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Research paper

Monitoring Lognormal Parameters with Max-EWMA Chart: Revisitation of An Existing Work

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Abstract

Almulhim et al. (2024) proposed, developed, and evaluated a Max-EWMA chart on the metrics of average run length (ARL) and standard deviation (SD) of the run length distribution. The review of Almulhim et al. (2024), to the best of my knowledge, overestimated the performance of the chart. The overestimation is in terms of reporting smaller out-of-control average run lengths. (ARL_1) than they should be, which makes the chart unnecessarily too good. Again, the results discussion in the work is not detailed, which limits better understanding of the chart's monitoring behavior. This study re-evaluates the performance of the Max-EWMA chart for joint monitoring of lognormal parameters with a detailed discussion to reflect reality. The result confirms the overestimation of the performance of the Max-EWMA chart and provides improved insight into the sensitivity reality of the Max-EWMA chart.

Keywords: *Lognormal Process; Max-EWMA Chart; Joint Monitoring; Simulation; Chart Performance; Normality.*

1. Introduction

In manufacturing processes, changes are monitored and controlled with quality control charts (Montgomery, 1991). The majority of the control charts in the literature are developed to monitor and control for the process mean and variance independently (McCracken & Chakraborti, 2013). The most conventional among them include the Shewhart charts and the traditional types of the cumulative sum chart (CUSUM) and exponentially weighted moving average (EWMA) chart. But in a process, both location and dispersion parameters of the quality characteristics can change simultaneously, and these control charts become unable to correctly track such shifts. Hence, to ensure product quality and process stability through statistical process monitoring, monitoring the process mean and variation jointly is important. Roberts (1959) first introduced the EWMA control chart designed to monitor changes in the process mean. The chart was more sensitive to small changes in the process mean than the standard Shewhart chart. Afterward, Crowder and Hamilton (1992) modified Roberts (1959) to monitor for the process variance independently. Obviously, now, two different EWMA charts- one for the process mean and the other for the process variance are required to monitor the process mean and variability of a single process (Chen et al., 2001). The researchers did not develop one EWMA control chart with the capacity to monitor both the process mean and variability simultaneously. However, Gan (1995) highlighted the inadequacy of the performance of a system with two independent EWMA charts and suggested a simple design procedure that combines the two charts. Chen et al. (2001) proposed a new Max-EWMA chart to address this need, and the findings proved that the Max-EWMA chart is good in identifying small to moderate shifts in both process parameters.

A lot of quality control charts in the literature are developed to monitor the parameters of normal processes (Huang et al., 2017 & Almulhim et al., 2024). However, in practical application, a process of interest usually deviates from normality and exhibits some significant degree of positive or right skew. A lognormal distribution is popular and relevant among the host of right-skewed distributions. The works of Morrison (1958), Jofle and Sichel (1968), and Kotz and Lovelace (1998) comprise a sample of research on processes that adhere to the lognormal distribution. The literature revealed that some of these works are either for the mean of the lognormal distribution (see; Morrison, 1958; Omar et al.,2021; Joffe & Sichel, 1968) or the variability of the lognormal, see (Yang, 2013; Huang et al., 2017) while others are both for the mean and variability of the lognormal distribution (Iqbal et al., 2023; & Huang, 2021). The ones on joint monitoring of the mean and variability mainly applied separate charts for mean and variance, but Almulhim et al. (2024) advanced the research to simultaneous monitoring of the scale and shape parameters in a lognormal process using a single EWMA statistic. This is an extension of Chen et al. (2001) for a lognormal distribution.

Evaluation of the performance of control charts is mainly based on the characteristics of the run length (RL) distribution, predominantly the average and the standard deviation of the RL distribution. Almulhim et al. (2024) evaluated the performance of the Max-EWMA chart on the basis of these metrics. However, as already said, to the best of our knowledge, the evaluation of Almulhim et al. (2024) overestimated the performance of the chart by reporting too small an ARL_1, which made the chart excessively sensitive. Secondly, the discussion of the

results in Almulhim et al. (2024) was not detailed enough to showcase the monitoring behaviour of the chart very well. Because of this, there is a need to re-evaluate the performance for a more reliable conclusion of the Max-EWMA chart.
Onwards, the presentation of this work follows. Section 2 presents the methodology of the Max-EWMA chart under a lognormal distribution. Monte Carlo simulation is in Section 3; Section 4 is the results presentation; Section 5 is the discussion of results and real-life application of the Max-EWMA control chart, while the conclusion of the study is in Section 6.

