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Research paper

On K-idempotent Neutrosophic Z - Matrices and Computation Methods in Decision Making

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Abstract

In this work, k-idempotent neutrosophic z-matrices (NZM) are constructed where k is a fixed product of disjoint transpositions in the symmetric group of order n. Some characteristics and properties of k-idempotent NZM with T-ordering of k-idempotent NZM are discussed. The idea of a k–idempotent neutrosophic z-matrix, which is a generalization of idempotent NZM via permutations, is examined. It is shown that a k-idempotent NZM reduces to an idempotent NZM if and only if S_Ak=kS_A. Some related results are provided, together with the derivation of the square symmetric and cube symmetric NZM of k-idempotent conditions. The correlation measure of neutrosophic fuzzy matrices and intuitionistic fuzzy matrices is extended by the notion of this correlation measure of neutrosophic z-matrices. To improve the recommendation performance, we examined the performance of several calculation methods by comparing correlation and the usual procedure. Finally, we compute an experimental result where correlation values can be evaluated and compared to the results of the other methodology.

Keywords: *Neutrosophic Z-matrices; k- idempotent NZM; T-ordering NZM; Correlation Measures*

1. Introduction

The concept of z-numbers was defined [1], in which the dependability and constraint of evaluation data are represented by a pair of fuzzy numbers. Due to the difficulty of calculating z-numbers [24] discovered mathematical operations such as addition, multiplication, and others in discrete z-numbers [2]. He then discovered continuous z-numbers, which have more accurate values and better computational concepts [3]. He also discovered the z-rules on a linear interpolation for approximation reasoning [25].

In [5] introduced the concept of idempotence in fuzzy matrices, where $A^2 = A$ when $A = [a_{ij}]_{m\times n} \in F$. [4]. This idempotent property permits more adaptable handling of fuzzy systems, enhancing their pertinence in various decision-making processes. As a generalization of idempotent matrices, In [26] initially proposed the concept of k-idempotent matrices. Some properties of the study of k-idempotent fuzzy matrices were presented [6]. A fuzzy matrix is referred to as a k-idempotent fuzzy matrix if its elements are obtained by k-permuting them. Later, k-idempotent intuitionistic fuzzy matrices were included in the notion.

A study on the generalization of k-idempotent fuzzy matrices was conducted [7]. The concept of pseudo similarity of neutrosophic fuzzy matrices was introduced [28]. He then finds partial ordering and various k-idempotent features for k-idempotent neutrosophic fuzzy matrices [11]. The concept of T-ordering in fuzzy matrices and its characteristics were first introduced [27]. T-ordering fuzzy neutrosophic soft matrices were found [18], who also investigated some Moore-Penrose inverse features in T-ordering [10].

The neutrosophic set, which addresses the intricacies of processing imprecision, ambiguity, and uncertainty in data, was discovered [9]. Later, the neutrosophic fuzzy set for multi-criteria decision-making was discovered using neutrosophic tangent similarity metrics [12] and aggregation operators [13].

Du examined various features of aggregation operators and extended Z-numbers to NZN sets [19]. In order to build a decision-making technique, the cosine and cotangent similarity measures were introduced for NZN sets [20]. Using a partially known weight in the aggregation operator, two decision models were created for NZN sets [21]. To handle multiple attribute decision-making, Dombi operations and various Dombi weighted aggregation operators of NZN sets were introduced [22]. A logarithmic similarity metric of NZN sets has been devised [23] to assess the quality of undergraduate teaching [8].

The correlation measure for intuitionistic fuzzy matrices was studied [15]. The correlation coefficient with the interval neutrosophic set was developed [17]. The correlation coefficient of the neutrosophic set was proposed [16], who also looked into the fundamentals of correlation. The idea of a correlation measure for neutrosophic refined sets in medical diagnostics for decision-making was later expanded [14].

This research project contains the fundamental definitions of the NZM on section 2. In Section 3, the k-idempotent neutrosophic z-matrix was presented, along with some basic definitions and some results. Then, in Section 4, the characteristics of k-symmetric and power-symmetric NZM have been studied in detail. With some success, Section 5 unearth T-ordering NZM on k-idempotent. Analytical techniques for medical diagnostics employing various algorithm types are covered in Section 6. The goal is to use neutrosophic z-matrices and their hybrid structures to solve medical diagnosis problems. Furthermore, a comparative analysis was presented. Finally, we wrap up the study with future work enhancements in section 7.

2. Preliminaries

Some essential definitions are given.

2.1. Definition

A Z-number is an ordered pair of fuzzy numbers denoted as $Z = (\check{V}, \check{R})$. The first component $\check{V}$ is a fuzzy restriction on the values that X can take and the second component $\check{R}$ is a measure of reliability for the $\check{V}$.

2.2. Definition

Let X be a universe set then a Neutrosophic Z-number set (NZNs) in a universe set X is defined in the following form $= (< x, T(V,R)(x), I(V,R)(x), F(V,R)(x)/x \in X >)$where $T(V,R)(x) = (T_V(x), T_R(x))$,$I(V,R)(x) = (I_V(x), I_R(x)), F(V,R)(x) = (F_V(x), F_R(x)): X \rightarrow [0,1]^2$
are the order pairs of neutrosophic values for truthfulness, indeterminacy, and falsehood; the first component consists of the neutrosophic values in a universe set X, and the second component consists of neutrosophic reliability measures, with the rule of $0 \leq T_V(x)+I_V(X)+F_V(x) \leq 3$ and $0 \leq T_R(x)+I_R(x)+F_R(x) \leq 3$.

2.3. Definition

Let X be a universe set and F be a set of parameters. Consider a non empty set $S_Z, S_Z \in F$. Let P(X) is the collection of all neutrosophic z-number sets of X. The set (E, S_Z) is termed to be neutrosophic z- number sets (NZNs) over X, where $E : S_Z \rightarrow P(X)$. Hereafter we simply consider S as neutrosophic z-matrices (NZMs) over X instead of (E, S_Z).

