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Research paper

A modified class of exponential-type estimator of population-mean in simple random sampling

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Abstract

The problem of obtaining better ratio estimators of the population means are dominating in survey sampling. This paper provides a modified class of exponential type estimators using combinations of some existing estimators. Expressions for the bias and Mean Square Error (MSE) with the optimality conditions for this class of estimators have been established. Both analytical and numerical comparison with some existing estimators shows better performances from members of the proposed class.

Keywords: *Simple Random Sampling; Auxiliary Information; Exponential Ratio Estimator; Mean Square Error; Optimality Conditions; Efficiency.*

1. Introduction

1.1. Background of study

The use of auxiliary information at both selection and estimation stages to increase the efficiency of estimators has been employed with several improvements at both selection and estimation stages since the work of [4]. Some estimation method that uses auxiliary variables includes ratio, product and regression estimators. Although, the first two methods give rise to biased estimators, the bias can be reduced by increasing the sample size. The classical ratio is most applicable when the auxiliary variable X is highly and positively correlated with the study variable Y while, the product estimator proposed by [14] and [12] is most preferred when the auxiliary variable is highly and negatively correlated with the study variable.

It is also known that the ratio is less efficient than the linear regression estimator, but when the regression of the study variable Y on the auxiliary variable X is a straight line passing through the origin, their efficiencies become the same. In practice, the regression of the study variable on the auxiliary variable is linear but sometimes, does not pass through the origin. This prompts the need for an appropriate transformation of the auxiliary variable to estimate the population total or mean of the study variable which may be asymptotically equal or close to the mean square error of the linear regression estimator [11].

In light of the above shortcomings, several authors have studied to improve the existing classical ratio and product estimators to increase efficiency and also give better options in decision making.[17],[19], [20], [15], [1], [21], [8], [9], [10], [18], [2], [3], [16], [7] and [6] are amongst authors who have contributed in one way or the other to the modification of the classical ratio estimator for the population mean of the study variable under simple random sampling without replacement (SRSWOR) scheme with extensions to other sampling techniques. This study is another attempt to propose an alternative estimator for the population mean using a combination of necessary scalars which is expected to have smaller relative bias and compare favourably with the linear regression estimator.

1.2. Notations and some existing estimators

Suppose we have $U=\{U_1, U_2, \ldots, U_N\}$ to be a finite population of N-identifiable units. Let a simple random sample of size n be drawn without replacement and the auxiliary variable is known. Let Y and X represent the study and auxiliary variable with sample means $\bar{y}$ and $\bar{x}$ respectively and assume that information on the population mean $\bar{X}$ of the auxiliary variable is known. Several estimators have been proposed when there exists either a positive or a negative relationship between the study and the auxiliary variable as seen in Table 1.

Notations used in Table 1 are defined as follows;

$\bar{X}=\frac{1}{N}\sum_{i=1}^{N} x_i$; Population mean of the auxiliary variable

$\bar{Y}=\frac{1}{N}\sum_{i=1}^{N} y_i$; Population mean of the study variable

$\bar{x}=\frac{1}{n}\sum_{i=1}^{n} x_i$; Sample mean of the auxiliary variable

$\bar{y}=\frac{1}{n}\sum_{i=1}^{n} y_i$; Sample mean of the study variable

$C_x=\frac{S_x}{\bar{X}}$; Coefficient of variation of the auxiliary variable

$C_y=\frac{S_y}{\bar{Y}}$; Coefficient of variation of the study variable

$\rho = \frac{S_{xy}}{S_x S_y}$; Correlation coefficient between the auxiliary and study variables

$b = \frac{S_{xy}}{S_x^2}$; Regression coefficient

$k = \rho \frac{C_y}{C_x}$; Population constant

$\lambda = \frac{1-f}{n}$

While, $f = \frac{n}{N}$ is the sampling fraction.