2. Methodology

2.1. The design of max-EWMA chart for lognormal distribution

Let $X_{i1}, X_{i2}, ..., X_{ij}$; i = 1, 2, . . ., j = 1, 2, . . ., n be random observations taken from a lognormal distribution with parameters (μ_x, σ_x). The probability density and the cumulative density functions are given by $f(x) = \frac{1}{x\sigma_x\sqrt{2\pi}}\exp\left(-\frac{(\ln x - \mu x)^2}{2\sigma_x^2}\right)$ and $F(x) = \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{\ln x - \mu_x}{\sigma_x\sqrt{2}}\right)\right]$ respectively, where $\operatorname{erf}(\cdot)$ denotes the error function. To enable the application of normal-based control charts, a logarithmic transformation is needed. Define $Y = \ln(x)$, then, Y follows a normal distribution with parameters (μ_y, σ_y^2), where $\mu_y = \ln\left(\frac{\mu_x^2}{\sqrt{\mu_x^2 + \sigma_x^2}}\right)$ and $\sigma_y^2 = \ln\left(1 + \frac{\sigma_x^2}{\mu_x^2}\right)$. These expressions connect the relationship between the mean and variance of the lognormal distribution to the associated normal distribution. Therefore, for each observed x, $Y = \ln(x)$ produces data that are approximately normally distributed with mean μ_y and variance σ_y^2.

Let $K_i = \frac{(\overline{X_i} - \mu_0)}{\sigma_0/\sqrt{n}}$ and $L_i = \Phi^{-1}\left[H\left\{\frac{(n-1)S_i^2}{\sigma_0^2}, (n-1)\right\}\right]$, where $\overline{X_i} = \frac{\sum_{j=1}^{n} X_{ij}}{n}$, $S_i^2 = \frac{\sum_{j=i}^{n}\left(Y_{ij} - \overline{Y_i}\right)^2}{n-1}$, Φ^{-1} is the inverse function of the standard normal distribution and $H\{\cdot\}$ is the cumulative density function of a chi-square random variable for every i^{th} sample drawn from Y. Then, it is known that. $K_i \sim N(0,1)$ and $L_i \sim N(0,1)$ and the EWMA statistics for the mean and variance of the process are respectively defined in (1) and (2)

$$M_i = \lambda K_i + (1 - \lambda)M_{i-1} \tag{1}$$

$$N_i = \lambda L_i + (1 - \lambda)L_{i-1} \tag{2}$$

Where $0 < \lambda \leq 1$ is the smoothing parameter. The values of both M_0 and N_0 are set to zero (0) for the first sample, which is the mean of their distribution. M_{i-1} and L_{i-1} are the preceding values of M_i and L_i quantities respectively. Under an in-control (IC) state, $E(M_i) = E(N_i) = 0$ and $V(M_i) = V(N_i) = \sqrt{\frac{\lambda}{(2-\lambda)}}$. Therefore, under the IC state, $M_i \sim N\left(0, \frac{\lambda}{(2-\lambda)}\right)$ and $N_i \sim N\left(0, \frac{\lambda}{(2-\lambda)}\right)$. However, the charting statistic is as in (3), which is the maximum of the absolute values of the EWMA statistics.

$$T_i = \max\{| M_i |, | N_i |\} \tag{3}$$

T_i is a non-negative quantity and as such, only the upper control limit of the chart will be used. Therefore, the chart signals an out-of-control (OOC) of the process if the value of (3) exceeds the upper control limit (UCL) of the chart (Chen et al., 2001). According to Xie (1999) and Chen et al. (2001), the UCL is constructed as follows. Standardize M_i and N_i as $Z_{1i} = \frac{M_i}{\sqrt{\lambda/((2-\lambda)}}$ and $Z_{2i} = \frac{N_i}{\sqrt{\lambda/((2-\lambda)}}$ and redefine (3) as in (4)

$$T_i = \sqrt{\frac{\lambda}{(2-\lambda)}}\max\{| Z_{1i} |, | Z_{2i} |\} \tag{4}$$

Let $\max\{| Z_{1i} |, | Z |\}$ be W, the expected value and variance of W are 1.12839, and the standard deviation is 0.602810; therefore, the UCL is as in (5)

$$UCL = (1.12839 + 0.602810K) * \sqrt{V(M_i)} \tag{5}$$

Where K is the control chart multiplier needed to achieve the desired ARL performance for the chart and

3. Monte Carlo simulation

The in-depth analysis and evaluation of the performance of the Max-EWMA control is presented in this section. The simulation procedures are structurally similar to Almulhim et al. (2024), and the steps are:

a) Select the parameters of the lognormal distribution (μ_x, σ_x) and transform them into the parameters of a normal distribution by using the guideline in Section 2.
b) By using the transformed parameters, generate samples each of size (n) from the standard normal distribution
c) Calculate the EWMA statistics for the mean and variance using (1) and (2) for each sample and thereafter, obtain the max-EWMA evaluating (3).
d) Choose the initial values of the parameters (K, λ), and evaluate (4). For each sample, compare the result in (3) with (4) to declare the sample IC or OOC. Record the number of samples before the first out-of-control and repeat (a-d) for 30,000 iterations to obtain 30,000 run lengths (RLs).
e) Obtain the IC (AR_0) and the IC ($SDRL_0$) by calculating the average and the standard deviation of the RLs in step (d). If the targeted ARL_0 is not achieved, repeat (a-e) with a different value of K and continue in the same fashion until the target is achieved. The value

of K for which the target will be used in calculating the OOC ARL (ARL_1)by redoing the entire steps in (a), but shifted lognormal parameters will be used.

