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***Research paper***

# The beta-burr type v distribution: its properties and application to real life data

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**Abstract**

A new distribution called the beta-Burr type V distribution that extends the Burr type V distribution was defined, investigated and established. The properties examined provide a comprehensive mathematical treatment of the distribution. Additionally, various structural properties of the new distribution verified include probability density function verification, asymptotic behavior, Hazard Rate Function and the cumulative distribution. Subsequently, we used the maximum likelihood estimation procedure to estimate the parameters of the new distribution. Application of real data set indicates that this new distribution would serve as a good alternative distribution function to model real- life data in many areas.

***Keywords***: *Burr Type V; Beta Burr V; Hazard Rate; Maximum Likelihood Estimation*.

## 1. Introduction

Burr's type V is one of the twelve distributions introduced in 1942 by [5], which can be used to fit basically any given set of unimodal data [7]. So many researchers have established beta –G distribution; these include among others: Nadarajah and Gupta [20] defined the beta-Frechet distribution; Famoye, et al. [5] defined the beta-Weibull; Nadaraja and Kotz [19] defined the beta-exponential distribution; Kong et al. [15] proposed the beta-gamma distribution. Fischer and Vaughan [12] introduced the beta-hyperbolic secant distribution; beta-Gumbel (BGU) distribution was introduced by Nadarajah and Kotz [18]; the beta-Pareto distribution was defined and studied by Akinsete et al. [4]; the beta-Rayleigh distribution was proposed by Akinsete and Lowe [3]; beta-Burr XII by Paranaiba et al. [21]; and very recently, Merovci et al. [17] developed the beta-Burr type X distribution leaving the rest types of Burr distributions with little or no interest from researchers. This is the case with Burr type V distribution. It is clear therefore, that this distribution is yet to catch the eyes of researchers. Burr Type V distribution can be used to model real lifetime data.
Ever since the introduction of the Burr distribution, it has somewhat been neglected as an option in the analysis of lifetime data. Even though there are researchers that indicate that this distribution possesses sufficient flexibility to make it a possible model for various types of data; for example, it was used to examine the strengths of 1.5 cm glass fibers [17], Paranaiba et al. [21] studied cancer recurrence by using the real data set.
Burr's type V distribution extension with two parameters is through the beta-G generator pioneered by Eugene et al. [8]. We investigate and discern the properties of the proposed distribution from G(x) which was used by Eugene et al [8]. It is known as the beta generalized class of distribution, and it has two shape parameters in the generator, and it is given in equation 3.
The probability density function of Burr type V distribution is:

$$f(x)=\begin{cases}\alpha\beta e^{-\tan x}\sec^2 x\left(1+\beta e^{-\tan x}\right)^{-\alpha-1} & \frac{-\pi}{2}<x<\frac{\pi}{2}\\ 0 & \text{elsewhere}\end{cases} \tag{1}$$

The cumulative distribution function of Burr type V distribution is:

$$F(x)=\left(1+\beta^{-\tan x}\right)^{-\alpha} \tag{2}$$

Where α and β are the location parameters of the distribution.
The rest of the paper is organized as follows: in section two, we briefly introduce the beta-burr type V (BBV) distribution, its properties is studied in section 3 while estimates of the parameters of the distribution using the maximum likelihood estimation technique is obtained in section 4, in section 5 we look at the comparison of our new distribution with other known distributions and we finally conclude in section 6.

## 2. Beta- burr type V (BBV)

Beta distribution takes on several problems in reliability analysis since it has been widely acclaim to be very powerful and applicable probability distribution. In recent years, development focuses on new techniques for building meaningful distributions, which include the use of the logit of beta (the link function of the beta generalized distribution which was introduced by Jones [14] and elsewhere [10]). The logit of beta introduced by Jones has probability density function (pdf).