2. The proposed class of estimators

The proposed estimator is an exponential ratio-product type estimator denoted by

$$\bar{y}_{pr} = \bar{y}\left[\begin{array}{l} \alpha_1 \frac{\bar{x}}{\bar{X}} \exp\left\{\frac{\delta_1(\bar{x}-\bar{X})}{\bar{x}+\bar{X}}\right\} \\ + \alpha_2 \frac{\bar{X}}{\bar{x}} \exp\left\{\frac{\delta_2(\bar{x}-\bar{X})}{\bar{x}+\bar{X}}\right\} \end{array}\right] \quad (1)$$

Where $\alpha_1 + \alpha_2 = 1$ while δ_1 and δ_2 suitably chosen scalars, which is a linear combination of [4], [14] and [12] with [1] product estimator.

2.1. Bias and MSE of the proposed estimator

To obtain the bias and MSE for the estimator $\bar{y}_{pr}$, let

$\bar{x} = \bar{X}(1+e_x)$, $\bar{y} = \bar{Y}(1+e_y)$,

And

$E(e_x) = E(e_y) = 0$, $E(e_x^2) = \frac{(1-f)}{n} C_x^2$, $E(e_y^2) = \frac{(1-f)}{n} C_y^2$,

$E(e_x e_y) = \frac{(1-f)}{n} \rho C_x C_y = \frac{(1-f)}{n} k C_x^2$

Simplifying the first part of (1) $\alpha_1 \frac{\bar{x}}{\bar{X}} \exp\left\{\frac{\delta_1(\bar{x}-\bar{X})}{\bar{x}+\bar{X}}\right\}$, we obtain

$$\alpha_1 \frac{\bar{x}}{\bar{X}} \exp\left\{\frac{\delta_1(\bar{x}-\bar{X})}{\bar{x}+\bar{X}}\right\} = \alpha_1 \frac{\bar{X}(1+e_x)}{\bar{X}} \exp\left\{\frac{\delta_1(\bar{X}(1+e_x)-\bar{X})}{\bar{X}(1+e_x)+\bar{X}}\right\}$$

$$= \alpha_1(1+e_x)\exp\left\{\frac{\delta_1}{2} e_x \left(1+\frac{e_x}{2}\right)^{-1}\right\}$$

Using Taylor's series expansion up to second order approximation and assuming higher orders are negligible, we obtain

$$\alpha_1 \frac{\bar{x}}{\bar{X}} \exp\left\{\frac{\delta_1(\bar{x}-\bar{X})}{\bar{x}+\bar{X}}\right\}$$

$$= \alpha_1 + \alpha_1 \frac{\delta_1}{2} e_x - \alpha_1 \frac{\delta_1}{4} e_x^2$$
$$+ \alpha_1 \frac{\delta_1^2}{8} e_x^2 + \alpha_1 e_x + \alpha_1 \frac{\delta_1}{2} e_x^2 \quad (2)$$

Similarly, the second term in (1) $\alpha_2 \frac{\bar{X}}{\bar{x}} \exp\left\{\frac{\delta_2(\bar{x}-\bar{X})}{\bar{x}+\bar{X}}\right\}$ is expanded as follows;

$$\alpha_2 \frac{\bar{X}}{\bar{x}} \exp\left\{\frac{\delta_2(\bar{x}-\bar{X})}{\bar{x}+\bar{X}}\right\} = \alpha_2 \frac{\bar{X}}{\bar{X}(1+e_x)} \exp\left\{\frac{\delta_2(\bar{X}(1+e_x)-\bar{X})}{\bar{X}(1+e_x)+\bar{X}}\right\}$$

Using Taylor's series expansion up to second order approximation that is, neglecting terms with power greater than two (2), we obtain

$$\alpha_2 + \alpha_2 \frac{\delta_2}{2} e_x - \alpha_2 \frac{\delta_2}{4} e_x^2$$
$$+ \alpha_2 \frac{\delta_2^2}{8} e_x^2 - \alpha_2 e_x + \alpha_2 \frac{\delta_2}{2} e_x^2 + \alpha_2 e_x^2 \quad (3)$$

