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Research paper

Optimal and efficient production of rose coco beans through the twenty-four points second order rotatable design

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Abstract

The yield results of the twenty four points response surface methodology (RSM) design permitted a response surface to be fitted easily and provided spherical information contours besides the realizations of an optimum combination of the fertilizers in *rose coco* beans, which resulted in economic use of scarce resources for optimal production of *rose coco* beans. In this study an existing A-optimum and D-efficient second order rotatable design in three dimensions was used to produce *rose coco* beans optimally and efficiently. The general objective of the study was to produce *rose coco* beans (*Phaseolus vulgaris*) optimally and efficiently using an existing A-optimum and D-efficient twenty four points second order rotatable design in three dimensions in a greenhouse setting using three inorganic fertilizers, namely, nitrogen, phosphorus and potassium. Thus the study was accomplished using the calculus optimum value of the free/letter parameter f=1.1072569. The specific objectives were to estimate the linear parameters, thereby making available for the yield response of *rose coco* beans at calculus optimum value design for the first time, fitted and tested the model adequacy via lack of fit test, and then found the setting of the experimental factors that produces optimal response using contour plots to assist visualizes the response surfaces. This study demonstrated the importance of statistical methods in the optimal and efficient production of *rose coco* beans. The results showed that the three factors: nitrogen, phosphorus, and potassium contributed significantly on the yield of *rose coco* beans ($p<0.05$). The regression coefficients were determined by employing least square's techniques to predict quadratic polynomial model for group 3 greenhouse (GP3G) for the three fertilizer combinations. In GP3G, the second-order model was adequate at 1% level of significance with a p-value of 0.0034. The analysis of variance (ANOVA) of response surface for *rose coco* yield showed that this design was adequate due to satisfactory level of a coefficient of determination, R^2, 0.8066 (GP3G) and coefficient variation, CV was 10.30. The canonical analysis showed that there was the saddle point for GP3G, meaning there was no unique optimum; therefore, ridge analysis was used to overcome the saddle problem. The result from ridge analysis provided the maximum yield of 70.25grams for the three fertilizer combinations at radii of one. We, therefore, recommend the use of GP3G design since it gave the required coefficient of determination (R^2=80.66) and the maximum yield (70. 25grams) was achieved.

Keywords: *Response Surface Methodology; Second Order Design; Optimality; Coded Levels; Natural Levels; Calculus Optimum Value; Rose Coco Beans.*

1. Introduction

In this study an existing A-optimum and D-efficient second order rotatable design in three dimensions was used to produce *rose coco* beans optimally and efficiently for parameter estimation. The research was done with a combination of independent treatment factors of inorganic fertilizers to check how the factors of fertilizers influence the yield of *rose coco* beans. The experiment was carried out in a greenhouse of size 15m x 10m during the period of February-July 2016 on the twenty four points second order rotatable design in three dimensions. The focus of this paper was to achieve an optimal and efficient production of *rose coco* beans (*Phaseolus vulgaris*) through an existing A-optimum and D-efficient second order rotatable design of twenty four points in three dimensions in a greenhouse setting using three inorganic fertilizers, namely, nitrogen, phosphorus and potassium. The study was geared to estimate the linear parameters, by making available for the yield response of *rose coco* beans at calculus optimum value design in one of the existing six specific second orders rotatable designs in three dimensions in which Mutiso [11] calculated the calculus optimal value to be 1.1072569 for the free/letter parameter. Koech [8] calculated the relative efficiencies for the six designs and their optimality criteria. Koech [8] showed that the twenty four point second order rotatable design was the most D-efficient and A-optimal design. Therefore, out of these researches we dwell much on the twenty four points, second order rotatable design and proceeded to have a practical greenhouse experiment to realize the optimal and efficient production of *rose coco* bean using the three inorganic fertilizer components. We also fitted and tested the model adequacy via lack of fit test, and then found the settings of the experimental factors that produced optimal response using contour plots to assist visualizes the response surfaces. Generally, agricultural researchers are in constant search for new or improved technologies to increase productivity. The researchers are usually interested in finding maximum or optimal yield using minimum cost. Therefore, achieving the production efficiency of *rose coco* has not been easy that this research tends to employ twenty four points second order rotatable design approach, focusing on the area of response surfaces, modeling *rose coco* yield production and highlighting the respective point of intercept and gradient levels of fertilizer with which the variety of *rose coco* beans were capable of delivering the bean production efficiency. The study founds how the yield fitted the second-order model and checked its model adequacy. The regression equation was fitted between the response variable, *rose coco* yield and the three fertilizer treatments, nitrogen (N), phosphorus (P) and potassium (K). The expected yield could be described as a continuous function of the application rate factor. If the fertilizer application rates are greater or smaller than the optimum application rates, they might result in a reduction in the yields. The purpose of implementing this RSM technique is to determine the optimum levels of fertilizer used in order to optimize *rose coco* yields. In any treatment arrangement, we sought a treatment or treatment combination that could be used to advance the current methods being used in farming in order to maximize the yield using the scarce resources available. It was seen how different components of fertilizers affect the output of the beans. The study therefore was used to determine; the best possible level of the identified fertilizer to maximize the yield in *rose coco* beans.

