*International Journal of Advanced Statistics and Probability*, 13 (1) (2026) 130-137

International Journal of Advanced Statistics and Probability

*Website: www.sciencepubco.com/index.php/IJASP*
*https://doi.org/10.14419/t09khb71*
***Research paper***

# Forecasting and Latent Heterogeneity in Multi-Departmental Manpower Systems: A Hidden Markov Framework

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*Received: 11-07-2026, Accepted: 16-08-2026, Published: 21-08-2026*

**Abstract**

Recent advances in manpower planning have emphasized the need to model workforce systems within departmentalized frameworks, reflecting the structural reality of most organizations. While departmentalization serves as a form of disaggregation that can address observable heterogeneity, it does not eliminate heterogeneity arising from latent, unobservable factors and may instead introduce additional forms of hidden variability associated with intra-departmental and inter-departmental transitions. This study aims to develop and apply a generalized hidden Markov manpower model for forecasting and analyzing latent heterogeneity in multi-departmental manpower systems. A recursive forecasting structure is derived and applied to a real departmentalized manpower dataset obtained from a university non-academic workforce, under the assumption of stable recruitment and promotion policies. The empirical results reveal a dominance of diagonal elements in the estimated transition probability matrix, indicating slow promotion dynamics across departments and grades. Forecasts over multiple periods show a gradual decline in lower-grade manpower stocks and relative stability in higher-grade categories, with marked differences in promotion propensities across latent subclasses within the same observable categories. These findings confirm the significant role of latent heterogeneity in shaping long-term manpower evolution. In summary, the generalized departmentalized hidden Markov manpower model developed in this study offers a more suitable framework for describing and forecasting manpower systems. By explicitly incorporating both departmental structure and unobservable differences in personnel behaviour, the model produces more reliable forecasts and provides insights that are useful for informed manpower planning and policy decisions.

***Keywords***: *Manpower Planning; Hidden Markov Manpower Model; Latent Heterogeneity; Departmentalized Manpower System; Workforce Forecasting.*

## 1. Introduction

The study of manpower systems using statistical techniques is motivated by the inherent unpredictability of human behaviour and the probabilistic nature of workforce dynamics [1]; [2]. This unpredictability influences personnel transitions and attrition, making it necessary to group individuals with similar transition tendencies to obtain reliable estimates for manpower description, prediction, and control [3]. Workforce heterogeneity is a central challenge in this process, arising from both observable and unobservable (latent) sources [4], with the latter playing a critical but difficult-to-measure role in shaping manpower dynamics [5]. Although traditional manpower models attempt to address heterogeneity by partitioning personnel into homogeneous categories based on observable characteristics such as grade, age, or seniority [6]; [7], empirical evidence shows that latent differences often persist within these categories, undermining the validity of classical models [8], [9]. Hidden Markov manpower models provide a principled framework for capturing such latent heterogeneity by inferring unobservable states from observable personnel movements, thereby improving model realism and enabling more reliable forecasting of future manpower structures [5]; [10].
Latent heterogeneity is evident when individuals within the same nominally homogeneous category exhibit different transition behaviours, such as rapid promotion, moderate progression, stagnation, or early exit, which classical Markov models cannot capture [8]; [9]. To address this, hidden Markov and Markov-switching models have been adopted. Early approaches, like the mover–stayer principle [11], classified individuals into high- and low-transition groups, while [12] extended this to multiple latent subclasses. [8] applied a Markov-switching framework for internal transitions and wastage, [9] further developed a three-state hidden Markov model; movers, mediocres, and stayers, within hierarchical manpower systems. [13] extended it to five hidden states representing: “high movers”, “movers”, “above mediocre”, “mediocre” and “stayers”.

Recent research shows the importance of modelling manpower systems within departmentalized frameworks, as most real-world organizations consist of functionally distinct departments with differing conditions and motivational environments [14 - 18]. Ignoring departmentalization can introduce additional heterogeneity and lead to misleading conclusions, whereas departmentalized models capture intra- and interdepartmental transitions, promotions, and wastage [14]. [19] extended Markov-switching manpower models to incorporate latent subclasses within each department and grade, accommodating both observable and unobservable heterogeneity and enabling optimal selection of hidden states. Hidden Markov models have also demonstrated strong predictive capabilities [10], though primarily for single-department systems. The present study combines these approaches by integrating latent subclass decomposition within a departmentalized framework, extending existing models to allow both inference and accurate forecasting of complex manpower structures. While departmentalization has addressed observable heterogeneity and the interaction with latent heterogeneity see [19], its interaction with forecasting manpower systems in hidden Markov models remains underexplored.

While the present framework builds on Markovian assumptions, latent heterogeneity in workforce transitions has also been addressed through non-Markovian approaches. Survival and proportional hazards models ([20]) capture heterogeneity through individual-specific hazard functions rather than discrete latent states, while semi-Markov models relax the memoryless assumption by allowing state-duration distributions to vary across individuals [3]. More recently, machine learning approaches, including latent class clustering and mixture-of-experts frameworks, have been applied to workforce mobility prediction, offering flexibility in capturing nonlinear transition patterns at the cost of the interpretable transition-probability structure that Markov-based models provide. The hidden Markov framework adopted here is chosen for its ability to yield directly interpretable, department- and grade-specific transition probabilities that connect naturally to manpower planning decisions, while these alternative approaches remain complementary directions for future comparative work.