$$g(x)=\frac{f(x)}{B(a,b)}[F(x)]^{a-1}[1-F(x)]^{b-1} \tag{3}$$

Where
F(x)is the cumulative distribution function of the baseline distribution and
f(x)is the probability density function of the baseline distribution
Many researchers have used this technique to come up with many compound distributions which include: [1],[3],[4],[6],[8],[9],[12],[15],[16],[17],[18],[19],[20],[21],[23] and many others. The aim of this paper is to propose a new model called beta- Burr type V distribution. Using the logit defined in (3), the pdf of the proposed distribution is given by equation (4):

$$f(x) = \frac{\alpha\beta e^{-\tan x}\sec^2 x}{\beta(a,b)}[1+\beta e^{-\tan x}]^{-\alpha a-1}[1-(1+\beta e^{-\tan x})^{-\alpha}]^{b-1} \quad (4)$$

## 3. Properties

We will examine the statistical properties of the distribution we have put forth in the previous section. It will be scrutinized by verifying the asymptotic behaviour, true probability function, cumulative density and hazard rate function.

### 3.1. Investigation of proper PDF

To verify that the proposed density function is a proper pdf, we integrate (4) with respect to x and see whether the integral is equal to unity or otherwise. If it is equal to unity, the density function is a proper pdf otherwise it is not.

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(x)dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\alpha\beta}{\beta(a,b)} e^{-\tan x}\sec^2 x[1+\beta e^{-\tan x}]^{-\alpha a-1}[1-(1+\beta e^{-\tan x})^{-\alpha}]^{b-1}dx \quad (5)$$

To integrate (5),

Let $p = \beta e^{-\tan x}$, then $dp = -p\sec^2 x\, dx$ and $dx = \frac{dp}{-p\sec^2 x}$

Hence,

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(x)dx = \int_0^\infty \frac{\alpha}{\beta(a,b)} p\sec^2 x\,[1+p]^{-\alpha a-1}[1-(1+p)^{-\alpha}]^{b-1}\frac{dp}{p\sec^2 x}$$

$$= \int_0^\infty \frac{\alpha}{\beta(a,b)}(1+p)^{-\alpha a-1}[1-(1+p)^{-\alpha}]^{b-1}dp$$

let $R = (1+p)^{-\alpha}$ Then $R^{-\frac{1}{\alpha}} = 1+p$

This implies

$dR = -\alpha(1+p)^{-\alpha-1}dp$ And $dp = \frac{dR}{-\alpha R^{1+\frac{1}{\alpha}}}$

Therefore,

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(x)dx = \frac{1}{\beta(a,b)}\int_0^1 R^{a-1}(1-R)^{b-1}\,dR = \frac{\beta(a,b)}{\beta(a,b)} = 1$$

Hence, f(x) is a proper pdf?
The graph of the BBV distribution for various parameter values (a, b, α and β) is given in fig. 1

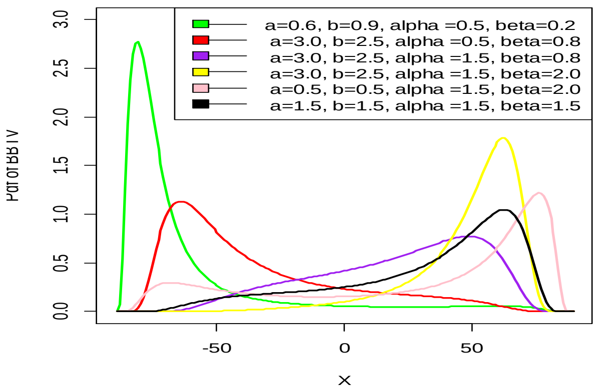


**Fig 1:** Graph of Beta-Burr Type V Distribution for Various Parameter Values.

### 3.2. Asymptotic behaviour

In order to investigate the asymptotic behavior of the proposed model (Beta –Burr type V), we find limit as $x \to \frac{\pi}{2}$ and limit as $x \to -\frac{\pi}{2}$ of the BBV distribution.

$$\lim_{x\to-\frac{\pi}{2}} f(x) = \lim_{x\to-\frac{\pi}{2}} \frac{\alpha\beta e^{-\tan x}\sec^2 x}{\beta\,(a,b)}(1+\beta e^{-\tan x})^{-\alpha a-1}[1-(1+\beta e^{-\tan x})^{-\alpha}]^{b-1}$$