2. Response surface methodology

The response surface methodology (RSM) is a collection of mathematical and statistical techniques useful for the modeling and analysis of problems in which a response of interest is influenced by several variables, and the objective is to optimize this response [10]. In most RSM problems, the form of the relationship between the response and the independent variables is unknown. Thus the first step in RSM is to find a suitable approximation for the true functional relationship between the response and the set of independent variables, which are subject under the control of the scientist or engineer. In order to get the most efficient result in the approximation of polynomials the proper experimental design must be used to collect data. The representation $\mathbf{y} = \mathbf{x}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$ was given where $\mathbf{y}$ is a vector of observations, $\boldsymbol{\varepsilon}$ is the vector of errors, $\mathbf{x}$ is the design matrix and $\boldsymbol{\beta}$ is a vector of unknown model coefficients. The design matrix was a set of combinations of the values of the coded variables, which specifies the settings of the design parameters to be performed during experimentation. The $\boldsymbol{\beta} = (\mathbf{x'x})^{-1}\mathbf{x'y}$ was used to estimate parameters in the polynomials by using the least squares' method. In this study, we concentrate on determining the optimum operating conditions for the system using response surfaces and statistical modeling to develop an appropriate approximating model between the response y and independent variables x_1 (nitrogen), x_2 (phosphorus), x_3 (potassium). The second-order model is likely required in these situations. In the case of three variables, the second-order model is given by:

$$\hat{y} = \beta_0 + \sum_i^k \beta_i x_i + \sum_i^k \beta_{ii} x_i^2 + \sum \sum_{i<j=2}^k \beta_{ij} x_i x_j \tag{1}$$

Where $\hat{y}$ is the measured **rose coco** response, β_0 is the intercept term, β_i are the linear coefficients, β_{ij} is the logarithmic coefficient, β_{ii} are the quadratic coefficients and x_{1u} denotes the coded level of the i^{th} factor (i=1, 2, 3) in the u^{th} run (u=1, 2... 24) of the experiment. The parameters of the model β_0, β_i, β_{ii}, β_{ij} are estimated by least squares estimation to provide $\hat{\beta}_0$, $\hat{\beta}_i$, $\hat{\beta}_{ii}$, $\hat{\beta}_{ij}$. The β's are a set of unknown parameters. To estimate the values of these parameters, we must collect data on the system we are studying. It is said to be SORD (second order rotatable design) if the variance of the estimate of the response $\hat{Y}_u$ is only a function of the distance $\left(d^2 = \sum_{i=1}^k x_i^2\right)$ of the point ($x_1, x_2 \ldots x_k$) from the origin (centre) of the design [7]. This model would likely be useful as an approximation to the true response surface in a relatively small region. The low and high factor settings are coded as negative and positive, the midpoint coded as 0.The second order model, usually fits by ordinary least squares, is here represented for a single response y_u, u=1…N.

3. Experiment layout of twenty four points calculus optimum value design

The *rose coco* plants were planted on spacing of 75x30cm with N, P and K fertilizers during planting, and thereafter no other supplements were added either by top dressing or foliar spray. The *rose coco* beans were subjected to inorganic fertilizer's N, P, K at different levels of a twenty four point second order rotatable design. Therefore, GP3G was given three replications; for example, in GP3G we had GP3GA, GP3GB and GP3GC. In GP3G a combination of 30 grams of nitrogen, 40grams of phosphorus and 50grams of potassium were the initial fertilizers applied to *rose coco* beans and acted as the center point. This group was given the straight N, P & K fertilizers, such that GP3GA, GP3GB and GP3GC were the three replications with each having twenty four *rose coco* plants. In the greenhouse, organic matter on the soil surface was cleared. The field was prepared by *Jembe* ploughing followed by harrowing until fine tillage was obtained. We had three replications of twenty four design points. Certified, viable and uniform seeds of *rose coco* beans were planted in the plots on February 2016. The beans were planted as a pure stand in a greenhouse. Before planting, the beans' seeds were dressed with Aldrin at the rate of 5g per kg of seeds, to control soil pests especially been flown (*Melanargromyza phaseoli*). Furadan (5% carbofuran) was applied in the bean rows at sowing to control cutworms (*Agrotisipsilon*). All the plots were given a drip irrigation using drip line pipes. The inorganic fertilizers (nitrogen, phosphorus, potassium) combinations were applied in each experimental unit before planting two seeds of *rose coco* beans, and one week after germination, they were thinned to one plant per experimental unit. First weeding was carried out on the greenhouse at two weeks after emergence. Second weeding was carried out four weeks later. We consider a set of twenty four point's rotatable designs as highlighted by Draper [4], Mutiso [11] and Koske [6] is given as:

$$D_1 = [\tfrac{1}{2}G(f, f, 0) + \tfrac{1}{4}G(c_1, 0, 0) + \tfrac{1}{4}G(c_2, 0, 0)] \tag{2}$$

4. A practical greenhouse example

The rotatable design (3) was set up in a greenhouse in an area in Saroiyot, Kesses, Uasin-Gishu County in Kenya in order to investigate the effects of three fertilizer ingredient's N, P, K on the yield of *rose coco* beans. The fertilizer ingredients and actual amount applied were nitrogen (N) x_{1u}, Ψ_1. =30 grams/hole; phosphorus (P) x_{2u}, Ψ_2.=40 grams/hole; and potassium (K) x_{3u},Ψ_3.=50 grams/hole. The response of interest was the average yield of *rose coco* in mg per plant. The set of twenty four points in (2) form a second-order rotatable arrangement in three factors. The calculations done by Koske [6] showed that f=1.1072569, c_1=0.7829487, c_2=1.2735263, hence from (2) the design D_1 yields an optimum design as:

$$D_2 = [\tfrac{1}{2}G(1.1072569, 1.1072569, 0) + \tfrac{1}{4}G(0.7829487, 0, 0) + \tfrac{1}{4}G(1.2735263, 0, 0)] \tag{3}$$

