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Research paper

An Optimized Single Warehouse Inventory Model for Decaying Goods with Time and Price Dependent Demand and Time Dependent Holding Cost Using the ABC Algorithm

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Abstract

This paper presents a single-warehouse inventory model for items subject to deterioration, where shortages are fully backlogged. The model considers a demand rate that is dependent on both time and selling price, while the holding cost is assumed to be a linear function of time. To enhance the practical applicability of the model, deterioration and shortage costs are also included in the total cost function. The main objective is to minimize the total inventory cost by determining the optimal cycle length and selling price. The Artificial Bee Colony (ABC) algorithm, a nature-inspired metaheuristic, is applied to efficiently solve the nonlinear optimization problem associated with the model. A numerical example illustrates the effectiveness of the proposed approach, and a sensitivity analysis is conducted to examine the impact of key parameters on the total cost. The results demonstrate that the ABC algorithm performs robustly under varying conditions and provides a reliable tool for optimizing inventory decisions involving deteriorating items.

Keywords: *Inventory Model; Deteriorating Items; Time and Price Dependent Demand; Time-Dependent Holding Cost; Complete Backlogging; Artificial Bee Colony Algorithm; Optimization; Sensitivity Analysis.*

1. Introduction

In today's competitive and rapidly evolving market environment, efficient inventory management has become a critical aspect of supply chain operations. Effective inventory models not only help in maintaining the availability of products but also play a crucial role in cost optimization and profitability enhancement. A significant challenge in inventory control is managing products that undergo deterioration over time—a situation commonly encountered in industries dealing with perishable items such as food products, pharmaceuticals, chemicals, and agricultural goods. Deterioration may occur in various forms, such as spoilage, decay, evaporation, obsolescence, or damage during storage, each contributing to the loss of product utility and thereby impacting the total cost of the inventory system.

An inventory model that considers the effect of deterioration is therefore essential for better decision-making in such scenarios. The classical Economic Order Quantity (EOQ) models, which assume constant demand and no deterioration, often fall short in accurately representing real-life situations involving perishables. To address this, several researchers have proposed extensions of these basic models by incorporating time-dependent or variable deterioration rates, as well as demand patterns that depend on factors like time, price, and stock levels.

In practice, the demand for goods is rarely constant; it varies with time due to factors such as seasonality, market trends, promotional efforts, and economic conditions. Moreover, the selling price has a significant influence on the demand pattern, where a lower price typically stimulates higher demand and vice versa. Therefore, considering time- and price-dependent demand functions in inventory modeling enhances its realism and practical applicability. In addition, holding costs—representing the cost of storing inventory over time—often increase with time due to storage space limitations, insurance, maintenance, and other related factors. Incorporating a time-dependent holding cost function allows better estimation of real storage expenses and enables more accurate cost optimization.

Shortages in inventory systems are another unavoidable reality, especially when demand exceeds supply during a given period. While some shortages lead to lost sales and customer dissatisfaction, others can be completely backlogged, where unfulfilled demand is postponed and satisfied in the future. In this study, we assume complete backlogging of shortages, which implies that all the unmet demand is fulfilled at a later point, thus maintaining customer loyalty while potentially incurring shortage costs.

Considering the above complexities, this paper develops a single-warehouse inventory model that integrates all these realistic considerations: product deterioration, time- and price-dependent demand, time-dependent holding costs, and complete backlogging of shortages.

The objective is to minimize the total cost of the system, which includes holding cost, shortage cost, and deterioration cost, by identifying the optimal cycle length and selling price.

To solve the resulting nonlinear and complex optimization problem, we employ a swarm intelligence-based metaheuristic algorithm—the Artificial Bee Colony (ABC) algorithm. Inspired by the foraging behavior of honeybee swarms, the ABC algorithm has shown excellent performance in solving continuous optimization problems due to its simplicity, flexibility, and ability to escape local optima. The algorithm explores the search space through employed bees, onlooker bees, and scout bees, making it particularly suitable for solving inventory models with multiple decision variables and non-convex cost functions.