Adding (2) and (3) together, simplifying the result and substituting for $\bar{y}$ the proposed estimator $\bar{y}_{pr}$ becomes

$$\bar{y}_{pr} = \bar{Y}\ \{1 + (\alpha_1\delta_1 + 2\alpha_1 + \alpha_2\delta_2 - 2\alpha_2)\frac{e_x}{2}$$
$$-(2\alpha_1\delta_1 - \alpha_1\delta_1^2 - 4\alpha_1\delta_1 + 2\alpha_2\delta_2 - \alpha_2\delta_2^2$$
$$+4\alpha_2\delta_2 - 8\alpha_2)\frac{e_x^2}{8} + e_y + (\alpha_1\delta_1 + 2\alpha_1 + \alpha_2\delta_2 - 2\alpha_2)\frac{e_x e_y}{2}\} \quad (4)$$

But $\alpha_2 = 1 - \alpha_1$

$$\bar{y}_{pr} \cong \bar{Y}\ \{1 + [\alpha_1\delta_1 + 4\alpha_1 + \delta_2 - \alpha_1\delta_2 - 2\ ]\frac{e_x}{2} - [2\alpha_1\delta_1 - \alpha_1\delta_1^2$$
$$+2\delta_2 - 2\alpha_1\delta_2 - \delta_2^2 + \alpha_1\delta_2^2 + 4\delta_2 - 4\alpha_1\delta_2$$
$$-8 + 8\alpha_1)]\frac{e_x^2}{8} + e_y + [\alpha_1\delta_1 + 4\alpha_1 + \delta_2 - \alpha_1\delta_2 - 2)]\frac{e_x e_y}{2}\} \quad (5)$$

The bias of the proposed estimator to its first order approximation is obtained from (5) as follows

$B(\bar{y}_{pr}) = E\left[\bar{y}_{pr} - \bar{Y}\right]$

$B(\bar{y}_{pr}) =$

$$\bar{Y}\ \{[\alpha_1\delta_1 + 4\alpha_1 + \delta_2 - \alpha_1\delta_2 - 2)]\lambda\frac{\rho C_y C_x}{2} - [\alpha_1\delta_2^2 - 2\alpha_1\delta_1 - \alpha_1\delta_1^2$$
$$+2\delta_2 - 2\alpha_1\delta_2 - \delta_2^2 + 4\delta_2 - 4\alpha_1\delta_2 - 8 + 8\alpha_1)]\lambda\frac{C_x^2}{8}\} \quad (6)$$

The expression for the MSE of the proposed estimator is also obtained from (5) as follows;

$MSE\left[\bar{y}_{pr}\right] = E\left[\bar{y}_{pr} - \bar{Y}\right]^2$

$$= E\,\bar{Y}^2\{e_y^{\,2} + (\alpha_1\delta_1 + 4\alpha_1 - \alpha_1\delta_2 + \delta_2 - 2)\, e_y e_x$$

$$+ (\alpha_1\delta_1 + 4\alpha_1 - \alpha_1\delta_2 + \delta_2 - 2)^2 \frac{e_x^{\,2}}{4}\}$$

$$= \lambda\,\bar{Y}^2\{C_y^{\,2} + (\alpha_1\delta_1 + 4\alpha_1 - \alpha_1\delta_2 + \delta_2 - 2)\,\rho C_y C_x +$$

$$(\alpha_1\delta_1 + 4\alpha_1 - \alpha_1\delta_2 + \delta_2 - 2)^2 \frac{C_x^{\,2}}{4}\} \quad (7)$$

Table 1 shows some existing estimators derived as members of the family of the proposed estimators:

Table 1: Some Existing Estimators with Their MSE and Bias

Estimator	MSE	Bias
$\bar{y}_{cl_R} = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)$ Cochran [4] classical Ratio estimator	$\frac{(1-f)}{n}\bar{Y}^2\left[C_y^2 + C_x^2(1-2k)\right]$	$\frac{(1-f)}{n}\bar{Y}C_x^2(1-k)$
$\bar{y}_{cl_P} = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)$ Robson [14]and Murthy [12] Classical Product estimator	$\frac{(1-f)}{n}\bar{Y}^2\left[C_y^2 + C_x^2(1+2k)\right]$	$\frac{(1-f)}{n}\bar{Y}C_x^2 k$
$\bar{y}_{\exp(R)} = \bar{y}\exp\left\{\frac{(\bar{X}-\bar{x})}{\bar{x}+\bar{X}}\right\}$ [1] ratio-type exponential estimator	$\frac{(1-f)}{n}\bar{Y}^2\left[C_y^2 + \frac{C_x^2}{4}(1-4k)\right]$	$\frac{(1-f)}{8n}\bar{Y}C_x^2(3-4k)$
$\bar{y}_{\exp(P)} = \bar{y}\exp\left\{\frac{(\bar{x}-\bar{X})}{\bar{x}+\bar{X}}\right\}$ [1] product-type exponential estimator	$\frac{(1-f)}{n}\bar{Y}^2\left[C_y^2 + \frac{C_x^2}{4}(1+4k)\right]$	$\frac{(1-f)}{8n}\bar{Y}C_x^2(4k-1)$
$\bar{y}_{ch_R} = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)^2$ [8] chain ratio estimator	$\frac{(1-f)}{n}\bar{Y}^2\left[C_y^2 + 4C_x^2(1-k)\right]$	$\frac{(1-f)}{n}\bar{Y}C_x^2(1-2k)$
$\bar{y}_{ch_P} = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)^2$ Chain product estimator	$\frac{(1-f)}{n}\bar{Y}^2\left[C_y^2 + 4C_x^2(1+k)\right]$	$\frac{(1-f)}{n}\bar{Y}C_x^2(1+2k)$
$\bar{y}_{LR} = \bar{y} + b(\bar{X}-\bar{x})$ Linear regression estimator	0	$\lambda\bar{Y}^2 C_y^{\,2}\left(1-\rho^2\right)$

2.2. Optimality conditions for the proposed estimator

To get the optimal value of α_1 that will minimize the MSE, The partial derivative of (7) is taken with respect to α_1, equated to zero and the value of α_1 is obtained.

$$\frac{\partial MSE}{\partial \alpha_1} = (\delta_1 + 4 - \delta_2)\rho C_y C_x$$

$$+ 2(\alpha_1\delta_1 + 4\alpha_1 - \alpha_1\delta_2 + \delta_2 - 2)\quad(\delta_1 + 4 - \delta_2)\frac{C_x^{\,2}}{4} = 0$$

$$\alpha_1 = \frac{2 - \delta_2 - 2k}{\delta_1 + 4 - \delta_2} \quad (8)$$

Substituting the value of α_1 in (7), we obtain the optimal MSE of $\bar{y}_{pr}$ as

$$MSE(\bar{y}_{pr})_{opt} = \lambda\bar{Y}^2\{C_y^{\,2} + (2 - \delta_2 - 2k)\rho C_y C_x + (\delta_2 - 2)$$

$$\rho C_y C_x + [(2-\delta_2-2k)^2 + 2$$

$$(2 - \delta_2 - 2k)(\delta_2 - 2) + (\delta_2 - 2)^2]\frac{C_x^{\,2}}{4}\}$$

$$MSE(\bar{y}_{pr})_{opt} = \lambda\bar{Y}^2 C_y^{\,2}\left(1-\rho^2\right) \quad (9)$$

From (9), it is observed that the optimum MSE of the proposed estimator is the same as the MSE of the linear regression estimator.

Table 2 shows the different members of the family of the proposed estimator $\bar{y}_{pr}$ obtained from suitably choosing δ_1, δ_2 and α_1. By substitution, various members of this family were formed with their respective biases while noting that $\alpha_2 = 1 - \alpha_1$. The MSE of all these members remained that of the linear regression estimator.