4. Results

To study the shift effects of the lognormal parameters, the shape parameter values of 0.5, 1, 1.5, 1.75, 1.9, 2, 2.16, 2.25, 2.5, 2.75, 3, 4 and 5 were considered while the scale parameter values of 0.25, 0.5, 0.75, 1, 1.25, 1.5, 1.649, 1.75, 2, 2.25, 2.5, 2.75, 3, 4 and 5 were used. At (1.65,2.16), the process is designed to IC with the associated normal process. According to Almulhim et al. (2024), for an $ARL_0 = 370$ with λ = 0.2 and 0.3, the associated values of K are 3.252 and 3.345, respectively. The results presented in Table 1 were evaluated for an $ARL_0 = 370$ with λ = 0.2 and associated K value of 3.252.

Table 1: ARL And SDRL of the Max-EWMA Control Chart for the Lognormal Process

		Shape parameter						
		0.5	1.0	1.25	1.5	1.75	1.9	2.16
α	n	ARL(SDRL)	ARL(SDRL)	ARL(SDRL)	ARL(SDRL)	ARL(SDRL)	ARL(SDRL)	ARL(SDRL)
0.25	3	1.00(0.02)	1.03(0.18)	1.07(0.25)	1.10(0.30)	1.12(0.33)	1.14(0.35)	1.16(0.36)
	5	1.00(0.00)	1.00(0.00)	1.00(0.00)	1.00(0.01)	1.00(0.03)	1.00(0.03)	1.00(0.05)
	10	1.00(0.00)	1.00(0.00)	1.00(0.00)	1.00(0.00)	1.00(0.00)	1.00(0.00)	1.00(0.00)
0.5	3	2.00(0.00)	2.00(0.14)	2.01(0.22)	2.01(0.29)	2.01(0.36)	2.01(0.39)	2.01(0.43)
	5	1.82(0.39)	1.68(0.47)	1.65(0.48)	1.64(0.48)	1.62(0.49)	1.61(0.49)	1.61(0.49)
	10	1.00(0.00)	1.00(0.03)	1.00(0.06)	1.01(0.11)	1.02(0.14)	1.03(0.17)	1.04(0.19)
0.75	3	3.01(0.23)	3.02(0.58)	3.04(0.68)	3.06(0.77)	3.08(0.85)	3.08(0.89)	3.11(0.95)
	5	2.08(0.28)	2.24(0.43)	2.28(0.46)	2.32(0.50)	2.34(0.53)	2.35(0.56)	2.36(0.60)
	10	1.99(0.11)	1.90(0.30)	1.85(0.35)	1.82(0.39)	1.79(0.42)	1.79(0.43)	1.77(0.45)
1.00	3	4.83(0.95)	5.75(1.62)	5.82(1.92)	5.85(2.16)	5.84(2.32)	5.85(2.45)	5.85(2.62)
	5	3.09(0.54)	3.90(0.84)	4.00(1.03)	4.03(5.45)	4.06(1.32)	4.06(1.37)	4.10(1.49)
	10	2.01(0.13)	2.60(0.52)	2.65(0.58)	2.67(0.64)	2.68(0.69)	2.70(0.72)	2.71(0.77)
1.25	3	5.30(1.46)	14.78(8.19)	19.31(12.24)	19.95(13.53)	19.06(13.23)	18.37(12.82)	17.22(12.19)
	5	3.13(0.58)	6.93(2.43)	9.33(4.03)	19.55(13.00)	10.85(6.03)	10.84(6.26)	10.52(6.25)
	10	2.01(0.15)	3.74(0.90)	4.94(1.42)	5.67(1.96)	5.92(2.36)	5.96(2.51)	5.94(2.66)
1.35	3	5.30(1.46)	17.49(11.12)	33.54(26.15)	45.05(37.83)	43.12(36.44)	40.18(33.60)	34.89(29.28)
	5	3.14(0.58)	7.41(2.96)	12.97(7.13)	19.55(32.86)	22.01(16.09)	21.80(16.18)	20.22(14.90)
	10	2.02(0.15)	3.83(1.00)	5.88(2.05)	8.54(3.79)	10.25(5.47)	10.43(5.82)	10.23(6.01)
1.649	3	5.30(1.45)	18.04(11.78)	43.21(35.92)	116.52(108.49)	293.73(287.06)	350.21(414.58)	369.63(364.79)
	5	3.14(0.58)	7.49(3.08)	14.73(8.98)	39.74(32.86)	138.69(133.50)	279.19(273.90)	371.55(367.16)
	10	2.01(0.15)	3.84(1.01)	6.25(2.45)	13.26(8.07)	47.63(41.17)	135.00(127.87)	369.66(365.96)
1.75	3	5.32(1.46)	18.07(11.81)	42.96(35.56)	112.59(105.76)	245.33(238.33)	301.12(294.66)	236.31(232.38)
	5	3.14(0.58)	7.48(3.08)	14.81(9.04)	39.24(32.22)	119.15(113.31)	187.57(181.84)	188.65(180.42)
	10	2.01(0.15)	3.84(1.02)	6.27(2.46)	13.24(7.99)	43.73(37.78)	89.25(82.70)	119.89(114.58)
2.00	3	5.31(1.45)	17.87(11.58)	36.56(29.21)	54.53(43.94)	54.46(47.05)	49.99(42.94)	42.60(36.75)
	5	3.13(0.58)	7.43(3.02)	13.53(7.75)	22.48(15.85)	26.98(20.68)	26.63(20.68)	24.57(19.27)
	10	2.01(0.15)	3.83(1.01)	6.00(2.17)	9.42(4.42)	11.99(6.87)	12.52(7.59)	12.24(7.69)
2.25	3	5.31(1.44)	14.27(7.66)	18.05(11.23)	18.65(47)	17.94(12.11)	17.28(11.95)	16.26(11.24)
	5	3.14(0.58)	6.78(2.31)	9.01(3.83)	10.07(5.04)	10.23(5.51)	10.18(5.71)	9.94(5.74)
	10	2.01(0.15)	3.71(0.88)	4.82(1.34)	5.43(1.86)	5.65(2.22)	5.68(2.33)	5.68(2.50)
2.50	3	5.28(1.41)	9.54(3.81)	10.13(4.66)	18.65(12.21)	9.96(5.30)	9.88(5.44)	9.57(5.42)
	5	3.13(0.58)	5.55(1.51)	6.10(2.01)	6.33(2.43)	6.36(2.66)	6.33(2.78)	6.34(2.97)
	10	2.01(0.15)	3.34(0.66)	3.69(0.88)	3.82(1.07)	3.88(1.22)	3.88(1.28)	3.91(1.39)
2.75	3	5.13(1.20)	6.95(2.20)	7.11(2.63)	7.11(2.97)	7.07(3.18)	7.03(3.27)	7.00(3.41)
	5	3.12(0.57)	4.50(1.05)	4.67(1.32)	4.73(1.54)	4.79(1.71)	4.78(1.77)	4.79(1.90)
	10	2.01(0.15)	2.91(0.53)	3.00(0.65)	3.05(0.76)	3.08(0.84)	3.08(0.89)	3.08(0.94)
3.00	3	4.31(0.95)	5.52(1.50)	5.59(1.79)	5.63(2.03)	5.61(2.23)	5.62(2.30)	5.62(2.45)
	5	3.07(0.52)	3.80(0.81)	3.87(0.97)	3.90(1.12)	3.95(1.25)	3.96(1.32)	3.96(1.41)
	10	2.01(0.12)	2.53(0.52)	2.57(0.56)	2.60(0.62)	2.61(0.66)	2.63(0.69)	2.64(0.73)
4.00	3	3.31(0.48)	3.44(0.69)	3.47(0.83)	3.48(0.93)	3.51(1.03)	3.51(1.08)	3.53(1.17)
	5	2.53(0.50)	2.56(0.53)	2.58(0.56)	2.59(0.62)	2.61(0.66)	2.63(0.69)	2.65(0.74)
	10	2.00(0.07)	1.98(0.14)	1.97(0.23)	1.95(0.30)	1.95(0.35)	1.94(0.38)	1.94(0.42)
5.00	3	2.85(0.36)	2.76(0.54)	2.77(0.62)	2.79(0.67)	2.81(0.74)	2.81(0.77)	2.84(0.83)
	5	2.00(0.07)	2.08(0.28)	2.12(0.34)	2.15(0.40)	2.17(0.44)	2.17(0.47)	2.19(0.51)
	10	1.86(0.35)	1.72(0.45)	1.68(0.46)	1.67(0.47)	1.64(0.48)	1.64(0.48)	1.63(0.49)

