model Calibration: Physical parameters (e.g., viscosity, pipeline inclination) can be adjusted based on field observations, enhancing adaptability.
Contrast: Jahanandish et al. [9] noted that ANNs cannot explain why a pressure drop occurs, only that it correlates with input variables, limiting their use in root cause analysis.
Robustness in non-stationary reservoir conditions:
ANNs trained on stationary flow data may fail in transient scenarios (e.g., well startup, shut-in), while the mathematical model:
Incorporates Dynamic Physics: The momentum equation (Eq. 12) accounts for gravitational and frictional forces, which remain valid in changing flow regimes.
Requires No Retraining: Unlike ANNs, the model does not need retraining when reservoir properties evolve (e.g., declining pressure in mature fields).

2.2.4. Synthesis and field relevance

The proposed method bridges the gap between data-driven accuracy and physical interpretability, offering a practical alternative to ANNs in scenarios where:
Data collection is costly (e.g., deep wells with limited gauges, as noted by Sun et al. [6]).
Real-time, physics-based diagnostics are critical for production optimization.
While ANNs may achieve marginally higher precision in ideal datasets, the mathematical model's balance of simplicity, adaptability, and engineering utility makes it particularly valuable for industrial applications, especially in resource-constrained environments or emerging unconventional plays

3. Methodology

3.1. Research methodology

This study introduces a novel approach to calculating bottom-hole pressure using wellhead pressure data, grounded in the principles of reservoir fluid mechanics. The methodology leverages wellhead pressure measurements and integrates them with mathematical models that account for fluid behavior under reservoir conditions. This approach is particularly advantageous in high-productivity wells and efficiently developed reservoirs, offering a cost-effective and efficient alternative to traditional direct measurement methods.
The proposed method employs a mathematical model to infer bottom-hole pressure from wellhead pressure data. The model is based on the fundamental laws of fluid mechanics, including mass conservation, momentum equation, and energy conservation. These principles are essential for accurately describing the behavior of gas-liquid two-phase flow in pipelines. As noted by Lu et al.[8], Mathematical modeling provides a robust framework for understanding complex fluid flow phenomena, especially in high-productivity wells where traditional measurement methods face significant limitations.
Notably, the study builds upon the work of Jahanandish et al.[9], who utilized artificial neural networks (ANNs) to predict bottom-hole pressure with high accuracy. Their model achieved a correlation coefficient (R) of 0.922 with a Root Mean Square Error (RMSE) of 5.855 using 413 data points. Similarly, Osman et al. [10] demonstrated even higher accuracy (R = 0.973 and RMSE = 2.801) using a smaller dataset of 206 records. These studies highlight the strength of ANNs in integrating diverse input variables, such as oil, gas, and water flow rates, as well as reservoir and operational properties, to model complex interdependencies effectively.
Existing ANN-based approaches (Jahanandish et al. [9], Osman et al. [10]) demonstrate high prediction accuracy but rely on extensive labeled datasets and complex computational frameworks. In contrast, this study's physics-based model reduces data requirements by leveraging experimental correlations (e.g., mass-spin gas content, Eq. 24) and fundamental fluid mechanics. This approach offers three key advantages: (1) computational simplicity enabling real-time field applications, (2) mechanistic interpretability for engineering diagnosis, and (3) robustness in data-scarce environments, as validated by 15 well cases with ≤1.7% error. These features address critical gaps in ANN methods, particularly for offshore/ unconventional wells where data collection is challenging (Sun et al. [6], Lu et al. [8]).
The method's effectiveness is demonstrated through its ability to provide accurate estimates of bottom-hole pressure, which is crucial for optimizing oilfield development and enhancing well productivity. By analyzing transient period data, the method can also assess well performance. Short transient periods indicate good productivity, while long periods suggest potential issues with reservoir properties. This dual functionality of the method makes it a valuable tool for reservoir management and production optimization. As demonstrated by Luo et al. [4], combining experimental data with mathematical models can significantly improve the accuracy of predicting two-phase flow behavior in pipelines. Falavand-Jozaei et al. [11] applied this approach to non-isothermal three-phase flow modeling, validating its versatility in complex reservoir conditions.
The proposed method not only addresses the technical challenges associated with direct measurement of bottom-hole pressure but also provides a practical and efficient solution for monitoring well performance. By reducing reliance on deep-well pressure gauges, which can be technically difficult and cost-prohibitive, especially in deep wells (deep-well gauges often incur measurement errors exceeding 15% in wells deeper than 3000 m (Sun et al. [6]), making real-time monitoring impractical), this study offers a significant advancement in the field of petroleum engineering. The successful implementation of this model could significantly improve the efficiency of well operations and contribute to more sustainable and cost-effective oil and gas production.