Table 2: Special Cases of Generalized Class of the Proposed Estimator

α_1	α_2	δ_1	δ_2	Estimator	Bias
1	0	$-(k+1)$	$-(k+1)$	$\bar{y}_{pr10}=\bar{y}\left[\frac{\bar{x}}{\bar{X}}\exp\left\{\frac{-(k+1)(\bar{x}-\bar{X})}{\bar{x}+\bar{X}}\right\}\right]$	$\frac{\bar{Y}\lambda}{8}\left\{3C_x^2+8\rho C_yC_x-3\rho^2C_y^2\right\}$
0	1	$1-k$	$1-k$	$\bar{y}_{pr11}=\bar{y}\left[\frac{\bar{X}}{\bar{x}}\exp\left\{\frac{(1-k)(\bar{x}-\bar{X})}{\bar{x}+\bar{X}}\right\}\right]$	$\frac{\bar{Y}\lambda}{8}\left\{3C_x^2-3\rho^2C_y^2\right\}$
$\frac{1}{2}$	$\frac{1}{2}$	$-2k$	$-2k$	$\bar{y}_{pr12}=\bar{y}\left[\frac{\bar{x}}{2\bar{X}}\exp\left\{\frac{-2k(\bar{x}-\bar{X})}{\bar{x}+\bar{X}}\right\}+\frac{\bar{X}}{2\bar{x}}\exp\left\{\frac{-2k(\bar{x}-\bar{X})}{\bar{x}+\bar{X}}\right\}\right]$	$\frac{\bar{Y}\lambda}{2}\left\{C_x^2+\rho C_yC_x-\rho^2C_y^2\right\}$
$\frac{1-k}{2}$	$\frac{1+k}{2}$	0	0	$\bar{y}_{pr13}=\bar{y}\left[\left(\frac{1-k}{2}\right)\left(\frac{\bar{x}}{\bar{X}}\right)+\left(\frac{1+k}{2}\right)\left(\frac{\bar{X}}{\bar{x}}\right)\right]$	$\frac{\bar{Y}\lambda}{2}\left\{C_x^2+\rho C_yC_x-2\rho^2C_y^2\right\}$
$\frac{1-2k}{4}$	$\frac{3+2k}{4}$	1	1	$\bar{y}_{pr14}=\bar{y}\left[\left(\frac{1-2k}{4}\right)\frac{\bar{x}}{\bar{X}}\exp\left\{\frac{(\bar{x}-\bar{X})}{\bar{x}+\bar{X}}\right\}+\left(\frac{3+2k}{4}\right)\frac{\bar{X}}{\bar{x}}\exp\left\{\frac{(\bar{x}-\bar{X})}{\bar{x}+\bar{X}}\right\}\right]$	$\bar{Y}\lambda\left\{C_x^2+\rho C_yC_x-4\rho^2C_y^2\right\}$
$\frac{-k}{2}$	$1+\frac{k}{2}$	2	2	$\bar{y}_{pr15}=\bar{y}\left[\left(\frac{-k}{2}\right)\frac{\bar{x}}{\bar{X}}\exp\left\{\frac{2(\bar{x}-\bar{X})}{\bar{x}+\bar{X}}\right\}+\left(1+\frac{k}{2}\right)\frac{\bar{X}}{\bar{x}}\exp\left\{\frac{2(\bar{x}-\bar{X})}{\bar{x}+\bar{X}}\right\}\right]$	$\bar{Y}\lambda\left\{-\rho^2C_y^2\right\}$