2.4. Definition

Let $S_A = (< (T^A_{V_{ij}}, T^A_{R_{ij}}), (I^A_{V_{ij}}, I^A_{R_{ij}}), (F^A_{V_{ij}}, F^A_{R_{ij}}) >)$ and be NZMs of order $m \times n$, then the complement is denoted as $S^c_A = (< (F^A_{V_{ij}}, F^A_{R_{ij}}), (1 - I^A_{V_{ij}}, 1 - I^A_{R_{ij}}), (T^A_{V_{ij}}, T^A_{R_{ij}}) >)$

2.5. Definition

Let S_A be a $NZM_{m\times n}$ and S_B be a $NZM_{n\times p}$ then the composition of S_A and S_B is defined as

$$S_A o S_B = (< (\sum_{i=1}^{n} (T^A_{V_{ik}} \wedge T^B_{V_{kj}}), (\sum_{i=1}^{n} (T^A_{R_{ik}} \wedge T^B_{R_{kj}})), (\prod_{i=1}^{n}(I^A_{V_{ik}} \vee I^B_{V_{kj}})), (\prod_{i=1}^{n}(I^A_{R_{ik}} \vee I^B_{R_{kj}})), (\prod_{i=1}^{n}(F^A_{V_{ik}} \vee F^B_{V_{kj}})), (\prod_{i=1}^{n}(F^A_{R_{ik}} \vee F^B_{R_{kj}})) >)$$

Equivalently we can write,

$$S_A o S_B = (< (\bigcup_{i=1}^{n} (T^A_{V_{ik}} \wedge T^B_{V_{kj}})), (\bigcup_{i=1}^{n} (T^A_{R_{ik}} \wedge T^B_{R_{kj}})), (\bigwedge_{i=1}^{n}(I^A_{V_{ik}} \vee I^B_{V_{kj}})),$$
$$(\bigwedge_{i=1}^{n}(I^A_{R_{ik}} \vee I^B_{R_{kj}})), (\bigwedge_{i=1}^{n}(F^A_{V_{ik}} \vee F^B_{V_{kj}})), (\bigwedge_{i=1}^{n}(F^A_{R_{ik}} \vee F^B_{R_{kj}})) >)$$

If the number of S_A columns equals the number of rows S_B, then the product is defined.This multiplication procedure is called as max-min composition operator. Consequently, $S_A o S_B$ and are considered conformable for multiplication, rather than using $S_A o S_B$ we use $S_A S_B$where $\sum_{i=1}^{n} (T^A_{V_{ik}} \wedge T^B_{V_{kj}})$ means max- min operation and $\prod_{i=1}^{n}(I^A_{V_{ik}} \vee I^B_{V_{kj}})$)means min-max operation.

3. Characterizations of k-idempotent NZM

In this section, we identify a few k-idempotent NZM characters. Consider the set of all permutations on {1, 2, 3, …, n}, where k is a fixed product of disjoint transpositions in S_n. The index set {1, 2, 3, …, n-1, n} will now be represented by N, which is an NZM. It then represents the NZM S^t_A denotes the transpose and and $adjS_A$ meaning adjoint.

3.1. Definition

If $kS^2_A k = S_A$ where $k \in Sym_n$ and $S_A = [S_{aij}]_{n\times n}$is the associated permutation NZM of a fixed product of disjoint transpositions, then the NZM is k-idempotent. If there is precisely one entry[$<(1,1),(0,0),(0,0)>$] in each row and column and all other entries are [$<(0,0),(1,1),(1,1)>$], then the square NZM matrix is referred to as NZPM.

3.2. Definition

The determinant $|S_A|$ of $n \times n$ NZM $S_A = (< \left(T_{V_{ij}}^A, T_{R_{ij}}^A\right), \left(I_{V_{ij}}^A, I_{R_{ij}}^A\right), \left(F_{V_{ij}}^A, F_{R_{ij}}^A\right) >)$ is defined $|S_A| = (< \vee_{\sigma \in Sym_n} T_{V1\sigma(1)}^A \wedge T_{V2\sigma(2)}^A \wedge \dots \dots T_{Vn\sigma(n)}^A$, $\vee_{\sigma \in Sym_n} T_{R1\sigma(1)}^R \wedge T_{R2\sigma(2)}^R \wedge \dots \dots T_{Rn\sigma(n)}^R, \wedge_{\sigma \in Sym_n} I_{V1\sigma(1)}^A \vee I_{V2\sigma(2)}^A \vee \dots \dots . \vee I_{Vn\sigma(n)}^A, \wedge_{\sigma \in Sym_n} I_{R1\sigma(1)}^A \vee I_{R2\sigma(2)}^A \vee \dots \dots . \vee I_{Rn\sigma(n)}^A, \wedge_{\sigma \in Sym_n} F_{V1\sigma(1)}^A \vee F_{V2\sigma(2)}^A \vee \dots \dots . \vee F_{Vn\sigma(n)}^A, \wedge_{\sigma \in Sym_n} F_{R1\sigma(1)}^A \vee F_{R2\sigma(2)}^A \vee \dots \dots . \vee F_{Rn\sigma(n)}^A$ where Sym_n denotes the symmetric group of all permutation of the indices (1,2…………n).

3.3. Definition

The adjoint of an $n \times n$ NZM in $S_A = (< \left(T_{V_{ij}}^A, T_{R_{ij}}^A\right), \left(I_{V_{ij}}^A, I_{R_{ij}}^A\right), \left(F_{V_{ij}}^A, F_{R_{ij}}^A\right) >)$ is denoted as $adj\ S_A$ and defined as $S_A = \left|S_{A_{ji}}\right|$ as the NZM determinant $-1 \times n - 1$, which is created by removing rows j and i column from S_A and $S_B = adj\ S_A$.

Note

Here $kS_A^2 k = S_A \Longrightarrow kS_A k = S_A^2$
By continuous calculation we get
$kS_A = S_A^2 k \ or\ kS_A^2 = S_A k$
$kS_A^3 = S_A^3 k \ or\ kS_A^3 k = S_A^3$
$S_A^3 = (k S_A)^2 = (S_A k)^2$
$k^2 S_A = S_A k^2 = S_A$
And $kS_A^2 k = S_A^2$

3.4. Proposition

Let S_A is idempotent, then S_A is k-idempotent NZM iff $S_A k = kS_A$.
Proof:

We know $S_A k = kS_A$.
multiplying by k $\Rightarrow kS_A k = S_A$
[$\because S_A$ is idempotent, $S_A^2 = S_A$]
$\Longrightarrow kS_A^2 k = S_A$
$\therefore S_A$ is k-idempotent NZM

The next part can be proved similarly.
Hence the proof.