Table 1: Continued

		Shape parameter						
		2.25	2.5	2.75	3	3.5	4	5
α	n	ARL(SDRL)	ARL(SDRL)	ARL(SDRL)	ARL(SDRL)	ARL(SDRL)	ARL(SDRL)	ARL(SDRL)
0.25	3	1.16(0.37)	1.18(0.38)	1.19(0.39)	1.20(0.40)	1.22(0.41)	1.24(0.43)	1.26(0.44)
	5	1.00(0.06)	1.01(0.07)	1.01(0.09)	1.01(0.10)	1.02(0.13)	1.02(0.15)	1.03(0.18)
	10	1.00(0.00)	1.00(0.00)	1.00(0.00)	1.00(0.00)	1.00(0.00)	1.00(0.00)	1.00(0.01)
0.5	3	2.01(0.45)	2.02(0.49)	2.01(0.51)	2.02(0.55)	2.02(0.59)	2.02(0.62)	2.03(0.68)
	5	1.60(0.50)	1.60(0.50)	1.60(0.50)	1.59(0.51)	1.60(0.52)	1.59(0.53)	1.60(0.55)
	10	1.04(0.20)	1.05(0.23)	1.07(0.25)	1.07(0.26)	1.09(0.29)	1.10(0.30)	1.12(0.33)
0.75	3	3.10(0.97)	3.13(1.04)	3.14(1.09)	3.14(1.12)	3.15(1.19)	3.16(1.24)	3.12(1.29)
	5	2.37(0.61)	2.38(0.65)	2.39(0.69)	2.40(0.72)	2.41(0.76)	2.41(0.80)	2.39(0.84)
	10	1.77(0.45)	1.76(0.47)	1.76(0.49)	1.76(0.50)	1.75(0.53)	1.74(0.54)	1.73(0.56)
1.00	3	5.83(2.62)	5.81(2.75)	5.77(2.77)	5.68(2.82)	5.52(2.79)	5.37(2.75)	4.97(2.55)
	5	4.10(1.52)	4.12(1.60)	4.10(1.66)	4.09(1.68)	4.01(1.72)	3.91(1.69)	3.64(1.58)
	10	2.72(0.78)	2.74(0.84)	2.74(0.86)	2.74(0.89)	2.71(0.91)	2.66(0.90)	2.50(0.85)