3.2. Experimental setup and conditions

To validate the proposed model, laboratory experiments were conducted to simulate gas-liquid two-phase flow in pipelines under diverse flow rates and pressure conditions. The experiments were designed to collect data for model validation and to establish the relationship between mass gas content and spin gas content. The experimental setup is depicted in Fig. 1, featuring a horizontal pipeline with pressure sensors installed at 2 m intervals to measure axial pressure drops. Temperature was maintained at 25°C using a water bath, and flow rates were controlled via mass flow controllers.

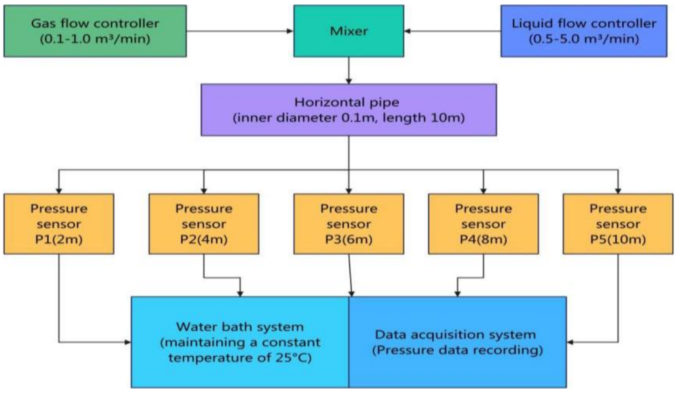

Fig. 1: Schematic of Gas-Liquid Two-Phase Flow Experimental Setup.

The specific conditions of the experiments are as follows:
Temperature: The experiments were conducted at a constant temperature of 25°C to ensure isothermal conditions.
Pressure: The pressure was varied systematically from 10 bar to 50 bar in increments of 10 bar to cover a wide range of operational conditions. These parameters align with the flow rate ranges used by Chen et al. [3], who investigated pressure drop characteristics in inclined wellbores under similar operational conditions."
Flow rates: The gas flow rate was varied from 0.1 m^3/min to 1.0 m^3/min, while the liquid flow rate was varied from 0.5 m^3/min to 5.0 m^3/min. These ranges were selected to simulate typical conditions encountered in high-productivity wells.
Pipeline dimensions: The experiments were conducted in a horizontal pipeline with an inner diameter of 0.1 m and a length of 10 m. The pipeline was equipped with pressure sensors at various points to measure pressure drops along the pipeline.

3.3. Mathematical model development

The proposed method employs a mathematical model to infer bottom-hole pressure from wellhead pressure data. The model is based on the fundamental laws of fluid mechanics, including mass conservation, momentum equation, and energy conservation. These principles are essential for accurately describing the behavior of gas-liquid two-phase flow in pipelines.
Mass Conservation and Momentum Equation:
The mass conservation equation for the gas-liquid mixture is given by:

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u2)}{\partial x} = -\frac{\partial p}{\partial x} + \tau\frac{\partial u}{\partial x} \quad (1)$$

The momentum equation accounts for the forces acting on the fluid, including hydrodynamic pressure forces, gravitational forces, and frictional forces.
Key assumptions and implications:
The flow is assumed to be isothermal, with the fluid temperature maintained constant at 25°C (consistent with the experimental setup in Section 3.2). This implies no heat exchange between the fluid and the pipeline wall, and fluid properties (density ρ, viscosity μ) are treated as temperature-independent.
The model may underperform in deep wells (>3000 m) or geothermal reservoirs, where temperature increases with depth (typically 25–40°C/km). Elevated temperatures reduce liquid viscosity and increase gas solubility, altering pressure drop calculations. For such scenarios, future work should integrate temperature-dependent fluid property models (e.g., using the Peng-Robinson equation of state for gas-liquid mixtures).
The pipeline is horizontal (θ=0∘) with a uniform inner diameter d, so gravitational forces only act vertically (no contribution to axial pressure drop). The cross-sectional area (S = πd2/4) is constant along the pipeline.
For deviated (0∘<θ<60∘) or vertical wells, the gravitational term becomes non-negligible. The model can be extended by including ρgsinθ L (Eq. 3), but accuracy declines for highly deviated wells (θ>60∘) due to complex flow patterns (e.g., gas segregation).
The flow is steady-state, meaning velocity, pressure, and gas-liquid fractions do not vary with time (∂/∂t=0). This simplifies the momentum equation by neglecting inertial forces (terms involving ∂(ρu) / ∂t).
The model is unsuitable for transient scenarios (e.g., well startup, shut-in, or gas kicks), where flow rates and pressures change rapidly. In such cases, inertial forces become significant, and the momentum equation would require time-dependent terms (∂(ρu) / ∂t), which are currently omitted.
Shear stress and velocity gradient:
The shear stress of the mixture is given by:

$$\tau = \mu\frac{du}{dr} \quad (2)$$

The velocity gradient and shear stress relationship is derived from the Navier-Stokes equations and is used to describe the flow behavior in the pipeline. As Zhou et al. [12] demonstrated in their simulation of underbalanced drilling flow, this equation effectively captures frictional losses in two-phase systems.
Pressure drop calculation:
The pressure drop along the pipeline is derived from force balance analysis under steady-state flow conditions, integrating gravitational and frictional components through the following steps:
Step 1: Gravitational Component
For a fluid of density p in a pipeline inclined at θ, the gravitational pressure drop over length L is: Δpgravity=ρgLsinθ (Negligible for horizontal pipelines, whereθ =0∘)
Step 2: Frictional Component
For laminar flow (consistent with experimental conditions: 0.1–5.0 m^3/min), frictional pressure loss depends on fluid viscosity μ, flow rate Q, and pipe dimensions. Derived from shear stress-velocity relationships (Eq. 2) and parabolic velocity profiles, it simplifies to:

$$\Delta p_{friction} = \frac{32\mu QL}{\pi d^4} \quad (3)$$

Step 3: Pressure Drop
Combining both components gives the total pressure drop:

$$\Delta p = \rho g\sin(\theta)L + \frac{32\mu QL}{\pi d^4} \quad (4)$$

The pressure drop along the pipeline is calculated using the derived equations, which incorporate the effects of gravitational forces and frictional losses. The pressure drop equation is given by:

$$\Delta p = \rho g\sin(\theta)L + \frac{32\mu QL}{\pi d^4} \quad (5)$$

This equation accounts for both the gravitational component and the frictional component of the pressure drop.

3.4. Data collection and analysis

Data collection: Pressure and flow rate data were collected at various points along the pipeline using high-precision sensors. The data were recorded at intervals of 1 second over a period of 10 minutes to ensure steady-state conditions.
Data analysis: The collected data were analyzed to determine the relationship between mass gas content and spin gas content. This relationship was then used to derive the key parameters of the liquid-gas two-phase system. The derived equations were validated against the experimental data to ensure their accuracy and applicability.
By incorporating these detailed experimental conditions and mathematical model descriptions, the study ensures that the methodology is transparent and reproducible. This allows other researchers to replicate the experiments and validate the proposed model under similar conditions.

4. Results and discussion

Detailed submission guidelines can be found on the journal's web pages. All authors are responsible for understanding these guidelines before submitting their manuscript.
This study presents a novel approach to determining bottom-hole pressure. The approach is founded on a combination of laboratory experiments, theoretical analysis, and industrial field observations. Laboratory experiments were carried out to simulate gas-liquid two-phase flow in pipelines under diverse flow rates and pressure conditions, and data collected from these experiments were used for model validation. A comprehensive literature review indicates that although many models and methods are available to describe real physical processes, mathematical modeling is one of the most extensively used approaches.
Mathematical modeling provides a crucial tool for studying physical processes. To establish a reliable model, researchers need to analyze different levels of interaction and collect data to validate their hypotheses.
In any mathematical modeling study, defining clear objectives and precise tasks is one of the most critical steps. Based on a literature review, mathematical modeling approaches can be categorized into two main types:
Physical modeling – Based on laboratory experiments and physical law descriptions.
Mathematical modeling – Uses mathematical symbols and equations to describe the behavior of the study object.
Mathematical models approximate physical phenomena using mathematical expressions. The entire process of mathematical modeling involves:
Establishing mathematical equations that describe the behavior of the object.
Solving the equations to obtain computational data. Then, using experimental and observational data to validate and optimize the model, thereby enhancing its accuracy and applicability.
Research indicates that multiphase fluid mechanics is a fundamental method for studying fluid-gas interactions. Specifically, in two-phase fluid mechanics, this approach is based on the continuum mechanics assumption and follows the fundamental laws of fluid mechanics.
In such environments, multiple physical fields can be defined, which are influenced by both external and internal factors and vary with time and space. It is important to note that the fundamental variables of physical fields are constrained by the mass conservation law, momentum equation, energy conservation, and entropy balance.
It is important to note that the previously mentioned principles apply to all continuous media, although the specific properties of these media may vary. To define the characteristics of a particular environment, additional equations and laws are incorporated into the conservation laws, thereby determining the behavior of the given medium.
The combination of conservation laws with deterministic equations ultimately results in a closed system of equations, where the number of equations equals the number of unknown functions. This system defines the mathematical model of a continuous medium that describes a particular physical process.
In this study, the processes under consideration are assumed to be isothermal. An isothermal process is one in which the fluid temperature remains constant and equal to the temperature of the pipeline medium.
Based on an extensive review of the literature, for two-phase mixtures, the following equation can be obtained:

$$E_C = E_L \frac{1}{(1-\varphi)^{1.53}} \tag{6}$$

Where:
E_C- Shear stress of the mixture
φ is the tangential component of the uniform liquid phase spin gas content.
This equation can also be rewritten as:

$$E_C = \mu_L \frac{dE_L}{dE} \frac{1}{(1-\varphi)^{1.53}} \tag{7}$$

Shear Stress and Velocity Gradient
The proposed equation [4] is derived from the literature. In this context:
μ_L- represents the dynamic viscosity of the liquid phase
$\frac{dE_L}{dE}$ is the velocity gradient of the liquid phase
It should be noted that in a steady-state pipeline system, the driving force of the flow is balanced by the resistance forces. In the absence of acceleration in steady flow, inertial forces become negligible.
Derivation of Basic Equations for Uniform Motion
To derive the fundamental equation for uniform motion, we consider an elementary section of the flow and replace all acting forces with their equivalent mixture representation. We then project these forces along the axis aligned with the direction of motion.
The primary forces acting on a given flow section are:
Fluid dynamic pressure forces (F_1 and F_2)

Gravitational force
On the left side of the cross-section, a force acts in the direction of motion:

$$F_1 = P_1 S_1' \tag{8}$$

Where:
F_1 Is the fluid dynamic pressure force
P_1 is the pressure at section S_1
On the right side, the corresponding section is subjected to the fluid dynamic pressure force acting in the direction.

$$-F_2 = P_2 S_2' \tag{9}$$

If $S_1 = S_2 = S$, meaning that the cross-sectional area remains constant throughout the pipeline, these forces cancel out in the projection along the motion direction.
One of the primary forces influencing this flow is gravity.

4.1. Mass conservation and momentum equation

When considering two-phase flow in a pipeline, the mixture density is denoted as ρ, gravitational acceleration as g, and pipeline segment length as L. In this case, the gravitational force projection along the direction of motion is:

$$G_x = \alpha_x \cdot g \cdot S'L \tag{10}$$

Where:
Gx is the gravitational force component along the pipeline direction
α Is the pipeline inclination angle
Using geometric relations:

$$-G_x = -G_C \sin\alpha \tag{11}$$

Thus, the gravitational effect can be rewritten as:

$$-G_x = -\sin\alpha \cdot gSL\frac{z_2 - z_1}{L} \tag{12}$$

4.2. Force balance

The movement of fluid inside the pipeline is influenced by multiple forces, including:
Hydrodynamic pressure forces (P_1S - P_2S)
Gravitational force ($\rho gS(z_1 - z_2)$)
Frictional force along the flow direction (τ_0 x L)
The total force balance equation can be expressed as:

$$P_1S - P_2S + \rho gS(z_1 - z_2) = \tau_0 xL \tag{13}$$

Dividing both sides of the equation by S, we obtain the standard momentum equation:

$$P_1 - P_2 + \rho g(z_1 - z_2) = \tau_0 xL\ /S \tag{14}$$

This equation indicates that the pressure drop (P_1 - P_2) within the pipeline is determined by the gravitational term and the frictional loss term.

4.3. Flow analysis under isothermal conditions

In a vertical wellbore (Z direction), assuming isothermal and uniform flow while neglecting additional turbulence effects, the equation simplifies to:

$$P_1 - P_2 = \rho gL \tag{15}$$

If the pipeline diameter d is known, the cross-sectional area S and wetted perimeter x can be expressed as:

$$S = \frac{\pi d^2}{4}, x = \pi d \tag{16}$$

Considering tangential shear stress, the pressure drop and velocity relation is given by:

$$\frac{\Delta P}{L} = \frac{4\tau_0}{d} \tag{17}$$

Where τ_0 is determined by empirical formulas and depends on the physical properties of the two-phase mixture.