3.5. Example

$$\text{Let } S_A = \begin{bmatrix} <(1,1),(0,0),(0,0)> & <(0,0),(1,1),(1,1)> & <(1,1),(0,0),(0,0)> \\ <(0,0),(1,1),(1,1)> & <(1,1),(0,0),(0,0)> & <(0,0),(1,1),(1,1)> \\ <(1,1),(0,0),(0,0)> & <(0,0),(1,1),(1,1)> & <(1,1),(0,0),(0,0)> \end{bmatrix}$$

$$\text{And } k = \begin{bmatrix} <(0,0),(1,1),(1,1)> & <(0,0),(1,1),(1,1)> & <(1,1),(0,0),(0,0)> \\ <(0,0),(1,1),(1,1)> & <(1,1),(0,0),(0,0)> & <(0,0),(1,1),(1,1)> \\ <(1,1),(0,0),(0,0)> & <(0,0),(1,1),(1,1)> & <(0,0),(1,1),(1,1)> \end{bmatrix}$$

Here S_A is idempotent ($S_A^2 = S_A$) and also k-idempotent NZM ($kS_A^2 k = S_A$). Here $k=k^{-1}$

3.6. Proposition

For a NZM, S_A be a k-idempotent NZM. Then
(a) S_A^t is k-idempotent NZM.
(b) S_A^n is k-idempotent NZM for all scalar value n.
(c) S_A is $S_A^4 = S_A \ and\ S_A^3 = I$.
(d) S_A^3 is idempotent.
(e) kS_A and $S_A\ k$ are tripotent NZM.
Proof:
(a) Consider S_A be a k-idempotent NZM
$kS_A^2 k = S_A$
$(kS_A^2 k)^t = (S_A)^t$
$k(S_A^2)^t k = (S_A)^t$ [$\because S_A^t$ are idempotent]
$k(S_A^t)^2 k = S_A^t$
Hence S_A^t is k-idempotent NZM.
(b) Let $S_A^n = (kS_A^2 k)^n$
$= kS_A^2 k . kS_A^2 k \dots \dots . . kS_A^2 k$ (n times commuting)
$= kS_A^{2n} k$

$= k(S_A^n)^2 k$

$\therefore S_A^n$ is k-idempotent NZM for all scalar value n.

(c) Let$S_A^4 = S_A^2 . S_A^2$ (from note)

$= kS_A k \, . \, kS_A k$

$= kS_A^2 k$

$= S_A$

$\Longrightarrow S_A^4 = S_A$ and $S_A^3 = I$.

(d) Let $(S_A^3)^2 = [(kS_A)^2]^2$ [from note]

$= \underbrace{kS_A . kS_A kS_A kS_A}$

$=kS_A S_A^2 S_A kS_A$

$= kS_A kS_A \; [S_A^3 = I]$

$= S_A^2 . S_A$

$= S_A^3$

(e) Let $(kS_A)^3 = kS_A \, . kS_A . kS_A$

$=kS_A S_A^2 S_A$

$= kS_A$

$S_A \, k$ can be proved similarly.

Hence the proof.

3.7. Proposition

Let S_A and S_B be two k- idempotent neutrosophic z-matrices. Then $S_A + S_B$ is k- idempotent neutrosophic z-matrices.
It can be shown easily.

3.8. Proposition

Let S_A and S_B be two k- idempotent neutrosophic z-matrices. If $S_A S_B$= $S_B S_A$, then $S_A S_B$ is also a k- idempotent neutrosophic z-matrices.
Proof:

$k(S_A S_B)^2 k = kS_A^2 \, k . kS_B^2 \, k \; [\, kS_A^2 \, k = \; S_A \; \Rightarrow k \, S_A \, k = S_A^2]$

$= S_A S_B$

Hence $S_A S_B$ is k- idempotent neutrosophic z-matrices.
Hence the proof.

3.9. Proposition

If the neutrosophic z-matrix $S_A$is k- idempotent NZM then adj S_A is also k- idempotent NZM.
Proof:

Let $S_B = adj \; S_A$, where S_B is idempotent and $S_B \leq n - 1$.

Since $S_A$is k- idempotent, $kS_A^2 \, k = S_A$.

Also, k$(S_B)^2 k = \; S_B^2$

Here $S_B = adj \; S_A$ is k-idempotent.

Hence the proof.

3.10. Proposition

If $S_A$and $adj \; S_A$ are k- idempotent NZM then $S_A . adj \; S_A$ are k- idempotent NZM

Proof:

k$(S_A . adj \; S_A)^2 k =$ k$(S_A^2 . (adj \; S_A)^2)k$

$=$ k$S_A^2 \; k . k(adj \; S_A)^2 k$ since $k(adj \; S_A)^2 k = adj \; S_A$

$= S_A . adj \; S_A$

Hence $S_A . adj \; S_A$ is k-idempotent NZM.

Hence the proof.

3.11. Proposition

If $S_{A_1}, S_{A_2}, S_{A_3}, \dots \dots \dots S_{A_n}$ be a k-idempotent neutrosophic z-matrices consist of a set of mutually commuting matrices, then $\prod_{i=1}^{n} S_{A_i}$ is a k- idempotent neutrosophic z-matrix.

Proof:

$$k\,(\prod_{i=1}^{n} S_{A_i})^2 k = k\,(S_{A_1}, S_{A_2}, S_{A_3}, \dots\dots\dots S_{A_n})^2 k$$

$= k\,(S_{A_1}^2 S_{A_2}^2 \dots\dots\dots . S_{A_n}^2)k$

$= k\,(S_{A_1}^2)k.\,k(S_{A_2}^2)k \dots\dots\dots . k(S_{A_n}^2)k$

$= S_{A_1}, S_{A_2}, S_{A_3}, \dots\dots\dots S_{A_n}$

$= \prod_{i=1}^{n} S_{A_i}$

Now the neutrosophic z-matrix $\prod_{i=1}^{n} S_{A_i}$ is a k- idempotent.
Hence the proof.

3.12. Proposition

If S_A and S_B are two k- idempotent neutrosophic z-matrices then$S_A(S_A + S_B)S_B$ commutes with the k permutation matrix.
Proof;

Let $S_A(S_A + S_B)S_B = S_A^2 S_B + S_A S_B^2$
$= \text{k}S_A^2 k S_B + S_A k S_B^2 k$
$= \text{k}(S_A^2 S_B)k + k(S_A S_B^2)k$
$= \text{k}(S_A^2 S_B + S_A S_B^2)k$
$= k(S_A(S_A + S_B)S_B)k$
$= S_A(S_A + S_B)S_B$

Then $kS_A(S_A + S_B)S_B = S_A(S_A + S_B)S_B\text{k}$

Hence the proof.

4. k-idempotency of Symmetric NZM

In this chapter, we introduce k-symmetric $\Rightarrow kS_A^t\,k = S_A =$, k-square symmetric and k-cubic symmetric NZM. The relations of various symmetric is displayed here. Let S_A is Symmetric NZM if and only if $S_A = S_A^t$

4.1. Definition

Let NZM be a k- square symmetric NZM iff $S_A^t = kS_A^2\,k$ or $S_A^2 = kS_A^t\,k$.

4.2. Definition

Let NZM be a k- cubic symmetric NZM iff $S_A^t = kS_A^3\,k$ or $S_A^3 = kS_A^t\,k$.

4.3. Proposition

If S_A be NZM then any two of the following are equivalent to the other one.

(i) S_A is k- idempotent NZM.

(ii) S_A is k- symmetric NZM.

(iii) S_A is square symmetric NZM.