1.25	3	16.85(12.00)	15.58(11.12)	14.55(10.33)	13.32(9.28)	11.37(7.70)	9.81(6.42)	7.81(4.91)
	5	10.44(6.29)	10.08(6.09)	9.47(5.69)	8.94(5.36)	7.62(4.36)	6.57(3.59)	5.24(2.67)
	10	5.94(2.70)	5.84(2.75)	5.66(2.67)	5.43(2.55)	4.74(2.11)	4.15(1.74)	3.36(1.32)
1.35	3	32.99(27.54)	28.35(23.42)	24.00(19.25)	20.50(16.05)	15.31(11.36)	12.36(8.71)	9.05(5.97)
	5	19.54(14.57)	17.65(12.92)	15.43(11.05)	13.18(9.03)	9.85(6.30)	7.90(4.69)	5.82(3.14)
	10	10.22(6.05)	9.65(5.71)	8.68(4.99)	7.65(4.17)	5.87(2.88)	4.77(2.17)	3.61(1.49)
1.649	3	293.56(288.78)	141.30(137.49)	76.29(71.29)	47.66(42.87)	25.24(19.88)	16.98(13.23)	10.83(7.68)
	5	279.89(271.91)	105.96(100.05)	48.46(43.67)	28.06(23.34)	14.63(10.71)	10.03(6.64)	6.61(3.80)
	10	252.93(248.24)	62.27(57.13)	24.52(19.73)	14.15(9.98)	7.65(4.40)	5.55(2.81)	3.88(1.69)
1.75	3	197(190.62)	110.72(106.54)	65.55(60.48)	42.73(37.75)	24.04(19.88)	16.65(12.80)	10.67(7.48)
	5	157.75(152.00)	78.32(72.89)	42.05(37.11)	25.92(21.25)	14.11(10.23)	7.80(4.69)	6.48(3.72)
	10	99.68(94.62)	44.25(38.96)	21.59(16.71)	13.44(9.21)	7.52(4.23)	5.45(2.74)	3.85(1.69)
2.00	3	39.41(33.70)	32.83(27.56)	27.44(22.42)	22.58(18.24)	16.45(12.64)	12.99(9.32)	9.22(6.13)
	5	23.61(18.37)	20.58(15.64)	17.38(12.79)	14.37(10.18)	10.48(6.78)	8.20(4.98)	5.91(3.21)
	10	12.09(7.70)	11.12(6.95)	9.77(5.81)	8.25(4.65)	6.14(3.06)	4.88(2.26)	3.66(1.53)
2.25	3	15.85(11.02)	14.78(10.20)	13.82(9.58)	12.81(8.86)	11.01(7.33)	9.59(6.29)	7.73(4.80)
	5	9.85(5.85)	9.59(5.69)	9.07(5.41)	8.53(4.99)	7.37(4.17)	6.44(3.51)	5.21(2.67)
	10	5.69(2.56)	5.61(2.55)	5.47(2.54)	5.25(2.42)	4.63(2.03)	4.09(1.71)	3.34(1.32)
2.50	3	9.55(5.53)	9.28(5.47)	9.06(5.40)	8.69(5.13)	8.07(4.88)	7.45(4.40)	6.42(3.71)
	5	6.32(2.96)	6.25(3.04)	6.14(3.05)	5.99(3.00)	5.60(2.82)	5.20(2.58)	4.51(2.17)
	10	3.91(1.41)	3.90(1.48)	3.89(1.51)	3.84(1.51)	3.65(1.43)	3.42(1.31)	2.99(1.11)
2.75	3	6.99(3.47)	6.91(3.53)	6.80(3.57)	6.72(3.60)	6.41(3.46)	6.11(3.33)	5.54(3.02)
	5	4.80(1.97)	4.77(2.01)	4.75(2.07)	4.73(2.10)	4.56(2.06)	4.40(1.20)	3.97(1.80)
	10	3.10(0.97)	3.11(1.03)	3.12(1.07)	3.11(1.09)	3.05(1.09)	2.94(1.04)	2.70(0.95)
3.00	3	5.62(2.50)	5.60(2.57)	5.58(2.64)	5.54(2.69)	5.36(2.67)	5.25(2.69)	4.88(2.51)
	5	3.97(1.45)	3.98(1.51)	3.97(1.57)	3.95(1.60)	3.88(1.62)	3.80(1.61)	3.57(1.53)
	10	2.64(0.75)	2.66(0.79)	2.67(0.82)	2.67(0.85)	2.66(0.89)	2.61(0.87)	2.47(0.83)
4.00	3	3.54(1.20)	3.56(1.26)	3.56(2.64)	3.56(1.37)	3.57(1.45)	3.54(1.48)	3.49(1.54)
	5	2.65(0.75)	2.66(0.79)	2.68(0.84)	2.68(0.87)	2.69(0.91)	2.68(0.96)	2.64(0.97)
	10	1.94(0.43)	1.94(0.46)	1.94(0.49)	1.93(0.52)	1.93(0.55)	1.93(0.58)	1.90(0.61)
5.00	3	2.84(0.83)	2.85(0.89)	2.86(0.95)	2.87(0.96)	2.88(1.03)	2.88(1.07)	2.87(1.14)
	5	2.19(0.53)	2.21(0.57)	2.22(0.60)	2.22(0.62)	2.22(0.67)	2.22(0.71)	2.22(0.75)
	10	1.63(0.49)	1.62(0.50)	1.62(0.50)	1.62(0.51)	1.62(0.52)	1.62(0.53)	1.60(0.55)