This is equal to zero, since

$$\lim_{x\to-\frac{\pi}{2}} \alpha\beta e^{-\tan x}\sec^2 x(1+\beta e^{-\tan x})^{-\alpha-1} = 0$$

Similarly, $\lim_{x\to\frac{\pi}{2}} f(x) = 0$

Since $\lim_{x\to\frac{\pi}{2}} sec^2 x = 0$

### 3.3. Cumulative density function (CDF) of BBV distribution

In this section, the cumulative density function of the BBV distribution will be obtained

$$F(x) = \int_{-\frac{\pi}{2}}^{x} \frac{\alpha\beta}{\beta(a,b)} e^{-\tan y}\sec^2 y[1+\beta e^{-\tan y}]^{-\alpha a-1}[1-(1+\beta e^{-\tan y})^{-\alpha}]^{b-1}dy \quad (6)$$

To integrate (6), *let* $p = \beta e^{\tan y}$, then

$dp = -p\sec^2 y dy$ And $dy = \frac{dp}{-p\sec^2 y}$

Therefore,

$$F(x) = \int_{\beta e^{-\tan x}}^\infty \frac{\alpha}{\beta(a,b)} p\sec^2 y[1+p]^{-\alpha a-1}[1-(1+p)^{-\alpha}]^{b-1}\frac{dp}{p\sec^2 y}$$

$$= \int_{\beta e^{-\tan x}}^\infty \frac{\alpha}{\beta(a,b)}[1+p]^{-\alpha a-1}[1-(1+p)^{-\alpha}]^{b-1}dp$$

Let $R = (1+p)^{-\alpha}$ then,

$dR = -\alpha R^{\frac{\alpha+1}{\alpha}}dp$ And $dp = \frac{dR}{-\alpha R^{\frac{\alpha+1}{\alpha}}}$

Hence,

$$F(x) = \int_0^{(1+\beta e^{-\tan x})^{-\alpha}} \frac{\alpha}{\beta(a,b)} R^{\frac{\alpha a+1}{\alpha}} [1-R]^{b-1} \frac{dR}{\alpha R^{\frac{\alpha+1}{\alpha}}}$$

$$F(x) = \int_0^{(1+\beta e^{-\tan x})^{-\alpha}} \frac{\alpha}{\beta(a,b)} R^{\frac{\alpha a+1-\alpha-1}{\alpha}} (1-R)^{b-1} \frac{dR}{\alpha}$$

$$F(x) = \frac{1}{\beta(a,b)} \int_0^{(1+\beta e^{-\tan x})^{-\alpha}} R^{a-1}(1-R)^{b-1} dR$$

Therefore,

$$F(x) = \frac{\beta[\{1+\beta e^{-\tan x}\}^{-\alpha}, a, b]}{\beta(a,b)} \quad (7)$$

Where$\beta[\{1+\beta e^{-\tan x}\}^{-\alpha}, a, b]$ is an incomplete beta function?

### 3.4. Hazard rate function

The Hazard Rate Function denoted by $h(x)$ is defined by:

$$h(x) = \frac{g(x)}{1-G(x)} \quad (8)$$

Substituting equations (4) and (7) in (8) gives the expression for the hazard function as:

$$h(x) = \frac{\dfrac{\alpha\beta e^{-\tan x}\sec^2 x\left[1+\beta e^{-\tan x}\right]^{-\alpha a-1}\left[1-\left(1+\beta e^{-\tan x}\right)^{-\alpha}\right]^{b-1}}{B(a,b)}}{1-\dfrac{B\left[\left\{1+\beta e^{-\tan x}\right\}^{-\alpha},a,b\right]}{B(a,b)}}$$

$$h(x) = \frac{\alpha\beta e^{-\tan x}\sec^2 x\left[1+\beta e^{-\tan x}\right]^{-\alpha a-1}\left[1-\left(1+\beta e^{-\tan x}\right)^{-\alpha}\right]^{b-1}}{B(a,b)-B(a,b)B\left[\left\{1+\beta e^{-\tan x}\right\}^{-\alpha},a,b\right]} \quad (9)$$

It is also known as failure rate or force of mortality.