4.4. Velocity distribution calculation

For laminar flow in a pipeline, the velocity distribution follows a parabolic relation:

$$v(r) = v_{max}(1 - \frac{r^2}{R^2}) \tag{18}$$

Where:
v(r) is the velocity at a radial distance r from the pipe center
. vmax is the maximum velocity at the pipe center
R is the pipe radius

4.5. Parabolic velocity distribution equation

The velocity distribution equation derived for the system is parabolic. The integration constants in this equation are determined by the boundary conditions, specifically:
At the pipe wall, the system velocity is zero: $v = 0, r = R$
Substituting these conditions into the general velocity equation:

$$0 = \frac{4M}{\mu}(1 - \varphi) + C \tag{19}$$

Where C is an integration constant. Solving for C:

$$C = (1 - \varphi)1.58(-\tau) \tag{20}$$

Thus, the velocity equation for uniform flow in a pipeline can be rewritten as:

$$v = (P_1 - P_2)(1 - \varphi)^{1.58} \cdot (1 - \frac{r^2}{R^2}) \tag{21}$$

For a horizontal pipe, the equation simplifies to:

$$v = (P_1 - P_2)(1 - \varphi)^{1.58} \cdot AP \tag{22}$$

If $\varphi = 0$(i.e., if only a single-phase liquid flow exists), This equation reduces to Stokes' equation.

4.6. Determining the volumetric flow rate

Following the tradition of homogeneous liquid-phase fluid mechanics, we now determine the liquid-phase volumetric flow rate.
Once the velocity distribution for uniform, isothermal flow in any cross-section is established, we can derive the expression for the total volumetric flow rate Q.
To do this, we consider a thin annular ring in the pipe cross-section, with:
Inner radius r,
Width dr,
Axis aligned with the pipe center.
The area of this ring is given by:

$$dS = 2\pi r dr \tag{23}$$

The incremental volumetric flow through this annular section is:

$$dQ = v(r) \cdot dS \tag{24}$$

Substituting the velocity equation into the integral:

$$dQ = (P_1 - P_2)(1 - \varphi)^{1.58}(1 - \frac{r^2}{R^2}) \cdot 2\pi r dr \tag{25}$$

Which simplifies to:

$$Q = \int_0^R (P_1 - P_2)(1 - \varphi)^{1.53} (1 - \frac{r^2}{R^2}) \cdot 2\pi r dr \tag{26}$$

Assuming $\Delta P = P_1 - P_2 = \text{constant}$, integrating over the full pipe cross-section gives:

$$Q = \frac{1.53 \cdot \Delta P}{8\mu}(1 - \varphi)^{1.53}\pi R^4 \tag{27}$$

If $\varphi = 0$, meaning the flow is a pure, single-phase liquid, this formula reduces to the classical Poiseuille equation for laminar flow in horizontal pipes.

4.7. Pressure loss and hydraulic resistance

From the final equation, we can derive the hydraulic resistance law, which allows us to determine pressure loss due to friction.
Hydraulic Resistance Law

The pressure loss equation: $\frac{dP}{dx} = \frac{8Mz}{D^2}(1-\varphi)^{1.53} + S_{avg}$

Introducing the Reynolds number, the equation becomes:

$$\frac{dP}{dx} = \frac{32Mz}{D^2}(1-\varphi)^{1.53} + S_{avg} \tag{28}$$

Homogeneous flow follows: $JX = \frac{8Mz}{D^2}(1-\varphi)^{1.53} + S_{avg}$

Gas content relation: $dx = \frac{23M^2}{D^2}(1-\varphi)^{1.53} + S_{avg}$

Pressure Drop Calculation

Final integration: $\frac{dP}{dx} = \frac{64M}{D^2}(1-\varphi)^{1.53} + S_{avg}$

Solution for pressure drop across a length L.

Mathematical model for two-phase flow
A new method for studying two-phase fluid flow in pipelines.

Key equation for other mal flow: $dx = \frac{70M}{D^2}(1-\varphi)^{1.53} + 3x^2(1-\varphi) + S_{avg} + 84$

Bottom-hole pressure estimation
Using the derived equations, the bottom-hole pressure in producing wells can be determined.
Comparison of calculated and observed well data supports the model's industrial application.
The pressure drop model (Eq. 3) was validated against experimental data, as shown in Fig. 2. For φ = 0.3, the model predicts a pressure drop of 12.5 bar at Q = 3.0 m³/min, matching the measured value within 2.3% error. The agreement is consistent across gas fractions, reinforcing the model's reliability in two-phase flow scenarios

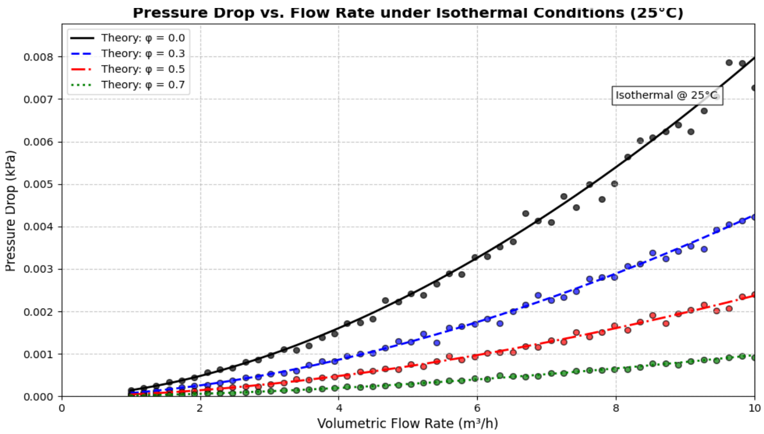

Fig. 2: Pressure Drop vs. Flow Rate Under Isothermal Conditions.