Let the scale parameters S_i, assume s_1=0.5, s_2=0.3 and s_3=1. According to Box [2] and Box and Wilson [3] it can be reverted to the natural levels denoted by Ψ_{iu} where Bose and Draper [1] scaling condition fixes a particular design when λ_2=1 where the actual form of the coding operation for each value of a variable is given by:

$$x_{iu}=\frac{\psi_{iu}-\psi_{i\bullet}}{s_i},\ \psi_{i\bullet}=\frac{\sum_{u=1}^{N}\psi_{iu}}{N},\ s_i=\left[\frac{\sum_{u=1}^{N}(\psi_{iu}-\psi_{i\bullet})^2}{N}\right]^{\frac{1}{2}},\ \psi_{iu}=x_{iu}s_i+\psi_{i\bullet},$$

$$\sum_{u=1}^{N}x_{iu}^2=N,\ \sum_{u=1}^{N}x_{iu}=0. \qquad (4)$$

The graphical visualization is very helpful in understanding the second-order response surface. Specifically, contour plots can help characterize the shape of the surface and locate the optimum response roughly. The experimenter tries to quantify the relationship between a set of 3 predictor variables $\Psi_i=(\Psi_1,\Psi_2,\Psi_3)$ and the response variable y. Often the goal of the experiment has been to maximize or minimize E(y), the expected value of the response. In most cases, the Ψ_i are transformed into coded x_{iu} by $(\Psi_{iu}-\Psi_{i.})/S_i$, i=1, 2, 3 where Ψ_{iu} and $S_i>0$ are the centering and scaling constants, respectively. Often, a second order model fit to the experimental data, including all linear, quadratic and cross product terms for the x_{iu}.

The table 1 below shows the yield which was obtained by Tum [15] in the research for twenty four point's rotatable design of coded levels and natural levels with the yield of *rose coco* beans where x_1u, x_2u and x_3u are coded values while Ψ_1u, Ψ_2u and Ψ_3u are natural values. The natural values (Ψ_iu) of fertilizers at the ratio of 30:40:50 of N: P: K fertilizers, were measured using a sensitive weighing scale and planted in a greenhouse which gave the observed yield in grams -y_i and predicted yield $\hat{y}$ using the second order model of GP3G in (5).

The statistical significance of the model equation (5) was determined using Fisher's test value (p value) and significance of each coefficient was determined by t-test, and the extent of variance that could be explained by the model was determined by the multiple coefficient of determination, R squared (R^2) value, this assess the fitness of the polynomial model [13]. When R^2 approaches unity, the better the empirical model fits the actual data. The smaller the value of R^2, the less relevant the dependent variables in the model have in explaining the behavior variation [9]. Later an experiment was run using the optimum values for the variables given by response optimization in order to validate the predicted optimum values of variable response of the *rose coco*.

Table 1: Twenty Four Point's Rotatable Design of Coded Levels and Natural Levels with the Yield of Rose Coco Beans-GP3G at the Ratio of 30:40:50 N: P: K Fertilizers

(x_1u	x_2u	x_3u)	Ψ_1u	Ψ_2u	Ψ_3u	Observed Yield- y_i	Predicted yield- $\hat{y}$
1.1072569	1.1072569	0	30.553628	40.332177	50	47	53.3952
-1.1072569	1.1072569	0	29.446372	40.332177	50	82	75.6594
1.1072569	-1.1072569	0	30.553628	39.667823	50	68	75.4937
-1.1072569	-1.1072569	0	29.446372	39.667823	50	50	44.7579
1.1072569	0	1.1072569	30.553628	40	51.107257	69	63.6074
-1.1072569	0	1.1072569	29.446372	40	51.107257	53	59.8717
1.1072569	0	-1.1072569	30.553628	40	48.892743	87	78.5079
-1.1072569	0	-1.1072569	29.446372	40	48.892743	70	73.7722
0	1.1072569	1.1072569	30	40.332177	51.107257	67	60.8035
0	-1.1072569	1.1072569	30	39.667823	51.107257	73	67.9020
0	1.1072569	-1.1072569	30	40.332177	48.892743	84	86.7041
0	-1.1072569	-1.1072569	30	39.667823	48.892743	67	70.8025
0.7829487	0	0	30.391474	40	50	60	55.2395
-0.7829487	0	0	29.608526	40	50	51	52.2443
0	0	0.7829487	30	40	50.782949	51	53.2638
0	0	-0.7829487	30	40	49.217051	72	63.4465
0	0.7829487	0	30	40.234885	50	51	56.6047
0	-0.7829487	0	30	39.765115	50	54	53.4924
1.2735263	0	0	30.636763	40	50	57	59.4536
-1.2735263	0	0	29.363237	40	50	55	54.5818
0	0	1.2735263	30	40	51.273526	55	60.9417
0	0	-1.2735263	30	40	48.726474	75	77.5047
0	1.2735263	0	30	40.382058	50	63	63.0059
0	-1.2735263	0	30	39.617942	50	58	57.9435

Table 2: The Estimated Effects & Coefficients of the Empirical Model for GP3G

Parameter	DF	Estimate	Standard Error	t Value	Pr> |t|
Intercept	1	51.751435	3.410664	15.17	<.0001***
N	1	1.912726	1.725989	1.11	0.2865
P	1	1.987578	1.725989	1.15	0.2688
K	1	-6.502796	1.725989	-3.77	0.0021**
N*N	1	3.247054	2.335319	1.39	0.1861
P*N	1	-10.807347	2.659765	-4.06	0.0012**
P*P	1	5.378536	2.335319	2.30	0.0371*
K*N	1	-0.203912	2.659765	-0.08	0.9400
K*P	1	-4.689981	2.659765	-1.76	0.0997•
K*K	1	10.772613	2.335319	4.61	0.0004***

Signif. Codes: '***'significant at 0.001, '**'significant at 0.01, '*'significant at 0.05,'●'significant at 0.1, '' significant at 1.