## 4. Maximum likelihood estimation of the parameters of the proposed model

Let $x_1, x_2, \dots, x_n$ be a random sample from a population X with probability density function$f(x;\ a,b,\alpha,\beta)$, where $a,b,\alpha,\beta$ are unknown parameters. The likelihood function$L(a,b,\alpha,\beta)$, is defined [11, 13, 22] to be the joint density of the random variables$x_1, x_2, \dots, x_n$. That is,

$$L(a,b,\alpha,\beta) = \prod_{i=1}^{n} f(x_i; a,b,\alpha,\beta) \quad (10)$$

Hence, the likelihood of BBV distribution is given by:

$$L(x;a,b,\alpha,\beta) = \prod_{i=1}^{n} \frac{\alpha\beta e^{-\tan x_i} sec^2 x_i}{\beta(a,b)} (1 + \beta e^{-\tan x_i})^{-\alpha a-1} [1 - (1+\beta e^{-\tan x_i})^{-\alpha}]^{b-1}$$

$$= \frac{\alpha^n \beta^n e^{-\sum_{i=1}^n \tan x_i}}{(\beta(a,b))^n} \prod_{i=1}^n sec^2 x_i (1+\beta e^{-\tan x_i})^{-\alpha a-1} [1 - (1+\beta e^{-\tan x_i})^{-\alpha}]^{b-1} \quad (11)$$

Taking log of (11), we have the log-likelihood given by:

$$l(x;a,b,\alpha,\beta) = n\log\alpha + n\log\beta - \sum_{i=1}^n \tan x_i - n\log(\Gamma a) - n\log(\Gamma b) + n\log(\Gamma a+b) + \sum_{i=1}^n \log(sec^2 x_i) - (\alpha a+1)\sum_{i=1}^n \log(1+\beta e^{\tan x_i}) + (b-1)\sum_{i=1}^n \log[1-(1+\beta e^{-\tan x_i})^{-\alpha}] \quad (12)$$

Differentiating (12) partially with respect to $a, b, \alpha and \beta$ gives (13), (14), (15) and (16):

$$\frac{\partial l(x{:}a,b,\alpha,\beta)}{\partial a} = n\psi(a) + n\psi(a+b) - \alpha \quad (13)$$

$$\frac{\partial l(x{:}a,b,\alpha,\beta)}{\partial b} = -n\psi(b) + n\psi(a+b) + \sum_{i=1}^n \log[1-(1+\beta e^{-\tan x_i})^{-\alpha}] \quad (14)$$

$$\frac{\partial l(x{:}a,b,\alpha,\beta)}{\partial \alpha} = \frac{n}{\alpha} - a\sum_{i=1}^n \log(1+\beta e^{-\tan x_i}) + (b-1)\sum_{i=1}^n \frac{(1+\beta e^{-\tan x_i})^{-\alpha}\log(1+\beta e^{-\tan x_i})}{1-(1+\beta e^{-\tan x_i})} \quad (15)$$

$$\frac{\partial l(x{:}a,b,\alpha,\beta)}{\partial \beta} = \frac{n}{\beta} - (\alpha a-1)\sum_{i=1}^n \frac{e^{-\tan x_i}}{1+\beta e^{-\tan x_i}} + (b-1)\sum_{i=1}^n \frac{\alpha e^{-\tan x_i}(1+\beta e^{-\tan x_i})^{-\alpha-1}}{1-(1+\beta e^{-\tan x_i})^{-\alpha}} \quad (16)$$

Equating (13), (14), (15) and (16) to zero and solving for a, b, α and β gives the MLEs of the respective parameters$\hat{a}$, $\hat{b}$, $\hat{\alpha}$ and $\hat{\beta}$.