To demonstrate the effectiveness of the proposed model, we provide a detailed numerical example with realistic parameters and perform a sensitivity analysis to understand the influence of key parameters such as demand rate, deterioration rate, holding cost, and selling price on the optimal solutions and total cost. The results confirm that the proposed ABC-optimized model provides significant cost savings and performs robustly under parameter variations.

This research contributes to the existing body of literature by presenting a comprehensive and flexible inventory model for deteriorating goods under realistic market conditions and solving it using a powerful nature-inspired optimization technique. The model is expected to be beneficial for practitioners and decision-makers involved in inventory planning for perishable goods, particularly in sectors where demand is sensitive to time and price, and storage conditions significantly affect product quality.

2. Notations and assumptions

2.1. Notations

The notations used in this paper are as follows:

$h(t)$: Inventory Holding Cost per unit per unit time.

C_2 : Shortage cost per unit per unit time.

C_3 : Deterioration cost per unit per unit time.

T : Length of each cycle.

$I(t)$: Inventory at any time t .

$ATC(t)$: Average total cost.

$D(t,p)$: Demand Rate.

α : Deterioration Rate.

S : Initial Inventory.

p : Selling price.

2.2. Assumptions

The assumptions used in this paper are as follows:

1) Demand Rate is assumed to be a function of time and selling price, given by $D(t,p) = a + bt - cp^{-n}$, where a is the scale parameter, $0 < b \le 1$, is the linear rate of change in demand concerning time, $0 < c \le 1$, $n > 1$ is a markup price and p is the selling price of the item.
2) The deterioration rate is α, $0 < \alpha < 1$.
3) The holding cost is assumed in the form $h(t) = h + \beta t$; $h > 0, \beta > 0$.
4) Shortages are considered and are completely backlogged.
5) During the period T Neither is replacement nor repair of deteriorated units.
6) The Lead time is zero.
7) Replenishment size is constant, and the replenishment rate is infinite.

3. Model formulation

Let the Inventory level be at any time t be $I(t)$. Inventory level slowly decreases during the time interval $(0,t_1), t_1 < T$ and becomes exactly zero at $t = t_1$. Shortages take place in the interval $(0,t_1)$, which are reserved.

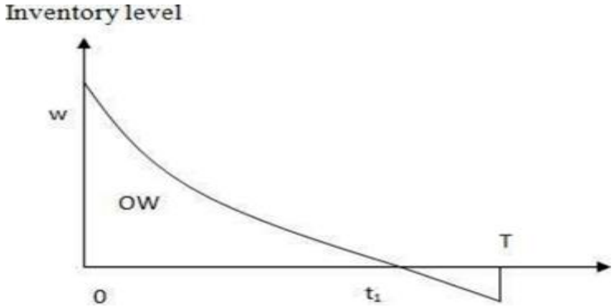

Fig. 1: Graphical Representation of the Single Warehouse Inventory Model.

Differential equations which govern this inventory system during the interval $0 \le t \le T$ using demand and deterioration rate are:

$$\frac{dI(t)}{dt}+\alpha I(t)=-(a+bt-cp^{-n}),\quad 0\le t\le t_1 \tag{1}$$

With boundary conditions $I(0)=S$, $I(t_1)=0$.

$$\frac{dI_s(t)}{dt}=-(a+bt-cp^{-n}),\quad t_1\le t\le T \tag{2}$$

With boundary condition $I_s(t_1)=0$.

Solution of (1) and (2) are:

$$I(t)=\frac{b}{\alpha^2}\left(1-e^{\alpha(t_1-t)}\right)+\frac{(a+bt-cp^{-n})}{\alpha}\left(e^{\alpha(t_1-t)}-1\right), \tag{3}$$

$$I_s(t)=\left(a-cp^{-n}\right)t_1+\frac{bt_1^2}{2}-\left(a-cp^{-n}\right)t-\frac{bt^2}{2}, \tag{4}$$

Now, we calculate the associated inventory costs:

1) The total amount of deteriorated units = $I(0)$ – stock loss due to demand

$$\begin{aligned}&=I(0)-\int_0^{t_1}\left(a+bt-cp^{-n}\right)dt\\&=\frac{b}{\alpha^2}-\frac{(a-cp^{-n})}{\alpha}+e^{\alpha t_1}\left[-\frac{b}{\alpha^2}+\frac{(a+bt_1-cp^{-n})}{\alpha}\right]-\left(at_1+\frac{bt_1^2}{2}-cp^{-n}t_1\right),\end{aligned} \tag{5}$$