Table 2 lists the regression coefficients and the corresponding p-values for the second-order polynomial model given as:

$$\begin{aligned}\hat{Y}_{GP3G}&=51.7514+1.9127X_1+1.9876X_2\\&-6.5028X_3+3.2471X_1^2+5.3785X_2^2+10.7726X_3^2\\&-10.8073X_1X_2-0.2039X_1X_3-4.6900X_2X_3\end{aligned} \qquad (5)$$

The $\hat{Y}_{GP3G}$ is the predicted response for *rose coco* beans in group 3 (GP3G). The regression coefficients of the linear term for potassium (p=0.0021) have significant effects on the yield (p-value <0.05), the quadratic P^2 (p=0.0371) and K^2 (p=0.0004) have significant effects on the yield and the interaction terms in N*P (nitrogen and phosphorus, p=0.0012) is significant. Among these, K, K^2, N*P was significant at the 1% significance level, while P^2 was significant at the 5% level, those other terms of the model showed no significant effect on the yield. The positive coefficients of P^2, K^2 enhance the yield since they are the largest coefficients in the model eq. 5. The largest negative coefficient of K and NP minimizes the yield of *rose coco* at their respective fertilizer input. The X_1 represents nitrogen (N), X_2 represents phosphorus (P), X_3 represents potassium (K).

This suggests that potassium (K), quadratic effects of phosphorus (P^2), potassium (K^2) and interaction N*P were the determining significant factors on the *rose coco* beans yield as these had the largest coefficients and those other terms of the model showed no significant effect on the yield.

Table 3: The ANOVA Results in the Fertilizer Concentration on Rose Coco Bean-GP3G

Regression	DF	Type I Sum of Squares	Mean Sq	R-Square	F Value	Pr> F
First Order	3	712.40067	237.467	0.2314	5.58	0.0099
Pure Quadratic	3	936.32622	312.109	0.3041	7.34	0.0034
Two-Factor Interaction	3	834.75000	278.250	0.2711	6.54	0.0054
Residuals	14	595.48000	42.534			
Lack of fit	14	595.48000	42.534			
Pure error	0	0.000000				
Total Model	9	2483.4769		0.8066	6.49	0.0011

Coefficient of variation (CV)= 10.3044, coefficient determination (R^2)= 0.8066, Adjusted R-squared =0.6823, correlation coefficient (r) =0.8981, root MSE=6.521839, response mean=63.291667, PRESS=2645.6089489

Table 3 is the results of the averaged data of the three replicates of GP3G. The statistical testing of the model was done by the Fisher's statistical test for analysis of variance (ANOVA), and the results are shown in table 3. In the table 3, all the linear (p=0.0099), quadratic (p=0.0034) and cross product (p=0.0054) terms were significant at 1%; therefore, the total model was significant with p values of 0.0011. The second-order model for GP3G was highly significant. The analysis of variance (F-test) showed that the second model fits well with the experimental data. The goodness of fit into the model was checked by the determination coefficient (R^2) and correlation coefficient (r). The determination coefficient (R^2) implies that the sample variation of 80.66% with the mean response of 63. 2917gms of *rose coco* bean production was attributed to the independent variables, and about 19.34% of the total variation couldn't be explained by the model, implying that R^2 in GP3G of second order model explained 80.66% of the variation in the model. The value of r (0.8981) for eq. 5 being close to 1 indicated a close agreement between the experimental results and the theoretical values predicted by the model equation.

5. The setting of the experimental factors that produces the optimal response

Response surface models are frequently used method for exploring the relationship of variables and yields. Choosing a model, and assessing the fit into the models, are the questions which one comes up every time one employs the technique. In RSM, it is common practice to code the original input variables to get dimensionless factors, $x_1, ..., x_k$, having zero mean and the same standard deviation [12]. The correspondences between these coded, actual values and design points of the experiments are given in table 1. Twenty four experimental runs were conducted linearly with the experiment performed in triplicate. The *rose coco* beans were labeled in the greenhouse to facilitate the identification during the entire analysis process. The average yield was computed for the GP3G. The SAS software [14] was used to generate the critical value of the response and also was used to generate response surfaces, while holding one variable constant in the second-order polynomial model. The R-software was used to come up with contour plots. Canonical analysis [5] was carried out to determine the location and nature of the stationary point of the model. When the results showed a saddle point in response surfaces, the ridge analysis of SAS RSREG procedure was employed to compute the estimated ridge of the maximum response for increasing radii from the center of original design. The fitted polynomial equations were generated to the response surface, and contour plots to visualize the relationship between the response and experimental levels within each factor.

5.1. Three-dimensional surfaces of nitrogen and phosphorus- GP3G

In the figure 1, maximum yield of 78gms was achieved with lower levels of nitrogen and higher levels of phosphorus, also a yield of about 71gms with high nitrogen and low phosphorus.

5.2. The contour plot of nitrogen and phosphorus

The contour plot is a two-dimensional and the third design variable must be held constant to construct the graph. The contour plot for nitrogen and phosphorus when potassium was held constant is given as in the figure 2, the maximum yield of 80gm per *rose coco* plant was obtained by lowering the nitrogen input from the center of 30gms and increasing the phosphorus to the higher scale from the centre point of 40gms, the same maximum yield of 80gms was obtained by higher nitrogen and lower phosphorus.

5.3. Three dimensional surfaces of nitrogen and potassium- GP3G

In figure 3, high yield of 84.42gms was obtained when we have high level of nitrogen from the center point of 30gms and low level of potassium from the center point of 50gms.