## 5. Application

A survival data set is applied to demonstrate that the proposed distribution is flexible and better to fit lifetime data in this section. The data used represent the remission times (in months) of a random sample of 128 bladder cancer patients. The data will be used to fit our proposed distribution (BBV) and some other distributions. The data are: 0.08, 2.09, 3.48, 4.87, 6.94 , 8.66, 13.11, 23.63, 0.20, 2.23, 3.52, 4.98, 6.97, 9.02, 13.29, 0.40, 2.26, 3.57, 5.06, 7.09, 9.22, 13.80, 25.74, 0.50, 2.46 , 3.64, 5.09, 7.26, 9.47, 14.24, 25.82, 0.51, 2.54, 3.70, 5.17, 7.28, 9.74, 14.76, 26.31, 0.81, 2.62, 3.82, 5.32, 7.32, 10.06, 14.77, 32.15, 2.64, 3.88, 5.32, 7.39, 10.34, 14.83, 34.26, 0.90, 2.69, 4.18, 5.34, 7.59, 10.66, 15.96, 36.66, 1.05, 2.69, 4.23, 5.41, 7.62, 10.75, 16.62, 43.01, 1.19, 2.75, 4.26, 5.41, 7.63, 17.12, 46.12, 1.26, 2.83, 4.33, 5.49, 7.66, 11.25, 17.14, 79.05, 1.35, 2.87, 5.62, 7.87, 11.64, 17.36, 1.40, 3.02, 4.34, 5.71, 7.93, 11.79, 18.10, 1.46, 4.40, 5.85, 8.26, 11.98, 19.13, 1.76, 3.25, 4.50, 6.25, 8.37, 12.02, 2.02, 3.31, 4.51, 6.54, 8.53, 12.03, 20.28, 2.02, 3.36, 6.76, 12.07, 21.73, 2.07, 3.36, 6.93, 8.65, 12.63, 22.69

In order to compare the models above with the proposed BBV, we consider criteria like log likelihood (LL), Akaike Information Criterion (AIC) ([2]), Consistent Akaike Information Criterion (CAIC) and Bayesian information criterion (BIC) for the data set. The better distribution corresponds to smaller LL, AIC, AICC and BIC values of these statistics.

Table 1 lists the MLEs, their standard errors (SE) in parentheses and the statistics and p values for the bladder cancer patients’ data. The table indicates that the BBV distribution has the lowest values for the AIC; BIC and CAIC statistics among the fitted models, and therefore it could be chosen as the best model. Moreover, the standard errors are much smaller compared with their estimates for the BBV distribution. Additionally, it is evident that the worst fit model was BDa distribution according to the data.

**Table 1:** Mles of the Model Parameters for the Bladder Cancer Patients Data, the Corresponding SE (Given in Parentheses) and the Measures AIC, BIC and CAIC

| Model | MLE | Loglikelihood | AIC | BIC | CAIC |
|---|---|---|---|---|---|
| Beta-Power Exponential | a = 1.174473 (0.131994)<br>b = 5.974162 (1.347064)<br>α = 0.017775 (0.004285) | -413.3579 | 832.7158 | 841.2719 | 832.9093 |
| Beta-Dagun | a = 15.83986 (1.26161)<br>b = 19.14655 (2.74046)<br>α = 0.56186 (0.08727)<br>β = 6.67282 (1.82561)<br>λ = 0.42873 (0.03835) | -412.4005 | 834.801 | 849.0612 | 835.2928 |
| Beta-Burr V | a = 6.7091 (2.2773)<br>b = 10.1948 (2.8099)<br>α = 10.3665 (1.4322)<br>β = 0.1126 (0.0289) | -25.98339 | 59.96678 | 71.3749 | 60.29198 |
| Burr V | α = 1.029e+03 (8.424e+00)<br>β = 1.131e-03 (1.006e-04) | -124.3364 | 252.6728 | 258.3769 | 252.7688 |

## 6. Conclusions

The beta –Burr type V distribution was introduced and investigated in this article. Some features of the distribution such as asymptotic behaviors, Cumulative density function and hazard function of the distribution were discussed. We also estimated the parameters of the proposed distribution via the method of maximum likelihood estimation technique. An application of the BBV distribution to a real data set indicates that this distribution outperforms both the burr type V and other generalized distributions.

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