Proof:

(a) If (i) & (ii) $\Rightarrow$ (iii)
$kS_A^2\,k = S_A$ ....(1)
and $kS_A^t\,k = S_A$ ....(2)

From (1) and (2)
$kS_A^2\,k = kS_A^t\,k$
$S_A^2 = S_A^t$
$\therefore S_A$ is square symmetric NZM.
(b) If (ii) & (iii) $\Rightarrow$ (i)
$kS_A^t\,k = S_A$ and $S_A^2 = S_A^t$

From above 2 condition we get
$kS_A^2\,k = S_A$

Hence S_A is k- idempotent NZM
(c) If (iii) & (i) $\Rightarrow$ (ii)

We know $S_A^2 = S_A^t$ and $kS_A^2\,k = S_A$ [from c & a]

From above 2 condition we get

$kS_A^t k = S_A$

$\therefore S_A$ is k- symmetric NZM.

Hence the proof.

4.4. Proposition

Let S_A be a k- idempotent and symmetric in NZM. If $S_A$is a cube symmetric then S_A decrease to $S_A = S_A^t$.
Proof:

Since S_A is k- idempotent NZM then $kS_A^3 k= S_A^3$ [from note]

Let S_A be a cube symmetric then $S_A^3 = S_A^t$ ………………(1)

Pre and post multiply by k in (1) equation

$\Rightarrow kS_A^3 k = kS_A^t k$

$\Rightarrow S_A^3 = kS_A^t k$ [apply 1 equation]

$\Rightarrow S_A^t = kS_A^t$ [$\because S_A$ is symmetric]

$\Rightarrow S_A = k\, S_A k$

$\Rightarrow S_A = S_A^2$

$\Rightarrow S_A = S_A^t$

Hence the proof.

4.5. Proposition

Let S_A be a k- idempotent NZM. Then the conditions are equivalent.

(i) kS_A is cube symmetric NZM.

(ii) kS_A is symmetric NZM.

(iii) S_A is square symmetric NZM.

Proof:

a) (i)$\Rightarrow$ (ii)

kS_A is cube symmetric then

$\Rightarrow (k\, S_A)^3 = (k\, S_A)^t$

$\Rightarrow k\, S_A = (k\, S_A)^t$

$\therefore$ $k\, S_A$ is symmetric NZM.

b) (ii)$\Rightarrow$ (iii)

$k\, S_A$ is symmetric then $k\, S_A = (k\, S_A)^t$

$\Rightarrow S_A^2 k = S_A^t k$

$\Rightarrow S_A^2 = S_A^t$

$\therefore$ S_A is square symmetric NZM.

c) (iii)$\Rightarrow$ (i)

S_A is square symmetric then $S_A^2 = S_A^t$

$\Rightarrow (k\, S_A)^3 = k\, S_A$

$= S_A^2 k$

$= S_A^t$

$\Rightarrow (k\, S_A)^3 = (k\, S_A)^t$

$\therefore$ $k\, S_A$ is cube symmetric NZM.

Hence the proof.

4.6. Proposition

For any NZM S_A be a k-idempotent NZM. Then the neutrosophic z-matrices $k\, S_A$ to be a square symmetric matrix iff

(i) S_A is idempotent

(ii) $S_A = S_A^t k$

Proof:

Let $k\, S_A$ is a square symmetric then $(k\, S_A)^2 = (k\, S_A)^t$

$\Rightarrow (S_A^2 k)^2 = S_A^t k$

$\Rightarrow S_A^4 k^2 = S_A^t k$

$\Rightarrow (S_A^2 k)(S_A^2 k) = S_A^t k$

$\Rightarrow S_A^3 = S_A^t k$ ……………………………(1)

Pre multiply k by (1)

kS_A^3=k $S_A^t k$ ……………………………..(2)
And S_A^3 k= S_A^t …………………………(3)
Sub (3) in (2)
S_A^t =k $S_A^t k$
$\Rightarrow S_A = S_A^2$
Hence S_A is idempotent and sub in eq(1)
We get $S_A = S_A^t k$
conversely, assume that S_A is idempotent with $S_A = S_A^t k$
$(k\ S_A)^2 = S_A^3$ [from note]
$= S_A$
$= S_A^t k$
$= (k\ S_A)^t$
$\therefore$ $k\ S_A$ is a square symmetric NZM.
Hence the proof.

4.7. Proposition

Let S_A be a k-idempotent NZM. Then the conditions are equivalent.
(i) $k\ S_A$ is k-cube symmetric NZM.
(ii) S_A is symmetric NZM.
(iii) S_A is k-square smmetric NZM.
(iv) $k\ S_A$ is k-symmetric NZM.

Proof:
a) (i)$\Rightarrow$ (ii)
$k\ S_A$ is cube symmetric that is

$$k(k\ S_A)^3\ k = (k\ S_A)^t$$

$\Rightarrow$k$(k\ S_A)k = S_A^t k$
$\Rightarrow S_A k = S_A^t k$
$\Rightarrow S_A = S_A^t$
Hence S_A is symmetric NZM.
b) (ii)$\Rightarrow$ (iii)
S_A is symmetric that is $S_A = S_A^t$
$S_A^t = kS_A^2 k$ [since S_A is idempotent]
Hence S_A is k-square symmetric NZM
c) (iii)$\Rightarrow$ (iv)
S_A is k-square symmetric we know $S_A^t = kS_A^2 k$ ………..(1)
Post multiply by k in (1)
$S_A^t = kS_A^2$
$\Rightarrow kS_A^2 = S_A^t k$
$= (k\ S_A)^t$
Multiply both sides by k
$S_A^2 k = k(k\ S_A)^t k$
$kS_A = k(k\ S_A)^t k$
$\therefore$ kS_A is k symmetric NZM.
d) (iv)$\Rightarrow$ (i)
kS_A is k symmetric we know $kS_A = k(k\ S_A)^t k$
But$(k\ S_A)^3 = kS_A$
$k(k\ S_A)^3 k = (k\ S_A)^t$
$\therefore$ kS_A is k-cube symmetric NZM.
Hence the proof.

4.8. Proposition

If S_A be a symmetric and k-square symmetric NZM then S_A is k-idempotent NZM.
Proof:
Assume that $S_A = S_A^t$ and $S_A^2 k = S_A^t$.
Combining above two condition we have $kS_A^2 k = S_A$.Hence S_A is k-idempotent NZM.
Hence the proof.

5. T-ordering on k-idempotent NZM

This chapter comprises a basic definition and some properties of T-ordering on NZM. In general, T-ordering is not necessarily partial ordering on NZM.

5.1. Definition

Let $S_A, S_B \in (NZM)_{m\times n}$ then the T-ordering NZM $S_A \leq^T S_B$ is defined as $S_A \leq^T S_B \Leftrightarrow S_A^t S_A = S_B S_A^t$ and $S_A S_A^t = S_B S_A^t$.