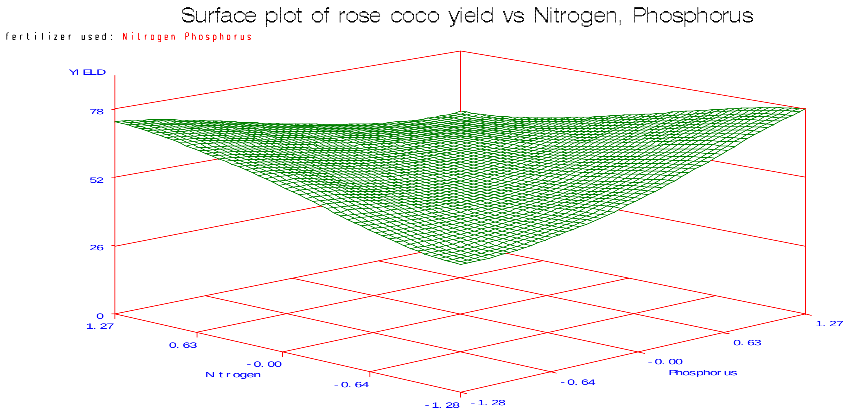

Fig.1: The Response Surface Plots for the Treatments of Nitrogen and Phosphorus Fertilizer Concentrations in GP3G.

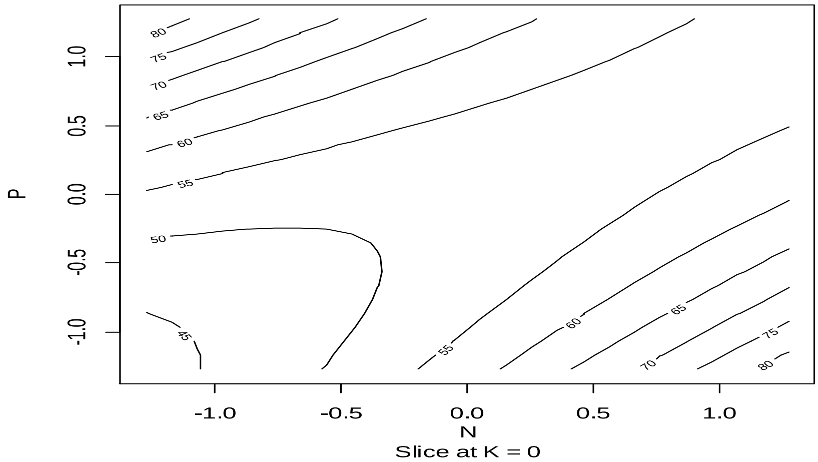

Fig. 2: The Contour Plot of Nitrogen and Phosphorus Fertilizers in GP3G of fig. 1.

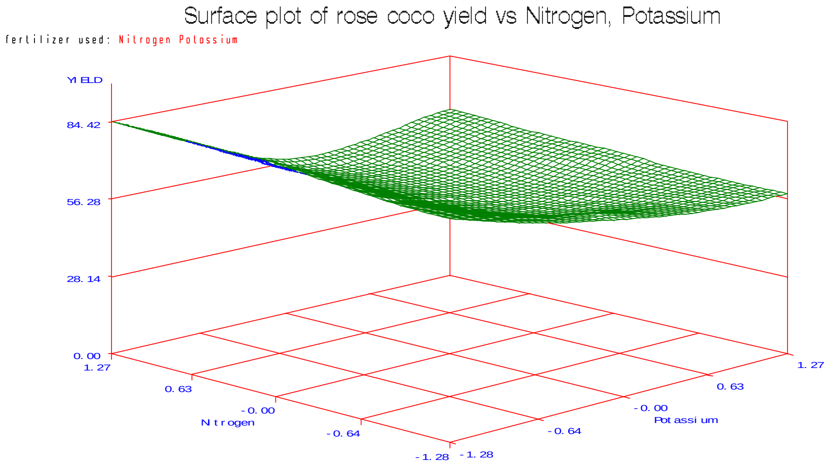

Fig. 3: The Response Surface Plots for the Treatments of Nitrogen and Potassium Fertilizer Concentrations in GP3G.

5.4. The contour plot of nitrogen and potassium

The contour plot for nitrogen and potassium when phosphorus was held constant is given as in the figure 4, the maximum yield of 85gms was obtained by increasing the nitrogen fertilizer input to the highest scale from the centre point 30gms and reducing the potassium to the lowest scale from the centre point of 50gms.

5.5. Three dimensional surfaces of phosphorus and potassium - GP3G

In the figure 5 maximum yields of 88.83gms of *rose coco* beans was achieved by high levels of phosphorus from the centre point of 40gms and low level of potassium fertilizers from the centre point of 50gms.

5.6. The contour plot of phosphorus and potassium

The contour plot for phosphorus and potassium when nitrogen was held constant is given as:

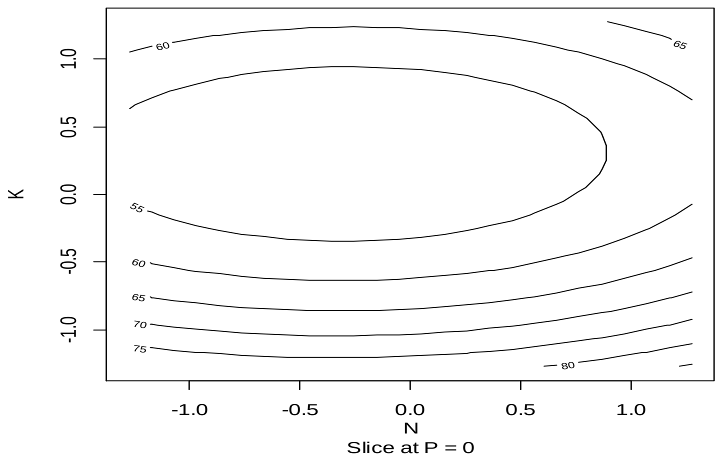

Fig. 4: The Contour Plot of Nitrogen and Potassium Fertilizers in GP3G of fig.3.