$\frac{2-3k}{4}$	$\frac{2+3k}{4}$	k	k	$\bar{y}_{pr16}=\bar{y}\left[\left(\frac{2-3k}{4}\right)\frac{\bar{x}}{\bar{X}}\exp\left\{\frac{k(\bar{x}-\bar{X})}{\bar{x}+\bar{X}}\right\}+\left(\frac{2+3k}{4}\right)\frac{\bar{X}}{\bar{x}}\exp\left\{\frac{k(\bar{x}-\bar{X})}{\bar{x}+\bar{X}}\right\}\right]$	$\frac{\bar{Y}\lambda}{4}\left\{2C_x^2+\rho C_yC_x-5\rho^2C_y^2\right\}$
$\frac{3-2k}{2}$	$\frac{-1+2k}{2}$	-1	-1	$\bar{y}_{pr17}=\bar{y}\left[\left(\frac{3-2k}{2}\right)\frac{\bar{x}}{\bar{X}}\exp\left\{\frac{-1(\bar{x}-\bar{X})}{\bar{x}+\bar{X}}\right\}+\left(\frac{-1+2k}{2}\right)\frac{\bar{X}}{\bar{x}}\exp\left\{\frac{-1(\bar{x}-\bar{X})}{\bar{x}+\bar{X}}\right.\right.$	$\frac{\bar{Y}\lambda}{8}\left\{24\rho C_yC_x-3C_x^2-16\rho^2C_y^2\right\}$
$\frac{3-2k}{6}$	$\frac{3+2k}{6}$	-1	1	$\bar{y}_{pr18}=\bar{y}\left[\left(\frac{3-2k}{6}\right)\frac{\bar{x}}{\bar{X}}\exp\left\{\frac{-1(\bar{x}-\bar{X})}{\bar{x}+\bar{X}}\right\}+\left(\frac{3+2k}{6}\right)\frac{\bar{X}}{\bar{x}}\exp\left\{\frac{(\bar{x}-\bar{X})}{\bar{x}+\bar{X}}\right\}\right]$	$\frac{\bar{Y}\lambda}{8}\left\{12\rho C_yC_x+9C_x^2-8\rho^2C_y^2\right\}$
$\frac{1+2k}{2}$	$\frac{1-2k}{2}$	1	-1	$\bar{y}_{pr19}=\bar{y}\left[\left(\frac{1+2k}{2}\right)\frac{\bar{x}}{\bar{X}}\exp\left\{\frac{(\bar{x}-\bar{X})}{\bar{x}+\bar{X}}\right\}+\left(\frac{1-2k}{2}\right)\frac{\bar{X}}{\bar{x}}\exp\left\{\frac{-1(\bar{x}-\bar{X})}{\bar{x}+\bar{X}}\right\}\right]$	$\frac{\bar{Y}\lambda}{8}\left\{4\rho C_yC_x-3C_x^2+8\rho^2C_y^2\right\}$