5. Discussion of Results

The existing performance of the Max-EWMA chart for a lognormal process is evaluated on the basis of the ARL and the SDRL measure through Monte-Carlo simulation. While the ARL_0 is the expected number of samples collected before the chart raises an OOC alarm for the process, the SDRL measures the variability of the RL distribution. The SDRL is the square root of the variance of the RL distribution, and as such, the value tells the stability of the chart's detection behaviour. Ideally, to minimize the false alarm and large OOC alarm rates, an effective control chart is expected to have a large IC ARL (ARL_0) and a small OOC ARL (ARL_1). The value of the ARL_0 at this point serves as the baseline for evaluating the performance of the chart.

As earlier stated, in this study, the IC state is defined for the parameter combination (1.649, 2.16), while all other combinations represent a shift either in the scale parameter, the shape parameter, or both. The value of the ARL_0 at (1.649, 2.16) serves as the baseline for evaluating the performance of the chart. Table 1 shows that at an IC state, when $n = 3$, the Max-EWMA control chart recorded the (ARL_0, $SDRL_0$) of (369.63, 364.79), which indicates that the chart maintains an acceptable false alarm rate when the process is operating under a stable condition. But as n increases to 5 and 10, the values become (371.55, 367.16) and (369.66, 365.96) respectively, which are all significantly close to the target, $ARL_0 = 370$. Conclusively, it can be said that the Max-EWMA chart has a good IC performance around the target value.

Shifts in the row (scale parameter) away from the IC value of 1.649 influence the Max-EWMA chart by altering its monitoring statistics, which have consequent effects on the ARL values. For instance, consider the IC reference point (1.649, 2.16) for the scale and shape parameters with sample size ($n = 5$) kept constant, if we cause incremental reductions in the scale parameter from 1.35 to 0.25 while maintaining the shape parameter, Max-EWMA yields ARL_1 values of 20.22, 10.52, 4.10, 2.36, 1.61 and 1.00. This sequence shows a clear and sharp structured decline of the ARL from the IC value ($ARL_0 = 370$) which signifies that the Max-EWMA control chart is highly responsive to reductions in the scale parameter. The sequence as well tells that the rate of detection is directly proportional to the magnitude of shift from the IC scale parameter value of 1.649. A similar trend is identified with positive shifts of 1.75 to 5.0 away from the IC scale parameter value, which is proof that the Max-EWMA control chart tracks shifts on both sides of the IC scale parameter very well. To see the effects of the sample size (n) on the scale parameter, the ARL_1 for the incremental reductions in the scale parameter from 1.35 to 0.25 are reported again from Table 1 but for (n = 3 &10). While their respective values are 34.89, 17.22, 5.85, 3.11, 2.01 and 1.16 for n = 3, the values are 10.23, 5.94, 2.71, 1.77, 1.04 and 1.00 for n = 10. This shows that the ARL_1 values consistently declined from $n = 3$ to $n = 10$ which indicates that larger sample sizes boost the sensitivity of the Max-EWMA control chart in detecting process shifts more quickly, but smaller sample sizes introduce more sampling variability, which delays the detection of the process shifts by the chart. A similar trend is seen for any particular shift, either in the scale or shape parameters or in both.