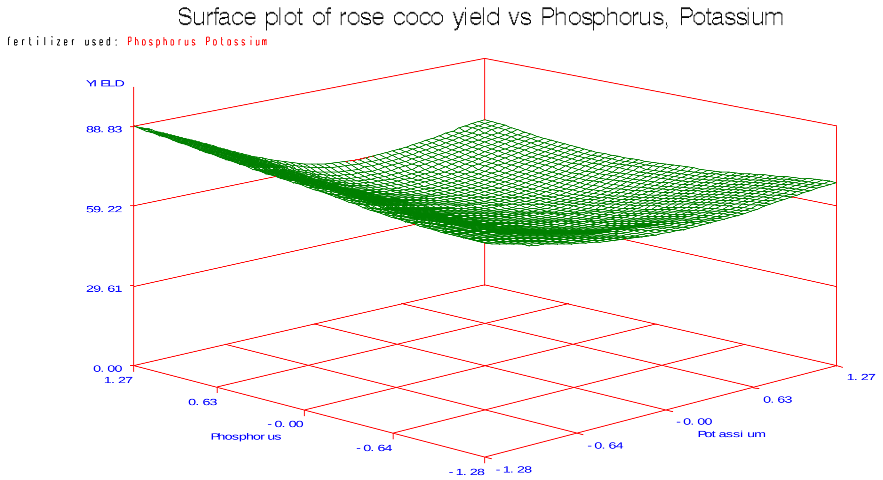

Fig. 5: The Response Surface Plots for the Treatments of Phosphorus and Potassium Fertilizer Concentrations in GP3G.

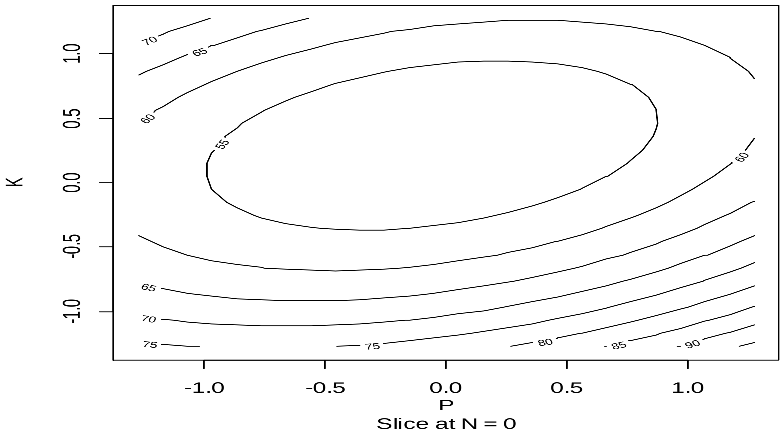

Fig. 6: The Contour Plot of Phosphorus and Potassium Fertilizers in GP3G of fig.5.

In the figure 6, the maximum yield of 95gms was obtained by increasing the phosphorus to the maximum scale from the center point 40gms and reducing the potassium to the lowest scale from the centre point of 50gms.

5.7. The canonical analysis of GP3G

Table 4: The Canonical Analysis of Response Surface, Eigenvectors and Eigenvalues-GP3G

	Eigenvectors		
Eigenvalues	N	P	K
12.224838	0.298898	-0.511793	0.805437
8.574872	-0.586456	0.567316	0.578120
-1.401508	0.752814	0.645152	0.130574
	Factor	Critical Value	
	N	0.444188	
	P	0.436329	
	K	0.401005	

The canonical analysis table 4 and figure 1,3,5 of response surface indicates that the predicted response surface was shaped a saddle point. The coded eigenvalues are λ_1=12.224838, λ_2=8.574872 and λ_3=-1.401508.The eigenvalue of 12.224838 shows that the valley orientation of the saddle was more curved, eigenvalue 8.574872 shows that the valley orientation of the saddle was less curved, the less hilly orientation was with an eigenvalue of 1.401508. The coefficients of the associated eigenvectors show that the valley in both of the first two was more aligned with potassium and the hilly with nitrogen. Because the canonical analysis resulted in a saddle point, the estimated surface does not have a unique optimum. The surface was more sensitive to the changes in amount of K & P, compared to fertilizers of N. The results from these indicated that under natural values 30.222094gms of N, 40.130899gms of P and 50.401005gms of K fertilizer were needed to achieve the saddle point of rose coco bean yield of 51.31gms per plant.

5.8. The stationary point

The sign of the stationary point is determined from the signs of the eigen-values of the matrix B. The standard quadratic model could be written in matrix notation as

$$y = \hat{\beta}_0 + \mathbf{x}'\mathbf{b} + \mathbf{x}'\mathbf{B}\mathbf{x} + \varepsilon \qquad (6)$$

Where xis a fixed combination of the levels of the k input variables, $\hat{\beta}_0$, b and B contains estimates of the intercept, linear and second order coefficients, respectively.
The fitted second-order model in matrix form is as follows:

$$\hat{y} = \hat{\beta}_0 + \mathbf{x}'\mathbf{b} + \mathbf{x}'\mathbf{B}\mathbf{x} \qquad (7)$$

The derivative of $\hat{y}$ with respect to the elements of the vector x is

$$\frac{\partial \hat{y}}{\partial x} = \mathbf{b} + 2\mathbf{B}x \qquad (8)$$

Therefore, setting the derivative vector to 0 yields the stationary point of the system:

$$x_s = -\frac{1}{2}\mathbf{B}^{-1}\mathbf{b} \qquad (9)$$

This may be a maximum, minimum, or a saddle point of the fitted surface. The eigenvalues (call them λs) and eigenvectors of **B** are the key to characterizing the shape. The $\mathbf{x}_s$ are a point of maximum if all λ's are negative, the point of minimum if all λ's are positive and saddle point if λ's are of mixed sign.