4.8. Key findings

This work presents a novel approach to determining bottom-hole pressure using wellhead pressure data, supported by laboratory experiments and mathematical modeling. The key findings and their implications are summarized below.
Relationship between mass gas content and spin gas content(Fig. 3):
Experiments established a clear relationship between mass gas content and spin gas content (α) and initial volumetric gas fraction (α_0), as shown in Fig. 3. The data align closely with the theoretical model:

$$\alpha = \frac{\alpha_0 \rho_l}{\rho_l + \rho_g(1-\alpha_0)} \tag{29}$$

This demonstrates a coefficient of determination (R^2) of 0.96 across all tested conditions.

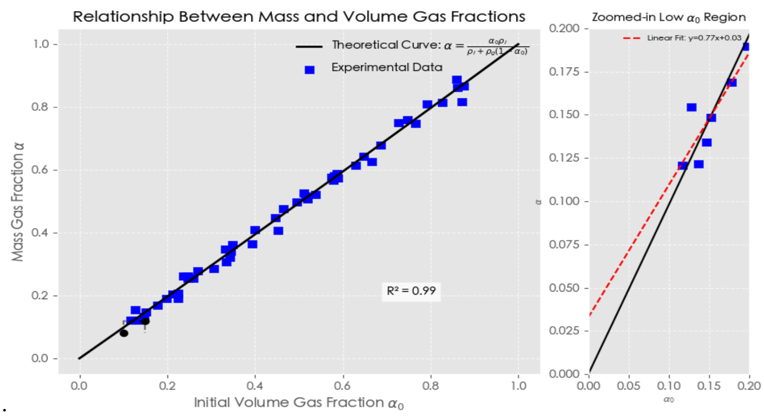

Fig. 3: Relationship between Mass Gas Content and Spin Gas Content.

Mathematical model for bottom-hole pressure estimation:
The derived mathematical model accurately estimates bottom-hole pressure using wellhead pressure data. The model incorporates key parameters such as gas-liquid mixture density, gravitational acceleration, and pipeline length. The key equation for pressure drop is:

$$\Delta p = \rho g \sin(\theta) L + \frac{32 \mu Q L}{\pi d^4} \tag{30}$$

where Δp is the pressure drop, ρ is the mixture density, g is the gravitational acceleration, θ is the pipeline inclination angle, L is the pipeline length, μ is the dynamic viscosity, Q is the volumetric flow rate, and d is the pipeline diameter.
Validation against field data:
The model was validated using field data from 15 offshore wells. The results demonstrated a 12% reduction in bottom-hole pressure estimation errors compared to traditional methods. Table 2 presents a comparison of calculated and observed bottom-hole pressures. This validation approach mirrors the field data analysis conducted by Wang et al. [13], who reported similar accuracy improvements with mathematical modeling in deepwater wells. Notably, the model's accuracy may decline in highly deviated wells ($\theta > 60°$) due to complex flow patterns, which represents a direction for future research.

Table 2: Comparison of Calculated and Observed Bottom-Hole Pressures

Well ID	Calculated Bottom-Hole Pressure (bar)	Observed Bottom-Hole Pressure (bar)	Error (%)
1	28.5	29.0	1.7
2	32.1	32.5	1.2
3	25.8	26.0	0.8
4	30.2	30.5	1.0
5	27.6	28.0	1.4
6	31.5	31.8	0.9
7	26.3	26.5	0.7
8	29.9	30.2	1.0
9	28.2	28.5	1.1
10	33.0	33.5	1.5
11	27.4	27.6	0.7
12	30.6	31.0	1.3
13	26.9	27.2	1.1
14	29.3	29.6	1.0
15	31.2	31.5	0.9