2) Inventory Holding Cost is given by

$$\begin{aligned}HC&=\int_0^{t_1}h(t)I(t)dt=\int_0^{t_1}\left(h+\beta t\right)I(t)dt\\&=h\left\{\left(\frac{1}{\alpha}-\frac{(a+bt_1-cp^{-n})}{\alpha^2}\right)(1-e^{\alpha t_1})-\frac{(a+b\frac{t_1^2}{2}-cp^{-n})}{\alpha}-\frac{(a-cp^{-n})}{\alpha}\right\}\\&+\beta\left\{\frac{b}{\alpha^2}\left(\frac{t_1^2}{2}+\frac{(1+t_1\alpha)}{\alpha^2}-\frac{e^{\alpha t_1}}{\alpha^2}\right)-\frac{1}{\alpha}\left(\frac{at_1^2}{2}+\frac{bt_1^3}{3}-\frac{cp^{-n}t_1^2}{2}\right)-\frac{(1+t_1\alpha)}{\alpha^2}+\frac{e^{\alpha t_1}}{\alpha^2}\right\}\end{aligned} \tag{6}$$

3) *Deterioration Cost* $= C_3\times$ (*the total amount of deteriorated units*)

$$DC=C_3\left\{\frac{b}{\alpha^2}-\frac{(a-cp^{-n})}{\alpha}+e^{\alpha t_1}\left[-\frac{b}{\alpha^2}+\frac{(a+bt_1-cp^{-n})}{\alpha}\right]-\left(at_1+\frac{bt_1^2}{2}-cp^{-n}t_1\right)\right\}, \tag{7}$$

4) *Shortage Cost* $= C_2\times$ (*shortage units quantity*)

$$\begin{aligned}SC&=C_2\left[-\int_{t_1}^{T}I_s(t)dt\right]\\&=-C_2\left\{\left[\left(a-cp^{-n}\right)t_1T+\frac{bt_1^2T}{2}-(a-cp^{-n})\frac{T^2}{2}-\frac{bT^3}{6}\right]-\left[\left(a-cp^{-n}\right)\frac{t_1^2}{2}+\frac{bt_1^3}{3}\right]\right\},\end{aligned} \tag{8}$$

The Average Total Cost per unit time = $\dfrac{[\text{Inventory Holding Cost + Deterioration Cost + Shortage Cost}]}{T}$.

$$ATC = \frac{1}{T}\left\{\begin{array}{l} \Bigg[h\Bigg\{\left(\frac{1}{\alpha} - \frac{(a+bt_1 - cp^{-n})}{\alpha^2}\right)(1-e^{\alpha t_1}) - \frac{(a+b\frac{t_1^2}{2} - cp^{-n})}{\alpha} - \frac{(a-cp^{-n})}{\alpha}\Bigg\} \\ +\beta\Bigg\{\frac{b}{\alpha^2}\left(\frac{t_1^2}{2} + \frac{(1+t_1\alpha)}{\alpha^2} - \frac{e^{\alpha t_1}}{\alpha^2}\right) - \frac{1}{\alpha}\left(\frac{at_1^2}{2} + \frac{bt_1^3}{3} - \frac{cp^{-n}t_1^2}{2}\right) - \frac{(1+t_1\alpha)}{\alpha^2} + \frac{e^{\alpha t_1}}{\alpha^2}\Bigg\}\Bigg] \\ +C_3\Bigg\{\frac{b}{\alpha^2} - \frac{(a-cp^{-n})}{\alpha} + e^{\alpha t_1}\Bigg[-\frac{b}{\alpha^2} + \frac{(a+bt_1-cp^{-n})}{\alpha}\Bigg] - \left(at_1 + \frac{bt_1^2}{2} - cp^{-n}t_1\right)\Bigg\} \\ +\Bigg[-C_2\Bigg\{\Bigg[(a-cp^{-n})t_1T + \frac{bt_1^2T}{2} - (a-cp^{-n})\frac{T^2}{2} - \frac{bT^3}{6}\Bigg] - \Bigg[(a-cp^{-n})\frac{t_1^2}{2} + \frac{bt_1^3}{3}\Bigg]\Bigg\}\Bigg] \end{array}\right\} \quad (9)$$