Where

$$\mathbf{b} = \begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \hat{\beta}_3 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} \hat{\beta}_{11} & \hat{\beta}_{12} & \hat{\beta}_{13} \\ & \hat{\beta}_{22} & \hat{\beta}_{23} \\ sym & & \hat{\beta}_{33} \end{bmatrix} \tag{10}$$

b is a (3x1) vector of the first-order regression coefficients and **B** is a (3x3) symmetric matrix whose main diagonal elements are the quadratic coefficients and whose off-diagonal elements are one-half the mixed quadratic coefficients [10].The estimated response value at the stationary point is

$$\hat{y}_s = \hat{\beta}_0 + \frac{1}{2} x_s^{'} \mathbf{b} \tag{11}$$

5.9. The stationary point-GP3G

Locating stationary points for the yield-GP3G

$$\mathbf{b} = \begin{bmatrix} 1.9127 \\ 1.9876 \\ -6.5028 \end{bmatrix} \mathbf{B} = \begin{bmatrix} 3.2471 & -5.40365 & -0.10195 \\ -5.40365 & 5.3785 & -2.3450 \\ -.10195 & -2.3450 & 10.7726 \end{bmatrix}$$

$$\mathbf{B}^{-1} = \begin{bmatrix} -0.3570 & -0.3979 & -0.0900 \\ -0.3979 & -0.2380 & -0.0556 \\ -0.0900 & -0.0556 & 0.0799 \end{bmatrix} \tag{12}$$

The stationary point using the equation

$x_s = -\frac{1}{2}\mathbf{B}^{-1}\mathbf{b}$ is

$$x_s = \begin{bmatrix} 0.4442 \\ 0.4363 \\ 0.4010 \end{bmatrix} \tag{13}$$

We could find the natural values of nitrogen, phosphorus and potassium using the stationary points.

$$0.4442 = \frac{\psi_{2u} - 30}{0.5}, \ \psi_{1u} = 30.2221, \ 0.4363 = \frac{\psi_{2u} - 40}{0.3}, \ \psi_{2u} = 40.13089$$

$$0.4010 = \frac{\psi_{3u} - 50}{1}, \ \psi_{3u} = 50.4010 \tag{14}$$

Using the equation $\hat{y} = \hat{\beta}_0 + x'\mathbf{b} + x'\mathbf{B}x$, we can find that estimated maximum response yield of *rose coco* beans at the stationary point was

$\hat{Y} = 51.3 grams$

Thus, it can be concluded that this level of main factors setting resulted in saddle optimum yield for the *rose coco* bean for the given amount of predictor variables.

The maximum yield of *rose coco* bean was obtained when under this combination of fertilizers in GP3G

$$-0.077342 = \frac{\psi_{1u} - 30}{0.5}, \ \psi_{1u} = 29.961329, \ 0.365469 = \frac{\psi_{2u} - 40}{0.3},$$

$$\psi_{2u} = 40.1096407 \quad -0.927605 = \frac{\psi_{3u} - 50}{1}, \ \psi_{3u} = 49.072395 \tag{15}$$

5.10. The ridge analysis

The ridge analysis in RSM is a contour-based technique for investigating a quadratic response surface. In general, this classical response-surface optimization method used for locating the extreme optimum of the predicted response, imposed on a sphere of a certain radius in the spherical region of experimentation. It is a search for a new stationary point S_R on a given radius R such that the second-order model has a minimum at this stationary point. Then, the maximum or minimum response value at different locations from the design center can be determined by comparing each 'constrained' stationary point [12] and [5]. This procedure was conducted by the SAS program [14]. An RIDGE statement computes the ridge of optimum response. The ridge starts at a given point x_0, and the point on the ridge at radius R from x_0 is the collection of factor settings that optimizes the predicted response at this radius. The ridge analysis can be used as a tool to help interpret an existing response surface or to indicate the direction in which further experimentation should be performed. The default starting point, x_0, has each coordinate equal to the point midway between the highest and lowest values of the factor in the design. The default radii at which the ridge is computed are 0, 0.1... 0.9, 1. If the ridge analysis is based on the response surface fit to coded values to the factor variables, then these results in a ridge that starts with the point with a coded zero value for each coordinate and extended toward, but not beyond, the edge of the range of experimentation. Alternatively, both the Center Point of the ridge and the radii at which it is to be computed can be specified. The coded radii give the distances from the ridge starting point at which to compute the optimal [14].

5.11. The ridge analysis-GP3G

The ridge analysis was performed to determine the critical levels of the design variables that produce the maximum and minimum response.

Table 5: The Estimated Ridge of Minimum Response for Variable YIELD-GP3G

Radius	Estimated Response	Standard Error	Factor Values		
			N	P	K
0.0	51.751435	3.410664	0	0	0
0.1	51.142673	3.398892	-0.038479	-0.031963	0.086589
0.2	50.696721	3.364302	-0.106285	-0.080759	0.148935
0.3	50.337171	3.309146	-0.193912	-0.147243	0.175264
0.4	49.998756	3.237429	-0.282585	-0.218021	0.180589
0.5	49.653355	3.155286	-0.368113	-0.288100	0.177458
0.6	49.290454	3.071483	-0.451052	-0.357014	0.170567
0.7	48.905533	2.997910	-0.532192	-0.424991	0.161723
0.8	48.496347	2.949793	-0.612079	-0.492276	0.151737
0.9	48.061651	2.945100	-0.691071	-0.559047	0.141024
1.0	47.600701	3.002620	-0.769404	-0.625432	0.129819

Therefore, the ridge analysis estimated maximum response for the variable yield was 70.25gms per rose coco bean with combination nitrogen of 29.96gms, phosphorus of 40.11gms and potassium of 49.07gms for GP3G.