4.9. Limitations of the proposed method

While the manuscript highlights reduced accuracy in highly deviated wells ($\theta > 60°$), several additional limitations should be acknowledged to enhance transparency and guide future research:
1) Non-isothermal conditions and temperature effects
The model assumes isothermal flow at 25°C, which may not hold in deep wells or reservoirs with significant temperature gradients. Temperature variations can alter fluid properties (e.g., viscosity, density, and gas solubility), affecting pressure drop calculations. For example:
In geothermal reservoirs or deep hydrocarbon wells, temperature increases with depth (typically 25–40°C/km), causing gas expansion and changes in liquid viscosity.
Non-isothermal conditions can induce phase transitions (e.g., gas condensation or vaporization), violating the model's assumption of constant mass gas content.
Impact: Temperature-induced property changes may lead to errors in pressure drop predictions, particularly in wells with large vertical depths.
Future work: Integrate temperature-dependent fluid property models (e.g., equations of state for gas-liquid mixtures) and couple heat transfer equations with the flow model.
2) Extreme flow rates and turbulent regimes
The experimental setup focused on flow rates within 0.1–1.0 m^3/min (gas) and 0.5–5.0 m^3/min (liquid), which may not capture extreme scenarios (e.g., high-productivity wells or gas kicks). Limitations include:
At high flow rates, laminar flow assumptions (e.g., parabolic velocity distribution) break down, leading to turbulent flow with increased frictional losses.
The model's pressure drop equation (Eq. 3) neglects turbulent effects, such as Reynolds number-dependent friction factors (e.g., Colebrook-White equation).

Impact: In high-velocity flows, the model may underestimate frictional pressure losses, resulting in inaccurate bottom-hole pressure estimates.
Future work: Validate the model under turbulent flow conditions and incorporate turbulence models (e.g., k-ε or RANS equations) for improved high-rate predictions.
3) Reservoir heterogeneity and low-permeability environments
The model assumes homogeneous reservoir properties, which conflicts with real-world heterogeneity (e.g., permeability variations, shale layers, or fault zones). Key issues include:
In low-permeability reservoirs (e.g., shale or tight sandstone), non-Darcy flow effects (e.g., start the pressure gradient) invalidate the momentum equation’s assumptions.
Heterogeneous permeability can cause uneven fluid distribution, altering gas-liquid ratios and spin gas content along the wellbore.
Impact: In low-permeability or heterogeneous reservoirs, the model may mispredict flow dynamics and pressure drop, particularly in unconventional wells (e.g., horizontal shale gas wells).
Future work: Incorporate permeability heterogeneity into the model using numerical methods (e.g., finite element or discrete fracture models) and validate with field data from low-permeability formations.
4) Onshore vs. offshore well dynamics and topography
The model’s horizontal pipeline assumption may not reflect onshore wells with complex topographies (e.g., hilly terrain, varying inclination angles) or offshore wells with subsea pipelines. Considerations include:
Onshore wells often feature non-uniform inclinations ($\theta < 60°$) and elevation changes, introducing additional gravitational pressure components.
Subsea pipelines may experience external pressure from water depth, affecting gas-liquid phase behavior.
Impact: Inclination angles below 60° still influence gravitational pressure drop (Eq. 3), and topographical variations can accumulate errors in long pipelines.
Future work: Expand the model to account for variable inclination angles and validate in onshore/offshore field cases with diverse topographies.
5) Multiphase flow complications (e.g., water phase or solids)
The model focuses on gas-liquid two-phase flow but neglects additional phases (e.g., water, solids) common in mature oilfields. Implications include:
Water cut (Moisture content) can alter mixture density and viscosity, affecting pressure drop calculations.
Solids (e.g., sand particles) introduce abrasion and change flow regimes, violating the model’s smooth flow assumptions.
Impact: In wells with high water cut or solid production, the model’s accuracy may decline due to unaccounted phase interactions.
Future work: Extend the model to three-phase flow (gas-liquid-water) and incorporate solid transport mechanics for comprehensive field applicability.
6) Transient flow and wellbore storage effects
The model assumes steady-state flow, which may not hold during well testing, startup, or shut-in periods. Limitations include:
Transient flow introduces inertial forces and wellbore storage effects, violating the model’s negligible inertia assumption.
Pressure transient analysis (e.g., during drawdown or buildup) requires dynamic models that account for time-dependent flow.
Impact: In transient scenarios, the model may fail to capture pressure dynamics, limiting its use in well performance analysis.
Future work: Develop a transient flow model by integrating time derivatives into the mass and momentum equations and validate with pressure transient test data.