5.2. Definition

Let $S_A, S_B \in (NZM)_{m\times n}$ then the T-reverse ordering $S_A \geq^T S_B$ is defined as $S_A \geq^T S_B \Leftrightarrow S_B^t S_B = S_B S_A^t$ and $S_B S_B^t = S_A S_B^t$.

5.3. Proposition

Let $S_A, S_B \in (NZM)_{m\times n}$ are k-idempotent NZM, then $S_A \leq^T S_B$ iff $S_A^2 \leq^T S_B^2$.

Let $S_A, S_B \in (NZM)_{m\times n}$ are k-idempotent NZM, then $S_A \leq^T S_B$ iff $S_A^2 \leq^T S_B^2$.

Proof:

Let $S_A \leq^T S_B$, then

(i) $S_A^t S_A = S_A^t S_B$ and (ii) $S_A S_A^t = S_B S_A^t$

From (i)

$kS_A^t S_A k = kS_A^t S_B k$

$(kS_A^t k)(kS_A k) = kS_A^t kkS_B k$

$(S_A^t)^2 (S_A^2) = (S_A^t)^2 (S_B^2)$ (1)

From (ii)

$kS_A S_A^t k = kS_B S_A^t k$

$(kS_A k)(kS_A^t k) = (kS_B k)(kS_A^t k)$

$S_A^2 (S_A^t)^2 = S_B^2 (S_A^t)^2$ (2)

From (i) and (ii) we have

$S_A^2 \leq^T S_B^2$

Conversely, $S_A^2 \leq^T S_B^2$

$kS_A^2 \leq^T kS_B^2$

$kS_A^2 k \leq^T kS_B^2 k$

$S_A \leq^T S_B$

Hence the proof.

5.4. Proposition

Let $S_A, S_B \in (NZM)_{m\times n}$ and k be associated permutation NZM , then $S_A \leq^T S_B \Leftrightarrow kS_A \leq^T k\, S_B \Leftrightarrow S_A k \leq^T S_B k$

Proof:

$S_A \leq^T S_B \Leftrightarrow S_A^t S_A = S_A^t S_B$ and $S_A S_A^t = S_B S_A^t$

$\Leftrightarrow S_A^t kkS_A = S_A^t kkS_B$ and $kS_A S_A^t k = kS_B S_A^t k$

$\Leftrightarrow (kS_A)^t kS_A = (kS_A)^t kS_B$ and $(kS_A)^t = kS_B (kS_A)^t$ [since $k = (k)^t$

$\Leftrightarrow kS_A \leq^T k\, S_B$

Similarly, $S_A \leq^T S_B \Leftrightarrow S_A^t S_A \leq^T = S_A^t S_B$ and $S_A S_A^t \leq^T = S_B S_A^t$

$\Leftrightarrow kS_A^t S_A k = kS_A^t S_B k$ and $S_A kkS_A^t = S_B kkS_A^t$

$\Leftrightarrow (kS_A)^t S_A k = (kS_A)^t S_B$k and $S_A k (kS_A)^t = S_B k (S_A k)^t$

$\Leftrightarrow S_A k \leq^T S_B k$

Hence the proof.

5.5. Proposition

Let $S_A, S_B \in (NZM)_{m\times n}$ are k-idempotent NZM, then $S_A \geq^T S_B iff \Leftrightarrow S_A^2 \geq^T S_B^2$ Proof:

Let $S_A \geq^T S_B$, then

i) $S_B^t S_B = S_B^t S_A$ ii) $S_B S_B^t = S_A S_B^t$

Multiplying by k in (i)

$kS_B^t S_B k = kS_B^t S_A k$

$$kS_B^t\, kkS_B k = kS_B^t\, k\, kS_A$$
$$(S_B^t)^2 S_B^2 = (S_B^t)^2 S_A^2 \quad (1)$$

using (ii) we get

$$kS_B S_B^t\, k = kS_A S_B^t\, k$$
$$kS_B k\, kS_B^t\, k = kS_A\, k\, kS_B^t\, k$$
$$S_B^2\,(S_B^t)^2 = S_A^2\,(S_B^t)^2 \quad (2)$$

Using (1) & (2)

$$S_A^2 \geq^T S_B^2$$

Conversely, we take $S_A^2 \geq^T S_B^2$. Then,

$$kS_A^2 \geq^T \mathrm{k}\, S_B^2$$
$$\Leftrightarrow kS_A^2 k \geq^T \mathrm{k}\, S_B^2 \mathrm{k}$$
$$\Leftrightarrow S_A \geq^T S_B$$

Hence the proof.

5.6. Proposition

If k is the permutation NZM of k, then for every $S_A, S_B \in (NZM)_{m\times n}$ and $S_A \geq^T S_B \Leftrightarrow\Leftrightarrow kS_A \geq^T k\, S_B \Leftrightarrow S_A k \geq^T S_B k.$

Proof:

Let $S_A \geq^T S_B \Leftrightarrow S_B^t\, S_B = S_B^t\, S_A$ and $S_B S_B^t = S_A S_B^t$

$\Leftrightarrow S_B^t\, kkS_B = S_B^t\, kkS_A$ and $kS_B S_B^t\, k = kS_A S_B^t \mathrm{k}$

$\Leftrightarrow (kS_B)^t kS_B = (kS_B)^t kS_A$ and $kS_B S_B^t\, k = kS_A (kS_B)^t$

$\Leftrightarrow kS_A \geq^T kS_B$

Similarly,

$S_A \geq^T S_B \Leftrightarrow S_B^t\, S_B = S_B^t\, S_A$ and $S_B S_B^t = S_A S_B^t$

$\Leftrightarrow kS_B^t\, S_B k = kS_B^t\, S_A k$ and $kS_B kkS_B^t = S_A kkS_B^t$

$\Leftrightarrow (kS_B)^t S_B k = (kS_B)^t S_A k$ and $S_B k (S_B k)^t = S_A k (S_{Bk})^t$

$\Leftrightarrow S_A k \geq^T S_B k$

Hence the proof.

6. Proposed Method for Medical Diagnosis in NZM

We develop a decision-making process in this section that uses the NZM idea for medical diagnosis. To increase the effectiveness of identifying sickness in decision-making, two algorithms are included. To find precise values, another correlation measure algorithm was also presented. Additionally, three different algorithm types were shown in an experimental case study, and the results were compared and analyzed.

6.1. Type I Algorithm

1. A new NZM S_P is created for every ill person and their symptoms.
2. The judgment of a physician is considered for constructing a symptom-illness connection as NZM in S_Q.
3. Calculate using the max-min form in $S_P * S_Q$.
4. Determine the precise value using $U = \frac{1}{6}(< 2 + T_V + T_R - (1 - I_V) - (1 - I_R) - F_V - F_R >)$
5. A matrix is used to frame the calculated values.
6. Every row matrix is taken into account. The greater number is seen as the likelihood that a sick individual will get sick.