Table 6: The Estimated Ridge of Maximum Response for Variable YIELD-GP3G

Radius	Estimated Response	Standard Error	Factor Values		
			N	P	K
0.0	51.751435	3.410664	0	0	0
0.1	52.563118	3.398892	0.019147	0.027378	-0.094254
0.2	53.593652	3.364302	0.026607	0.055878	-0.190184
0.3	54.850777	3.309146	0.026256	0.087039	-0.285893
0.4	56.339123	3.237429	0.020297	0.121001	-0.380719
0.5	58.061791	3.155286	0.010145	0.157476	-0.474445
0.6	60.020980	3.071483	-0.003227	0.196086	-0.567045
0.7	62.218302	2.997910	-0.019119	0.236473	-0.658571
0.8	64.654972	2.949793	-0.037012	0.278327	-0.749109
0.9	67.331915	2.945100	-0.056519	0.321394	-0.838756
1.0	70.249853	3.002620	-0.077342	0.365469	-0.927605

6. Conclusions

The graphics and visualization techniques are some of the best tools for understanding response surfaces, for which this research work utilized in the expounding of the nature and shape of the response surface generated from the second-order models for the group 3(GP3G). We visualize mountain and valleys of the second-order response surface of *rose coco* beans for GP3G each at two combinations of fertilizers N, P, K. The curvature in GP3G showed that the second-order model fits the data well. Contour plots showing the contours on the surface, that is, curves of N, P, K pairs that have the same response value were generated, which depicts the pattern and nature of the combination of the predictive factors, nitrogen, phosphorus and potassium fertilizer concentration of the *rose coco* beans yield. In this research work, we have been able to successfully utilize response surface methodology to come up with a clear model for the relationship involving nitrogen, phosphorus and potassium as fertilizer's variables in the production of *rose coco* beans using the twenty four point second order rotatable design in a greenhouse setting. In this study, the average *rose coco* yield was obtained using three replications for the group GP3G. We could estimate the parameter coefficient for the GP3G. The analysis of variance (ANOVA) of response surface for *rose coco* yield showed that the twenty four second order rotatable design was adequate due to satisfactory levels of a coefficient of determinations, R^2 (0.80) for the GP3G which means that 80.03% of the total variability within the system was explained by the chosen factors' N, P, K and coefficient of variations was 10.30. In addition, linear, quadratic and cross product terms were all found to be significant at 1%. The group GP3G total model was significant at 0.1% (p = 0.0011). The results showed that GP3G fitted the second-order models well for the *rose coco* yield using the three fertilizer treatments, nitrogen (N), phosphorus (P) and potassium (K).

The eigenvalue were used to determine whether the solution gave a maximum, minimum or saddle point on the response curve. The 3-dimensional response surface plots were a good way to visualize the fertilizer interaction. The canonical analysis of response surface in the group, that is, GP3G indicated that the stationary point was a saddle point, implying that in the experimental region, there were no maximum or minimum points. The study uses the ridge analysis as an alternative solution to overcome the saddle point problem. Ridge analyses were performed to determine the critical levels of the design variables that could produce a maximum response.

The canonical analysis indicates that the directions of principal orientation for the predicted response surface are along the axes associated with the three factors. In GP3G uncoded values, the largest eigenvalue (79.989081) corresponds to the eigenvector (-0.471069, 0.876678, 0. 097616), -0.097616), the largest component of which (0.876678) is associated with P; similarly, the second-largest eigenvalue (10.972726) is associated with K. The third eigenvalue (-7.439426), associated with N. The coded form of the canonical analysis indicates that the estimated response surface was at a saddle point, in uncoded terms, the model predicts that the yield of rose coco saddles when N=30.222094grams, P=40.130899grams, and K=50. 401005grams. In this canonical analysis, we saw that in the GP3G was a valley orientation inclines towards potassium but inclined to phosphorus for uncoded. On the hilly or downward curvature in GP3G are inclined to nitrogen at 30gms. The optimal values for all the three variables (nitrogen, phosphorus and potassium) for optimal production of *rose coco* beans predicted in GP3G (51.3gms) by the model was achieved with 30.22gms nitrogen, 40.13gms phosphorus and 50. 40gms potassium fertilizers, we have to be cautious with the specific amount of each fertilizer applied to *rose coco* plant because it might be affecting the effectiveness of the other fertilizers' variable component. It was found in the current study that three factors, nitrogen concentration, phosphorus concentration and potassium concentration, were important fertilizers for the *rose coco* beans yield. The stationary points of the group were the saddle point, indicating that optimum conditions for GP3G did not exist in the experimental range. Since analysis of the surface response revealed that the stationary point in the group was saddled points, the conclusion cannot be well drawn. Hence, the study used ridge analysis to offer an alternative solution for the saddle problem. Under the three combinations of the fertilizers; nitrogen 29.96gms, phosphorus 40.11gms and potassium 49.07gms achieved 70.25gms of rose coco yield in GP3G. The results from this study show that a twenty four-points second order rotatable design was one of the suitable methods to optimize the best operating conditions in the multi-factor operating environment for the purpose of obtaining maximum *rose coco* beans yield. The study has demonstrated the applicability of the twenty four points, second order rotatable design for the optimal and efficient production of rose coco beans. We saw that GP3G was desirable for further researches and to act as a starting point for farmers to plant *rose coco* beans so as to achieve the maximum/optimal potential yield. We, therefore, recommend the use of GP3G since it gave the required coefficient of determination, R^2 of 80.66%, and the maximum yield of 70. 25gms was achieved. With the use of N, P and K fertilizers in this research study design, GP3G if adopted the farmers have a high potentiality to increase their yields or even triple the *rose coco* yield in the study area. The ministry of agriculture in Kenya should focus more researches of this kind, such as that production using inorganic fertilizers (N, P, K) concentration matches the crop and the soil requirements of the *rose coco* beans in the growing areas for maximum yield to be realized. The research needs to be tried in the open field with the found optimal fertilizer combinations to see its performance.

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