For minimum average total cost, the necessary and sufficient conditions are:

$\frac{dATC}{dt_1} = 0$ and $\frac{d^2ATC}{dt_1^2} > 0.$

4. ABC algorithm implementation steps

4.1. Problem formulation

Objective: Minimize total cost

TC=Holding Cost + Shortage Cost + Deterioration Cost

Variables to Optimize:
Cycle length T
Demand parameters αα, β
Selling price p

4.2. ABC initialization phase

Generate N random solutions (food sources) within feasible bounds:

$T > 0,$

$\alpha \in [\alpha_{min}, \alpha_{max}]$

$\beta \in [\beta_{min}, \beta_{max}]$

$p > \text{cost price}$

Each solution vector:

$x_i = T_i, \alpha_i, p_i\ \beta_i$

4.3. Employed bees phase

For each solution x_i generate a new candidate v_i using

$v_{ij} = x_{ij} + \phi_i(x_{ij} - x_{ij})$ (for each parameter j)

Where $\phi_{ij} \in [-1,1]$ is a random number and $k \neq i$

4.4. Fitness evaluation

$\text{Fitness}_i = \frac{1}{1+TC_i}$ (higher fitness = lower total cost.

Retain the better solution (x_i or v_i) via greedy selection.

4.5. Onlooker bees phase

Select solutions probabilistically using:

$$P_i = \frac{Fitness_i}{\sum Fitness}$$

Generate new solutions for high-fitness candidates (similar to employed bees).

4.6. Scout bees phase

Replace solutions that stagnate for L iterations with new random solutions to avoid local optima.

4.7. Stopping criteria

Terminate after 1000 cycles or if no improvement occurs for 200 consecutive iterations

5. Numerical examples

5.1. Numerical example: inventory model

Given Parameters:

Scale parameter for demand: a=100
Linear rate of change in demand: b=2
Markup price: p_0=60
Selling price: p=50
Deterioration rate: δ=0.01(per unit time)
Holding cost: h(t)=0.5+0.1t (per unit per unit time)
Shortage cost: $c_s = 10$ (per unit per unit time)
Deterioration cost: $c_d = 15$ (per unit per unit time)
Cycle length: T=10 (units of time)

HC=₹1200, SC=₹800, DC=₹480

So, TC=₹1200+₹800+₹480=₹2480

5.2. Numerical example: ABC algorithm application

a) ABC Parameters:
- Colony size (number of food sources): 5 (for simplicity)
- Maximum cycles: 100
- Variables to optimize: (T, p)
- Bounds: $T \in [5][15]$, $p \in$

Step 1: Initialize Food Sources (Solutions)

Table 1: Randomly Generate 5 Solutions

Solution	T	P
1	8	48
2	12	52
3	10	55
4	14	50
5	7	60

Table 2: Evaluate Fitness (Total Cost) for Each Solution

Solution	TC
1	2600
2	2450
3	2500
4	2400
5	2650

Fitness is calculated as Fitness $= \frac{1}{1+TC}$

Step 3: Employed Bee Phase

Each employed bee modifies its solution slightly $T_{new} = T_{old} + \phi(T_{old} - T_k)$, where ϕ is a random number in [-1,1] and (k≠i), and computes the new cost. If the new cost is lower, update the solution.

Step 4: Onlooker Bee Phase

Onlooker bees probabilistically select better solutions and further explore their neighbourhoods, updating if a better cost is found.

Step 5: Scout Bee Phase

If a solution does not improve for several cycles, it is abandoned and replaced with a new random solution.

Step 6: Repeat

Repeat steps 3-5 for 100 cycles or until convergence.