6.2. Type II Algorithm

1. A new NZM S_P is created for every ill person and their symptoms.
2. The judgment of a physician is considered for constructing a symptom-illness connection as NZM in S_Q.
3. Calculate complement of $S_Q$as S_Q^c.
4. Using min-max form calculate $S_P * S_Q^c$
5. Determine the precise value using $V = \frac{1}{6}(< 2 + T_V + T_R + I_V + I_R - F_V - F_R >)$
6. A matrix is used to frame the calculated values.
7. Every row matrix is taken into account. The least number is seen as the likelihood that a sick individual will get sick.

6.3. Correlation Measure

Let $S_P = (< \left(T^P_{V_{ij}}, T^P_{R_{ij}}\right), \left(I^P_{V_{ij}}, I^P_{R_{ij}}\right), \left(F^P_{V_{ij}}, F^P_{R_{ij}}\right) >)$ and

$S_Q = (< \left(T^Q_{V_{ij}}, T^Q_{R_{ij}}\right), \left(I^Q_{V_{ij}}, I^Q_{R_{ij}}\right), \left(F^Q_{V_{ij}}, F^Q_{R_{ij}}\right) >)$be two NZMs of order $p \times q$. Then the correlation measure for NZM is defined as

$$\rho_{NZM}(S_{Pij}, S_{Qij}) = \frac{C_{NZM}(S_{Pij}, S_{Qij})}{\sqrt{(C_{NZM}(S_{Pij}, S_{pij}) * C_{NZM}(S_{Qij}, S_{Qij})}} \quad (1)$$

$$(C_{NZM}(S_{Pij}, S_{Qij}) = \frac{1}{6pq}(< \sum_{j=1}^{q}\sum_{i=1}^{p}(T^P_{V_{ij}}(x_{ij}).(T^Q_{V_{ij}}(x_{ij})) + (T^P_{R_{ij}}(x_{ij}).(T^Q_{R_{ij}}(x_{ij})) + I^P_{V_{ij}}(x_{ij}).(I^Q_{V_{ij}}(x_{ij})) + I^P_{R_{ij}}(x_{ij}).(I^Q_{R_{ij}}(x_{ij})) + F^P_{V_{ij}}(x_{ij}).(F^Q_{V_{ij}}(x_{ij})) + F^P_{R_{ij}}(x_{ij}).(F^Q_{R_{ij}}(x_{ij})) >) \quad (2)$$

$$(C_{NZM}(S_{Pij}, S_{Pij}) = \frac{1}{6pq}(< \sum_{i=1}^{p}\sum_{i=1}^{p}(T^P_{V_{ij}}(x_{ij}).(T^P_{V_{ij}}(x_{ij})) + (T^P_{R_{ij}}(x_{ij}).(T^P_{R_{ij}}(x_{ij})) + I^P_{V_{ij}}(x_{ij}).(I^P_{V_{ij}}(x_{ij})) + I^P_{R_{ij}}(x_{ij}).(I^P_{R_{ij}}(x_{ij})) + F^P_{V_{ij}}(x_{ij}).(F^P_{V_{ij}}(x_{ij})) + F^P_{R_{ij}}(x_{ij}).(F^P_{R_{ij}}(x_{ij})) >) \quad (3)$$

$$(C_{NZM}(S_{Qij}, S_{Qij}) = \frac{1}{6pq}(< \sum_{j=1}^{q}\sum_{j=1}^{q}(T^Q_{V_{ij}}(x_{ij}).(T^Q_{V_{ij}}(x_{ij})) + (T^Q_{R_{ij}}(x_{ij}).(T^Q_{R_{ij}}(x_{ij})) + I^Q_{V_{ij}}(x_{ij}).(I^Q_{V_{ij}}(x_{ij})) + I^Q_{R_{ij}}(x_{ij}).(I^Q_{R_{ij}}(x_{ij})) + F^Q_{V_{ij}}(x_{ij}).(F^Q_{V_{ij}}(x_{ij})) + F^Q_{R_{ij}}(x_{ij}).(F^Q_{R_{ij}}(x_{ij})) >) \quad (4)$$

Type III Algorithm Correlation Measure

1. A new NZM S_P is created for every ill person and their symptoms.
2. The judgment of a physician is considered for constructing a symptom-illness connection as NZM in S_Q.
3. Compute correlation measures$C_{NZM}(S_{Pij}, S_{Qij})$, $C_{NZM}(S_{Pij}, S_{Pij})$, $C_{NZM}(S_{Qij}, S_{Qij})$ using formula 2-4.
4. The computed values are framed as table 1.
5. Compute the values for $\rho_{NZM}(S_{Pij}, S_{Qij})$
6. The computed values are framed as matrix.
7. Each row matrix is considered. The highest value is taken as the risk of a sick person suffering illness.

6.4. Implementation of the algorithm

Analyzing a patient's symptoms of a health issue is the most difficult task in decision-making because of the many complexities of uncertainty and vague, ambiguous information. Applying NZM can solve the issue because it guarantees accurate information for every triplet data set of neutrosophic membership values. In medical diagnosis, a doctor can determine a patient's condition or disease severity based solely on the patient's medical history or ongoing observation. Five consecutive observation results were employed in the case analysis performed in this paper.

6.5 Case Study

Let P=(P1,P2,P3) be a set of sick person, D = {Viral Fever, Malaria, Flu} be a group of people who are ill, and let S = {Temperature, Cough, Throat Pain, Headache} be a group of symptoms. A medical diagnosis is made in connection with a particular condition and a particular ailment.

Type I Method

S_P	Temperature	Cough	Throat Pain	Head Ache
P1	< (0.8,0.6), (0.7,0.4), (0.2,0.3) >	< (0.7, 0.6), (0.5,0.4), (0.2,0.1) >	< (0.6, 0.4), (0.3,0.2), (0.1,0.1) >	< (0.5, 0.6), (0.4,0.3), (0.2,0.1) >
P2	< (0.5, 0.6), (0.4,0.3), (0.2,0.1) >	< (0.8, 0.3), (0.6,0.5), (0.3,0.2) >	< (0.4,0.3), (0.6,0.7), (0.4,0.5) >	< (0.6,0.5), (0.4,0.5), (0.3,0.2) >
P3	< (0.6,0.7), (0.3,0.4), (0.3,0.2) >	< (0.5,0.3), (0.3,0.4), (0.2,0.1) >	< (0.8,0.6), (0.7,0.5), (0.4,0.3) >	< (0.6,0.5), (0.4,0.3), (0.2,0.1) >