As well, shifts in the columns (shape parameter) away from the IC value of 2.16 affect both the skewness and the dispersion measure of the lognormal distribution and, as such, affect the performance of the monitoring structure of the Max-EWMA control chart. For instance, consider the IC reference point (1.649, 2.16) for the scale and shape parameters with sample size ($n = 5$) kept constant; if we cause incremental reductions in the shape parameter from 1.9 to 0.5 while maintaining the scale parameter, Max-EWMA yields ARL_1 values of 279.19, 138.69, 39.74, 14.74, 7.49, and 3.14. This progressive decrease shows a clear and significant drift from the IC value ($ARL_0 = 370$), which proves that the Max-DEWMA chart is very responsive to reductions in the shape parameter. A close trend is identified for the upward shifts, which confirms that the chart effectively tracks shifts in both directions of the shape parameter. To see the effects of the sample size (n) on the shape parameter, the ARL_1 for the incremental reductions in the shape parameter from 1.9 to 0.5 are reported again from Table 1 but for (n = 3 &10). While their respective values are 350.21, 293.73, 116.52, 43.21, 18.04, and 5.30 for n = 3, the values are 135.00, 47.63, 13.26, 6.25, 3.84, and 2.01 for n = 10. This shows that the ARL_1 values consistently declined from $n = 3$ to $n = 10$, which indicates that larger sample sizes boost the sensitivity of the proposed Max-EWMA control chart in detecting process shifts more quickly, but smaller sample sizes introduce more sampling variability, which delays the detection of the process shifts by the chart. A similar trend is seen for any particular shift, either in the scale or shape parameters or in both.

5.1. Comparative study with Almulhim et al. (2024)

In Almulhim et al. (2024), when the IC reference point (1.649, 2.16) is considered for the scale and shape parameters, if incremental reductions in the scale parameter from 1.35 to 0.25 are caused while maintaining the shape parameter, Almulhim et al. (2024) reported ARL_1 values to be 18.10, 9.89, 3.99, 2.30, 1.54 and 1.17 for n = 3, 11.41, 6.46, 2.82, 1.73, 1.23 and 1.04 for n = 5 and 6.37, 3.90, 1.92, 1.24, 1.03 and 1.00 for n = 10. A similar trend is seen for every other parameter shift in the work. These values, when compared to ARL_1 that we presented in Table 1, are much smaller, which confirms that Almulhim et al. (2024) overestimated the Max-EWMA chat for the Lognormal distribution. The codes for the analysis and performance evaluation are presented in the Appendix for reference, especially for the verification of our results.

5.2. Real-life application of the Max-EWMA control chart

In this section, we provide the practical application of the Max-EWMA control chart. The dataset under consideration for analysis is an engine oil dataset, which is the percent viscosity increase (PVIS) measurements reported by Huang et al. (2018) and referenced by Almulhim et al. (2024). Specifically, the PVIS data for Ref Oil 434 with the IC (mean and standard deviation) of (150.672, 127.376) were considered. It is common to see such data in reliability engineering and condition monitoring, where the lubricants are positively skewed, which makes the lognormal distribution appropriate in modeling such datasets.
The observations were transformed using the natural logarithm ($Y = \ln(X)$) to facilitate the application of the proposed methodology. The transformation approximately normalized the data and stabilized the variance to normal parameters. (μ_y, σ_y^2), where $\mu_y = \ln\left(\frac{\mu_x^2}{\sqrt{\mu_x^2+\sigma_x^2}}\right)$ and $\sigma_y^2 = \ln\left(1+\frac{\sigma_x^2}{\mu_x^2}\right)$. For each sample of size n, obtain $K_i = \frac{(\overline{X}_i-\mu_0)}{\sigma_0/\sqrt{n}}$, $L_i = \Phi^{-1}\left[H\left\{\frac{(n-1)S_i^2}{\sigma_0^2},(n-1)\right\}\right]$ and UCL and progressively evaluate equations (1), (2), (3) and (5). Thereafter, plot the values of (3) against the sample numbers and the UCL to declare a sample point IC or out-of-control. Figs. 1 and 2 below are the plots of the Max-EWMA statistic against the sample number when the computation is not reset and when it is reset, respectively.

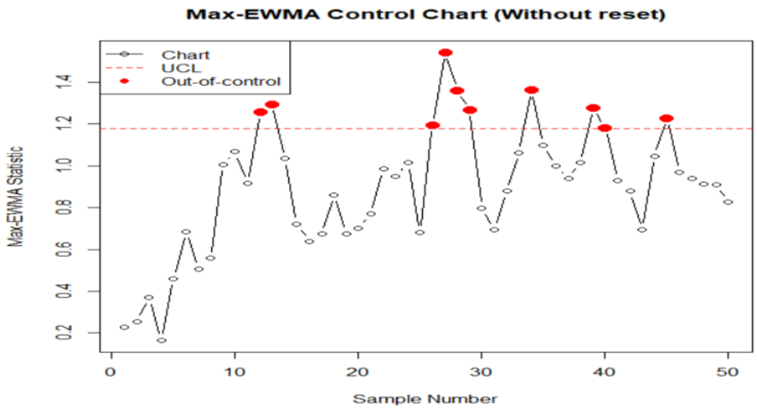

Fig. 1: Plot of the Max-EWMA Statistics Against Sample Number without Reset.