3. Efficiency comparison

The efficiency comparisons in this study are done using the mean square error of the proposed estimators and that of three existing estimators.

3.1. Efficiency comparison with classical ratio estimator

The mean square error of the classical ratio estimator is given as

$$MSE\left(\bar{y}_{cl_R}\right)=\frac{(1-f)}{n}\bar{Y}^2\left\{C_y^2-2\rho C_yC_x+C_x^2\right\}$$

The condition for the proposed estimators to be more efficient than the classical ratio estimator is defined by

$$MSE\left(\bar{y}_{pr}\right)-MSE\left(\bar{y}_{cl}\right)<0$$

$$\frac{(1-f)}{n}\bar{Y}^2C_y^2\left(1-\rho^2\right)-\frac{(1-f)}{n}\bar{Y}^2\left\{C_y^2-2\rho C_yC_x+C_x^2\right\}<0$$

$$C_y^2\left(1-\rho^2\right)-C_y^2+2\rho C_yC_x-C_x^2<0$$

$$\rho^2C_y^2-2\rho C_yC_x+C_x^2>0$$

$$\left(\rho C_y-C_x\right)^2>0 \qquad (10)$$

It implies from (10) that the proposed estimator is always more efficient than the classical ratio estimator.

3.2. Efficiency comparison with Bahl and Tuteja [1] exponential ratio estimator

The mean square error of the exponential ratio estimator is given by

$$MSE\left(\bar{y}_{exp(R)}\right)=\frac{(1-f)}{n}\bar{Y}^2\left[C_y^2+\frac{C_x^2}{4}(1-4k)\right]$$

The condition for the proposed estimators to be more efficient than [1] exponential estimator is defined by

$$MSE\left(\bar{y}_{pr}\right)-MSE\left(\bar{y}_{ex}\right)<0$$

$$\frac{(1-f)}{n}\bar{Y}^2C_y^2\left(1-\rho^2\right)-\frac{(1-f)}{n}\bar{Y}^2\left[C_y^2+\frac{C_x^2}{4}(1-4k)\right]<0$$

$$C_y^2\left(1-\rho^2\right)-C_y^2-\frac{C_x^2}{4}(1-4k)<0$$

$$4\rho^2C_y^2-4\rho C_yC_x+C_x^2>0$$

$$\left(2\rho C_yC_x-C_x\right)^2>0 \qquad (11)$$

It implies from (11) that the proposed estimator is always more efficient than the [1] estimator.

3.3. Efficiency comparison with Kadilar and Cingi [8] chain ratio estimator

The mean square error of the chain ratio estimator is given as

$$MSE\left(\bar{y}_{ch_R}\right) = \frac{(1-f)}{n}\bar{Y}^2\left[C_y^2 + 4C_x^2(1-k)\right]$$

The condition for the proposed estimators to be more efficient than the chain ratio estimator is defined by

$$MSE\left(\bar{y}_{pr}\right) - MSE\left(\bar{y}_{ch}\right) < 0$$

$$\frac{(1-f)}{n}\bar{Y}^2 C_y^2\left(1-\rho^2\right) - \frac{(1-f)}{n}\bar{Y}^2\left[C_y^2 + 4C_x^2(1-k)\right] < 0$$

$$4\rho^2 C_y^2 + 4C_x^2 - 4\rho C_y C_x > 0$$

$$\left(2\rho C_y - C_x\right)^2 > 0 \quad (12)$$

The condition in (12) shows that the proposed estimator is always more efficient than the chain ratio estimator.

4. Numerical illustration

To further authenticate the results of our study, data set are extracted from three existing works by different authors. This is to examine the performance of the proposed estimator in this study when compared with other existing estimators.

4.1. Data for population one (1)

$N = 104$

$\rho = 0.865$

$\lambda = 0.040$

$n = 20$

$C_x = 1.653$

$k = 104$

$C_y = 1.866$

$\bar{Y} = 625.37$

Source: Kadilar and Cingi [10]

4.2. Data for population two (2)

$N = 106$

$\rho = 0.82$

$\lambda = 0.0405$

$n = 20$

$C_x = 2.02$

$k = 1.697$

$C_y = 4.18$

$\bar{Y} = 15.37$

Source: Kadilar and Cingi [8]

4.3. Data for population three (3)

$N = 278$

$\rho = 0.7213$

$\lambda = 0.017236$

$n = 48$

$C_x = 1.6198$

$k = 0.643506$

$C_y = 1.4451$

$\bar{Y} = 39.068$

Source: Das [5]

Table 3: Estimators with Their Respective MSE, Bias and Relative Bias for Population 1

Estimators	MSE	Bias	Relative bias (%)
$\bar{y}_{cl_R}$	13870	1.6243	1.379
$\bar{y}_{exp(R)}$	23643	-7.8138	5.08172
$\bar{y}_{Ch_R}$	59057.1	-65.759	27.0595
$\bar{y}_{LR}$	13846.05	0	0
$\bar{y}_{pr1}$	13846.05	68.5875	58.28834
$\bar{y}_{pr2}$	13846.05	1.20395	1.02316
$\bar{y}_{pr3}$	13846.05	35.297	29.99
$\bar{y}_{pr4}$	13846.05	2.39833	2.038197
$\bar{y}_{pr5}$	13846.05	-126.8	107.758
$\bar{y}_{pr6}$	13846.05	65.7974	55.91721
$\bar{y}_{pr7}$	13846.05	-30.897	26.2574
$\bar{y}_{pr8}$	13846.05	44.6778	37.969
$\bar{y}_{pr9}$	13846.05	112.9118	95.9569
$\bar{y}_{pr10}$	13846.05	73.61116	62.55769

Table 4: Estimators with Their Respective MSE, Bias and Relative Bias for Population 2