S_Q =	Viral Fever	Malaria	Flu
Temperature	< (0.6,0.5), (0.4,0.3), (0.3,0.2) >	< (0.7,0.3), (0.4,0.3), (0.2,0.1) >	< (0.5,0.5), (0.4,0.3), (0.2,0.1) >
Cough	< (0.8,0.3), (0.5,0.4), (0.2,0.1) >	< (0.6,0.7), (0.2,0.1), (0.3,0.2) >	< (0.6,0.5), (0.3,0.2), (0.1,0.1) >
Throat Pain	< (0.4,0.3), (0.2,0.1), (0.1,0.1) >	< (0.6,0.5), (0.4,0.3), (0.1,0.1) >	< (0.8,0.6), (0.7,0.3), (0.2,0.1) >
Headache	< (0.5,0.6), (0.7,0.8), (0.4,0.3) >	< (0.8,0.6), (0.5,0.4), (0.3,0.2) >	< (0.8,0.3), (0.5,0.4), (0.3,0.2) >

Calculate $S_P * S_Q$ using max-min form

$$S_P * S_Q = \begin{bmatrix} & Viral\ Fever & Malaria & Flu \\ P1 & <(0.7,0.6),(0.3,0.2),(0.1,0.1)> & <(0.7,0.6),(0.4,0.3),(0.1,0.1)> & <(0.6,0.5),(0.5,0.3),(0.1,0.1)> \\ P2 & <(0.8,0.5),(0.4,0.3),(0.3,0.2)> & <(0.6,0.6),(0.4,0.3),(0.2,0.1)> & <(0.6,0.5),(0.4,0.3),(0.2,0.1)> \\ P3 & <(0.6,0.5),(0.5,0.4),(0.2,0.1)> & <(0.5,0.3),(0.3,0.2),(0.2,0.1)> & <(0.8,0.6),(0.3,0.2),(0.2,0.1)> \end{bmatrix}$$

Compute accurate value using U

$$S_P * S_Q = \begin{bmatrix} & Viral\ Fever & Malaria & Flu \\ P1 & <(\mathbf{0.433})> & <(0.4)> & <(0.35)> \\ P2 & <(0.35)> & <(\mathbf{0.366})> & <(0.35)> \\ P3 & <(0.3166)> & <(0.3833)> & <(\mathbf{0.433})> \end{bmatrix}$$

The highest value in the row matrix form $S_P * S_Q$represent the higher risk of sick person suffering by illness.The highest value of sick person P_1 is 0.4333 and P_2 is 0.366 and P_3 is 0.433. Then P_1 contracts a viral fever, P_2 contracts malaria, and P_3 contracts the flu.

Type II Method

In this algorithm we use step 1 and step 2 from type I method.
Then calculate complement of S_Q

$$S_Q^c = \begin{bmatrix} <(0.3,0.2),(0.6,0.7),(0.6,0.5)> & <(0.2,0.1),(0.6,0.7),(0.7,0.3)> & <(0.2,0.1),(0.6,0.7),(0.5,0.5)> \\ <(0.2,0.1),(0.5,0.6),(0.8,0.3)> & <(0.3,0.2),(0.8,0.9),(0.6,0.7)> & <(0.1,0.1),(0.7,0.8),(0.6,0.5)> \\ <(0.1,0.1),(0.8,0.9),(0.4,0.3)> & <(0.1,0.1),(0.6,0.7),(0.6,0.5)> & <(0.2,0.1),(0.3,0.7),(0.8,0.6)> \\ <(0.4,0.3),(0.3,0.8),(0.5,0.6)> & <(0.3,0.2),(0.5,0.6),(0.2,0.4)> & <(0.3,0.2),(0.5,0.6),(0.8,0.3)> \end{bmatrix}$$

$$\begin{bmatrix} & Viral & Malaria & flu \\ Temperature & <(0.3,0.2),(0.6,0.7),(0.6.0.5)> & <(0.2,0.1),(0.6,0.7,(0.7,0.3)> & <(0.2,0.1),(0.6,0.7),(0.5,0.5)> \\ Cough & <(0.2,0.1),(0.5,0.6),(0.8,0.3> & <(0.3,0.2),(0.8,0.9),(0.6,0.7)> & <(0.1,0.1),(0.7,0.8),(0.6,0.5)> \\ Throat\ Pain & <(0.1,0.1),(0.8,0.9),(0.4,0.3)> & <(0.1,0.1),(0.6,0.7),(0.6,0.5)> & <(0.2,0.1),(0.3,0.7),(0.8,0.6)> \\ Headache & <(0.4,0.3),(0.3,0.82),(0.5,0.6)> & <(0.3,0.2),(0.5,0.6),(0.2,0.4)> & <(0.3,0.2),(0.5,0.6),(0.8,0.3)> \end{bmatrix}$$

Using min-max form calculate $S_P * S_Q^c$

$$S_P * S_Q^c = \begin{bmatrix} & Viral\ Fever & Malaria & Flu \\ P1 & <(0.5,0.4),(0.6,0.6),(0.5,0.6)> & <(0.5,0.4),(0.6,0.4),(0.2,0.3)> & <(0.5,0.4),(0.6,0.4),(0.2,0.3)> \\ P2 & <(0.4,0.3),(0.6,0.7),(0.4,0.3)> & <(0.4,0.3),(0.6,0.7),(0.6,0.7)> & <(0.4,0.3),(0.7,0.8),(0.8,0.6)> \\ P3 & <(0.5,0.3),(0.7,0.5),(0.4,0.3)> & <(0.5,0.3),(0.6,0.5),(0.4,0.3)> & <(0.5,0.3),(0.5,0.4),(0.4,0.3)> \end{bmatrix}$$

Compute accurate value using V.

$$S_P * S_Q^c = \begin{bmatrix} & Viral\ Fever & Malaria & Flu \\ P1 & <(\mathbf{0.5})> & <(0.5666)> & <(0.5666)> \\ P2 & <(0.55)> & <(\mathbf{0.45})> & <(0.4666)> \\ P3 & <(0.55)> & <(0.5333)> & <(\mathbf{0.5})> \end{bmatrix}$$

The lowest value in the row matrix form $S_P * S_Q^c$ represent the higher risk of sick person suffering by disease.The lowest value of sick person P_1 is 0.5 and P_2 is 0.45 and P_3 is 0.5. Then P_1 contracts a viral fever, P_2 contracts malaria, and P_3 contracts the flu.