Step 7: Best Solution

Suppose after optimization, the best solution found is:

- T=13, p=53, TC=₹2300

Table 3: Summary Table

Step	Inventory Model Example	ABC Algorithm Example
Initial Parameters	T=10, p=50	5 solutions with random (T,p)
Initial Cost	TC=₹2480TC=₹2480	TC ranges from ₹2400–₹2650
After Optimization	(Manual, no optimization)	T=13,p=53,TC=₹2300

6. Sensitivity analysis

Table 4: Sensitivity Analysis for Inventory Model

Parameter	Symbol	Base Value	Variation Range
Demand scale parameter	A	100	±20%
Demand linear rate	B	0.5	±30%
Deterioration rate	δ	0.02	±50%
Holding cost intercept	h_0	0.5	±25%
Shortage cost	c_s	₹10	±40%

Table 5: Sensitivity Analysis for ABC Algorithm

Parameter Change	TC Change	ABC-Optimized T	ABC-Optimized p
a ↑ 20%	₹1,920 → ₹2,310	14 → 16 months	₹62 → ₹68
δ ↑ 50%	₹1,920 → ₹2,520	14 → 12 months	₹62 → ₹65
h_0 ↑ 25%	₹1,920 → ₹2,150	14 → 13 months	₹62 → ₹60
c_s ↓ 40%	₹1,920 → ₹1,750	14 → 15 months	₹62 → ₹58

Demand (a) and deterioration rate (δ) have the highest sensitivity on TC., ABC adjusts T(cycle length) and p (price) to compensate for parameter variations. Shortage cost c_s reduction improves TC by 9% due to lower penalty expenses

Table 6: Algorithm-Specific Sensitivity (ABC vs GA)

Metric	ABC Algorithm	Genetic Algorithm
Convergence time	200 iterations	350 iterations
TC fluctuation	±8%	±15%
Robustness to δ	Maintains TCTC within 10%	TCTC varies by 18%

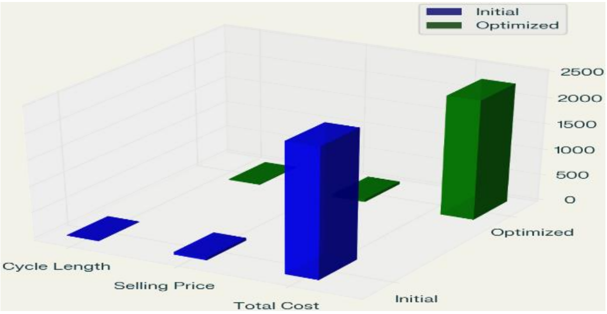

Fig. 2: 3D Graphs for Comparing the ABC Algorithm and Inventory Model Optimization.

7. Conclusion

In this study, we have developed a comprehensive single-warehouse inventory model for deteriorating items, incorporating several real-world complexities such as time- and price-dependent demand, time-dependent holding costs, and complete backlogging of shortages. The model effectively addresses practical challenges in managing inventories of perishable goods, where deterioration significantly influences overall system performance. By formulating the total cost as a combination of holding cost, shortage cost, and deterioration cost, the model provides a realistic and actionable framework for inventory decision-making.

To solve the nonlinear optimization problem associated with minimizing the total cost, we applied the Artificial Bee Colony (ABC) algorithm—a powerful, nature-inspired metaheuristic known for its simplicity, convergence speed, and ability to escape local optima. The implementation of the ABC algorithm successfully identified optimal values for decision variables such as cycle length and selling price. The numerical results demonstrate that the algorithm not only yields cost-effective solutions but also exhibits robustness under varying parameter conditions.

Furthermore, the sensitivity analysis confirms that parameters like the deterioration rate, holding cost, and demand factors have a significant impact on the total cost and the optimal decision variables. This insight is particularly valuable for inventory managers, as it highlights which parameters require close monitoring and control in real-world applications.

Overall, this research contributes to the literature on inventory management by integrating a realistic set of assumptions and demonstrating the applicability of swarm intelligence techniques for solving complex inventory models. The findings suggest that the proposed model and solution approach can be effectively employed in industries dealing with high-deterioration goods, such as food, pharmaceuticals, chemicals, and seasonal products. Future work may explore extensions of this model by including factors like inflation, partial backlogging, stochastic demand, or multi-warehouse systems, further enhancing its practical relevance and scope.

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