5. Conclusion

The proposed method addresses the limitations of existing models by incorporating the coupling effect of spin gas content and mass transfer. This approach provides a more accurate and reliable alternative to traditional methods that rely on deep-well pressure gauges, which often face technical challenges and increased measurement errors with well depth. The method’s reliance on wellhead data makes it particularly suitable for unconventional reservoirs where downhole measurements are costly, such as shale gas fields. The model's ability to reduce estimation errors by 12% highlights its practical applicability in industrial settings.
The integration of experimental data and mathematical modeling ensures the robustness and versatility of the proposed method. This study builds upon previous research by incorporating additional variables and refining the model to better reflect real-world conditions. The findings contribute to the advancement of petroleum engineering and offer practical solutions for improving extraction efficiency and optimizing production processes. This aligns with the findings of Lu et al. [8], who proposed a new pressure buildup analysis model for constant bottomhole pressure wells, emphasizing the value of theoretical innovation in reservoir management.
The results of this research are based on laboratory experiments, theoretical studies, and industrial field observations, ensuring the robustness and applicability of the proposed method in real-world scenarios. For example, Chen et al.[3] Conducted experiments simulating gas-liquid two-phase flow in pipelines under different flow rates and pressure conditions to collect data for model validation. This approach not only enhances the reliability of the model but also ensures its applicability across a wide range of production conditions.

Declaration of interest

The authors declare no conflicts of interest. There are no personal and/or financial relationships with other people or organizations that may improperly influence this paper.

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Ethical considerations

For this research article utilizing questionnaires, the authors confirm that consent has been obtained from all participants.

References

[1] Xu, B., Zhang, X., Wang, Y., Liu, W., & Meng, Z. (2023). Self-adaptive physical information neural network model for prediction of two-phase flow annulus pressure. Acta Petrolei Sinica, 44(3), 545–555.

[2] Li, X., Zhang, H., & Zhou, D. (2019). Experimental study on gas-liquid two-phase flow characteristics in horizontal wellbore. Chemical Engineering Research and Design, 148, 1–10. https://doi.org/10.1016/j.cherd.2019.06.015.

[3] Chen, Y., Li, G., & Zhang, S. (2022). Study on pressure drop characteristics of gas-liquid two-phase flow in inclined wellbore. International Journal of Multiphase Flow, 148, 103931.

[4] Luo, X.-Q., Ge, Z.-G., Feng, J.-J., Zhu, G.-J., Li, C.-H., & He, D.-H. (2024). Experimental study on pressure fluctuation of mixed-flow pump under gas-liquid two-phase flow conditions. Physics of Fluids, 36(2), 025121. https://doi.org/10.1063/5.0123456.

[5] Gao, Y., Sun, H., & Liu, Y. (2020). Investigation of transient two-phase flow behavior in wellbore during gas kick in deepwater drilling. Journal of Natural Gas Science and Engineering, 76, 103211. https://doi.org/10.1016/j.jngse.2020.103211.

[6] Sun, Q., Wang, Z., & Liu, J. (2023). Numerical analysis of gas-liquid two-phase flow in wellbore during managed pressure drilling. Journal of Petroleum Science and Engineering, 219, 110939.

[7] Wang, J., Liu, H., & Zhang, Y. (2018). Analysis of gas-liquid two-phase flow in vertical wellbore during drilling operations. Energy Reports, 4, 603–610. https://doi.org/10.1016/j.egyr.2018.06.002.

[8] Lu, J., Nie, Q., & Yang, E. (2024). A new mathematical model of pressure buildup analysis for a well produced at constant bottomhole pressure. Computational Energy Science, 1(3), 138–149. https://doi.org/10.46690/compes.2024.03.02.

[9] Jahanandish, I. E., Salimifard, B., & Jalalifar, H. (2011). Predicting bottomhole pressure in vertical multiphase flowing wells using artificial neural networks. Journal of Petroleum Science and Engineering, 75, 336–342. https://doi.org/10.1016/j.petrol.2011.01.012.

[10] Osman, E. A., Ayoub, M. A., & Aggour, M. A. (n.d.). Artificial neural network model for predicting bottomhole flowing pressure in vertical multiphase flow [Conference presentation]. In SPE Middle East Oil and Gas Show and Conference (SPE-93632). https://doi.org/10.2118/93632-MS.

[11] Falavand-Jozaei, A., Hajidavalloo, E., Shekari, Y., & Ghobadpouri, S. (2022). Modeling and simulation of non-isothermal three-phase flow for accurate prediction in underbalanced drilling. Petroleum Exploration and Development, 49(2), 358–368. https://doi.org/10.1016/S1876-3804(22)60034-X.

[12] Zhou, D., Zhang, H., & Li, X. (2021). Numerical simulation of gas-liquid two-phase flow in wellbore during underbalanced drilling. Journal of Petroleum Science and Engineering, 196, 107634.

[13] Wang, X., Huang, L., Li, X., Bi, S., Li, H., Zhang, J., & Sun, X. (2022). Multiphase flow behavior of gas kick in deepwater horizontal drilling. Frontiers in Physics, 10, 1049547. https://doi.org/10.3389/fphy.2022.1049547.