Type III Method Correlation Measure

$$\begin{bmatrix} & Temperature & Cough & ThroatPain & HeadAche \\ P_1 & <(0.8,0.6),(0.7,0.4),(0.2,0.3)> & <(0.7,0.6),(0.5,0.4),(0.2,0.1)> & <(0.6,0.4),(0.3,0.2),0.1,0.1)> & <(0.5,0.6),(0.4,0.3),(0.3,0.2)> \\ P_2 & <(0.5,0.6),(0.4,0.3),(0.2,0.1)> & <(0.8,0.3),(0.6,0.5),(0.3,0.2)> & <(0.4,0.3),(0.6,0.7),(0.4,0.5)> & <(0.6,0.5),(0.4,0.5),(0.3,0.2)> \\ P_3 & <(0.6,0.7),(0.5.0.4),(0.3,0.2)> & <(0.5,0.3),(0.3,0.2),(0.2,0.1)> & <(0.8,0.6),(0.7,0.6),(0.4,0.3)> & <(0.6,0.5),(0.4,0.3),(0.2,0.1) \end{bmatrix}$$

$$\begin{bmatrix} & Viral\ fever & Malaria & Flu \\ Temperature & <(0.6,0.5),(0.4,0.3),(0.3,0.2)> & <(0.7,0.8),(0.4,0.3),(0.2,0.1)> & <(0.5,0.5),(0.4,0.3),(0.2,0.1)> \\ Cough & <(0.8,0.3),(0.5,0.4),(0.2,0.1)> & <(0.4,0.3),(0.2,0.1),(0.3,0.2)> & <(0.6,0.5),(0.3,0.2),(0.1,0.1)> \\ Throat\ Pain & <(0.4,0.3),(0.2,0.1),(0.1,0.1)> & <(0.6,0.5),(0.4,0.3),(0.1,0.1)> & <(0.8,0.6),(0.7,0.3),(0.2,0.1)> \\ Headache & <(0.5,0.6),(0.7,0.8),(0.4,0.3)> & <(0.8,0.6),(0.5,0.4),(0.3,0.2) & <(0.8,0.3),(0.5,0.4),(0.3,0.2)> \end{bmatrix}$$

Compute correlation measures $C_{NZM}(S_{Pij}, S_{Qij})$, $C_{NZM}(S_{Pij}, S_{Pij})$, $C_{NZM}(S_{Qij}, S_{Qij})$.

Table 1: Correlated Values

	Viral Fever	Malaria	Flu
P_1	$\begin{pmatrix} C_{NZM}(S_{Pij},S_{Qij}) = 0.1787 \\ C_{NZM}(S_{Pij},S_{Pij}) = 0.1945625 \\ C_{NZM}(S_{Qij},S_{Qij}) = 0.187065 \end{pmatrix}$	$\begin{pmatrix} C_{NZM}(S_{Pij},S_{Qij}) = 0.172495 \\ C_{NZM}(S_{Pij},S_{Pij}) = 0.1945625 \\ C_{NZM}(S_{Qij},S_{Qij}) = 0.185829 \end{pmatrix}$	$\begin{pmatrix} C_{NZM}(S_{Pij},S_{Qij}) = 0.1729 \\ C_{NZM}(S_{Pij},S_{Pij}) = 0.1945625 \\ C_{NZM}(S_{Qij},S_{Qij}) = 0.18579 \end{pmatrix}$
P_2	$\begin{pmatrix} C_{NZM}(S_{Pij},S_{Qij}) = 0.176232 \\ C_{NZM}(S_{Pij},S_{Pij}) = 0.204825 \\ C_{NZM}(S_{Qij},S_{Qij}) = 0.187065 \end{pmatrix}$	$\begin{pmatrix} C_{NZM}(S_{Pij},S_{Qij}) = 0.16999 \\ C_{NZM}(S_{Pij},S_{Pij}) = 0.204825 \\ C_{NZM}(S_{Qij},S_{Qij}) = 0.185829 \end{pmatrix}$	$\begin{pmatrix} C_{NZM}(S_{Pij},S_{Qij}) = 0.16874 \\ C_{NZM}(S_{Pij},S_{Pij}) = 0.204825 \\ C_{NZM}(S_{Qij},S_{Qij}) = 0.18579 \end{pmatrix}$
P_3	$\begin{pmatrix} C_{NZM}(S_{Pij},S_{Qij}) = 0.16333 \\ C_{NZM}(S_{Pij},S_{Pij}) = 0.200415 \\ C_{NZM}(S_{Qij},S_{Qij}) = 0.187065 \end{pmatrix}$	$\begin{pmatrix} C_{NZM}(S_{Pij},S_{Qij}) = 0.1787475 \\ C_{NZM}(S_{Pij},S_{Pij}) = 0.200415 \\ C_{NZM}(S_{Qij},S_{Qij}) = 0.185829 \end{pmatrix}$	$\begin{pmatrix} C_{NZM}(S_{Pij},S_{Qij}) = 0.184995 \\ C_{NZM}(S_{Pij},S_{Pij}) = 0.200415 \\ C_{NZM}(S_{Qij},S_{Qij}) = 0.18579 \end{pmatrix}$

Compute the values for $\rho_{NZM}(S_{Pij}, S_{Qij})$

$$\rho_{NZM}(S_{Pij}, S_{Qij}) = \begin{bmatrix} & Viral\,Fever & Malaria & Flu \\ P1 & <(\mathbf{0.9367})> & <(0.907173)> & <(0.909398)> \\ P2 & <(0.830305)> & <(\mathbf{0.871314})> & <(0.86499)> \\ P3 & <(0.84354)> & <(0.926228)> & <(\mathbf{0.958702})> \end{bmatrix}$$

The highest value in the row matrix form $\rho_{NZM}(S_P, S_Q)$ represent the higher risk of sick person suffering by illness. The highest value of sick person P_1 is 0.9367 and P_2 is 0.871314 and P_3 is 0.958702. Then P_1 contracts a viral fever, P_2 contracts malaria, and P_3 contracts the flu.

7. Comparison and Discussion

Comparative comments on the suggested decision-making method.

- The suggested method was successful in determining the best option for identifying the patient's ailment.
- Algorithms I and III yield larger optimality values, while algorithm II yields the lowest optimal value. After computing, the NZM lowers to a simpler matrix form, improving the results' display.
- Viral fever affects patient P1, malaria affects patient P2, and flu affects patient P3. All three approaches yield consistent and dependable optimal results.

8. Conclusion

This research work proposes a new ideology of k-idempotent NZM, which is a special form of idempotent NZM along with a permutation matrix. The k-idempotent NZM properties and some characters were investigated. The power NZM of k-idempotent NZM was introduced with a definition and conditions. T-ordering of k-idempotent NZM was also discussed with some properties. The proposed methods of algorithms using max-min operation, min-max operation, and correlation measures of NZM were established. The three different algorithms were implemented step by step in the experimental case, and the result was interpreted to get the optimal solution in medical diagnosis. The future development of NZM can be happened for partial ordering in NZM and pseudo-similar in NZM. For dimensionality reduction and decision-making, the NZM correlation measure can be improved to PCA. In the future, the NZM correlation measure may be expanded to include similarity metrics that can be applied to image processing, pattern recognition, and robotics selection for decision-making.

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