Estimators	MSE	Bias	Relative bias (%)
$\bar{y}_{cl_R}$	73.8414	-1.7728	20.6309
$\bar{y}_{\exp(R)}$	110.866	-1.2044	11.4389
$\bar{y}_{Ch_R}$	58.4478	-6.0898	79.6559
$\bar{y}_{LR}$	54.8538	0	0
$\bar{y}_{pr1}$	54.8538	2.52408	34.07995
$\bar{y}_{pr2}$	54.8538	-1.7929	24.2074
$\bar{y}_{pr3}$	54.8538	-0.232	3.13289
$\bar{y}_{pr4}$	54.8538	-3.8946	52.5848
$\bar{y}_{pr5}$	54.8538	-22.44	302.977
$\bar{y}_{pr6}$	54.8538	7.32515	98.90388
$\bar{y}_{pr7}$	54.8538	-6.8051	90.8826
$\bar{y}_{pr8}$	54.8538	-2.6535	35.8271
$\bar{y}_{pr9}$	54.8538	2.012431	27.17175
$\bar{y}_{pr10}$	54.8538	8.529585	115.1661

Table 5: Estimators with Their Respective MSE, Bias and Relative Bias for Population 3

Estimators	MSE	Bias	Relative bias (%)
$\bar{y}_{cl_R}$	35.1279	0.62985	10.62704
$\bar{y}_{\exp(R)}$	27.7771	0.09408	1.785
$\bar{y}_{Ch_R}$	153.367	-0.5071	4.09464
$\bar{y}_{LR}$	26.3556	0	0
$\bar{y}_{pr1}$	26.3556	1.52513	29.70775
$\bar{y}_{pr2}$	26.3556	0.38819	7.561432
$\bar{y}_{pr3}$	26.3556	1.08605	21.15507
$\bar{y}_{pr4}$	26.3556	0.72024	14.02943
$\bar{y}_{pr5}$	26.3556	-0.0228	0.44371
$\bar{y}_{pr6}$	26.3556	0.73163	14.25128
$\bar{y}_{pr7}$	26.3556	0.2531	4.930027
$\bar{y}_{pr8}$	26.3556	1.28502	25.03072
$\bar{y}_{pr9}$	26.3556	2.961427	57.68518
$\bar{y}_{pr10}$	26.3556	0.637552	12.41878

5. Discussion

The expression for the MSE of the proposed class of estimator given in equation (9) is found to be the same as the regression estimator. This finding makes the proposed estimator a suitable alternative to the regression estimator. Analytical results show that the proposed alternative class is more efficient than classical ratio, [1], [8] estimators. Table 2 shows members of the family of the proposed class

Empirical results from Table 3, Table 4 and Table 5 show that the proposed class has a smaller MSE compared to classical ratio, [1], [8], [9], and [10] estimators for all three (3) populations used in this work. This can be attributed to the fact that it is as efficient as the regression estimator and confirms [4], [14], [12], [11] that the regression estimator is generally more efficient than the ratio and product estimators.

In the aspect of bias, for every population a member of the proposed class has the least bias when compared with some existing estimators. For populations 1,2 and 3, $\bar{y}_{pr2}$, $\bar{y}_{pr3}$ and $\bar{y}_{pr5}$ have the least bias respectively. The relative biases of some members of the proposed class shown in Table 3, Table 4 and Table 5 are less than 10% for every population under investigation showing that some members of the proposed estimator are almost unbiased. According to [13] any estimator with relative bias less than 10% is considered to have a negligible bias. It has also been observed from the tables that the performances of this proposed class of estimators in terms of bias, depends on the type of population under study.

6. Conclusion

Members of the modified class of estimators have been found to perform better in terms of bias and efficiency than the three existing estimators considered. This proposed class of estimator compares favourably with the regression estimator in terms of efficiency, but in terms of bias, some members of the proposed class of estimators performs less than linear regression estimator. Thus, those members with negligible bias, according to [13], provide good alternatives to the linear regression estimator of the population mean, as their performances, in terms of bias and efficiency, are almost equally significant.

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