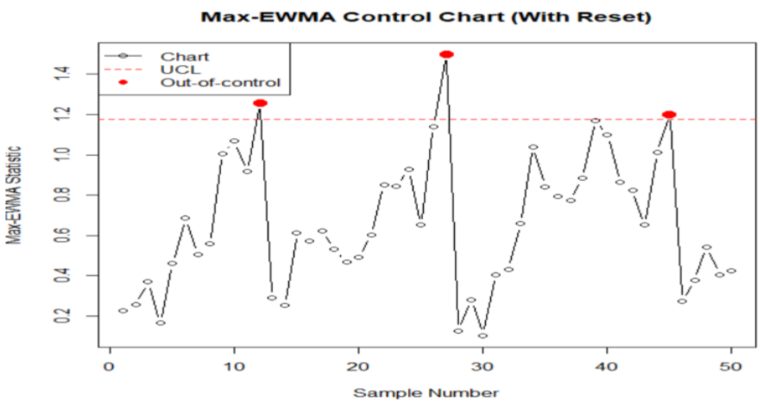

Fig. 2: Plot of the Max-EWMA Statistics Against Sample Number with Reset.

From Fig.1, it was noticed that the process was declared OOC at the twelfth (12) sample point. However, surprisingly, after this point, every other point was noticed to remain OOC. This is referred to as signal persistence, which is common with any EWMA-based control chart. This is because the computational structure of such charts, which the Max-EWMA chart is among, depends on the previous values. Therefore, since the Max-EWMA chart signalled an OOC at the twelfth (12) sample point, it has already declared the value of the Max-EWMA chart large relative to the control limit. The next value, say at $i = 13$ will remain large since its value depends on the already large value at $i = 12$ and it is continued in that order. So, this is the reason why the process shifted and remained OOC as shown in Fig 1.
Due to this behaviour, in practical applications, practitioners usually reset the chart after the first signal to make it begin afresh. What reset does is to separate each OOC condition, and as such, each signal is treated as a new independent event. Fig. 2 is the Max-EWMA chart with reset. The graph also indicates the process OOC at the twelfth (12) sample point, with the subsequent OOC points at 27 and 45, while

every other point remained IC, unlike in Fig 1. The signals at samples 12, 27, and 45 suggest possible changes in the operational settings of the process and resultant possible production of degraded product quality. In real-life applications, such signals call for timely investigation and corrective adjustments to avoid further deterioration in the process quality. Therefore, when the Max-EWMA chart is implemented with a resetting design can identify independent OOC situations, which provides a more realistic monitoring framework for industrial applications.

6. Conclusion

This study presents the re-evaluated performance of the Max-EWMA chart for jointly monitoring the lognormal parameters. Unlike the overestimated work of Almulhim et al. (2024) and incomprehensive result discussion of the results, this study re-evaluated the performance of Almulhim et al. (2024) and provided a more comprehensive discussion of the results. The result confirms the overestimation of the performance of the Max-EWMA chart and provides improved insight into the sensitivity reality of the Max-EWMA chart.

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Appendix

```
n <- 3
lambda <- 0.2
reps <- 30000
alpha0 <- 1.649
gamma0 <- 2.16
mu0 <- log((alpha0^2) / sqrt(alpha0^2 + gamma0^2))
sigma0_sq <- log(1 + (gamma0^2 / alpha0^2))
sigma0 <- sqrt(sigma0_sq)
sigma_shift <- 1.0
sigma_shift_sq <- log(1 + (sigma_shift^2 / alpha0^2))
sigma_shift_norm <- sqrt(sigma_shift_sq)

Var_EWMA <- lambda/(2-lambda)

L <- 3.252
UCL <- (1.128379 + 0.602810 * L) * sqrt(Var_EWMA)
alpha_vals <- c(0.25,0.5,0.75,1,1.25,1.35,1.649,1.75,
 2,2.25,2.5,2.75,3,4,5)

results_alpha <- matrix(NA, length(alpha_vals), 3)

for (a in 1:length(alpha_vals)) {

 mu_shift <- log((alpha_vals[a]^2) /
 sqrt(alpha_vals[a]^2 + gamma0^2))

 RL <- numeric(reps)
```

```
for (r in 1:reps) {

M_prev <- 0
N_prev <- 0
t <- 0

repeat {

t <- t + 1
Y <- rnorm(n, mean = mu_shift, sd = sigma_shift_norm)
K <- (mean(Y) - mu0) / (sigma0 / sqrt(n))

Ls <- qnorm(
pchisq((n - 1) * var(Y) / sigma0_sq, df = n - 1)
)

M_prev <- lambda * K + (1 - lambda) * M_prev
N_prev <- lambda * Ls + (1 - lambda) * N_prev

if (max(abs(M_prev), abs(N_prev)) > UCL) break
}

RL[r] <- t
}

results_alpha[a, ] <- c(mean(RL), sd(RL), median(RL))
}

colnames(results_alpha) <- c("ARL", "SDRL", "MRL")


cbind(alpha_vals, results_alpha)
```