The table below situates the Departmentalized Hidden Markov Model relative to existing approaches to latent heterogeneity in manpower systems, across three dimensions relevant to applied manpower planning.

**Table 1:** Comparison of Markov-Based Approaches to Latent Heterogeneity in Manpower Systems

| Model | Handling of Unobserved (Latent) Heterogeneity | Departmental Mobility | Computational Complexity |
|---|---|---|---|
| Classical Markov Model | None — assumes members within an observable grade/department are homogeneous. | Captured only if departments are modelled explicitly as separate states. | Low. |
| Mover-Stayer Model ([11]) | Two latent classes (movers, stayers) inferred from transition rates. | Not addressed in the original single-population formulation. | Low to moderate. |
| Standard Hidden Markov Model (e.g., [8]; [9]) | Multiple ordered latent classes (up to 5) per observable state, estimated via EM/Baum-Welch. | Typically formulated for a single department or non-departmentalized system. | Moderate; grows with number of latent states. |
| Departmentalized HMM | Multiple ordered latent classes per grade, jointly with department-specific transition structure. | Modelled explicitly — intra-departmental, inter-departmental, and wastage flows are distinguished. | Higher; parameter count scales with departments × grades × latent classes (see stability discussion below). |

## 2. Model Formulation

Consider a departmentalized manpower system consisting of k observable internal departments $D_1, D_2, \dots, D_k$, and an external environment $D_{k+1}$ representing wastage $D_i$ is further subdivided into u observable hierarchical grades $g_1, g_2, \dots, g_u$. Let $d_{is}$ represent the observable state of an individual in grade s within department i.

The system contains $k \times u$ active internal states and u non-active external states. Let $\{Y_t\}$ be the stochastic process representing the state membership. The transition probabilities between observable states are defined as:

$$P_{ij(sr)} = P\left(Y_{t+1} = d_{jr} \,\middle|\, Y_t = d_{is}\right) \tag{1}$$

Where:

- $j = i$: intra-departmental transition (promotion/demotion with $D_i$).
- $i \neq j$: inter-departmental transition (transfer from $D_i$ to $D_j$).
- $j = k + 1$ and $r = s$: Transition to the external environment (wastage).

Following the assumptions of [8], [9], [19] and [21], no grade transitions occur upon leaving the system ($p_{i,k+1(sr)} = 0$ for $r \neq s$) and all states in $D_{k+1}$ are absorbing ($p_{k+1,k+1(ss)} = 1$).

To account for intra-state heterogeneity, where individuals in the same grade $d_{is}$ exhibit different transition behaviours – we assume each grade is subdivided into m latent classes. Following [13], we define these as $S_1^{is}, S_2^{is}, S_3^{is}, S_4^{is}$, and $S_5^{is}$ , representing “stayers,” “mediocres,” “above mediocres,” “movers,” and “high movers” respectively.

The unobserved state of a member at time t is represented by the Markov chain $\{X_t\}$, the magnitude of an individual’s transition probability is determined by their membership in these latent subcategories.

To account for latent heterogeneity within each observable state $d_{is}$ $(i = 1, \dots, k; s = 1, \dots u)$,
members are classified into $k'$ subclasses governed by a hidden Markov chain $\{X_t^{is}\}$. For the general case where $k' = 5$, the latent states $S_l^{is}(l = 1, \dots, 5)$.

The hierarchy of these latent states is inferred based on the magnitude of the transition probabilities in equation (1).

Let $p_{ijr}^{is}$ be the probability of transiting from the latent state $S_l^{is}(l = 1, \dots, k')$ within the current grade $d_{is}$ to a destination $d_{jr}(j = 1, \dots, k; r = 1, \dots, u)$.

Formally, this emission probability is defined as the conditional probability of the system state membership process $\{Y_t\}$ switching to $Y_{t+1}$ given the current observable and hidden states:

$$p_{ijr}^{is} = P\left(Y_{t+1} = d_{jr} \,\middle|\, X_t^{is} = S_l^{is}\right) = P\left(Y_{t+1} = d_{jr} \,\middle|\, Y_t = d_{is}, X_t^{is} = S_l^{is}\right) \tag{2}$$

This probability dictates how the manpower system moves to the next appropriate distribution depending on the current states of the hidden Markov chain.

The probabilities $p_{ijr}^{is}$ are assembled into the state-specific emission probability matrix, denoted as $P^{is}$. This is a $(k' \times (uk+1))$ matrix where the rows correspond to the ordered latent classes and the columns correspond to all possible observable destination states (including wastage to $D_{k+1}$).

$$P^{is} = \begin{bmatrix} p_{1,11}^{is} & p_{1,12}^{is} & \cdots & p_{1,ku}^{is} & p_{1,k+1}^{is} \\ p_{2,11}^{is} & p_{2,12}^{is} & \cdots & p_{2,ku}^{is} & p_{2,k+1}^{is} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ p_{k',11}^{is} & p_{k',12}^{is} & \cdots & p_{k',ku}^{is} & p_{k',k+1}^{is} \end{bmatrix} \quad (3)$$

The complete model is defined by a set of parameters. $\lambda = \{A, B, \pi\}$ for each grade as follows:

- The hidden transition matrix (A): The probability of a member moving between latent classes within $d_{is}$ is given by the $5 \times 5$ matrix $A^{is} = \{a_{lm}^{is}\}$:

$$a_{lm}^{is} = P\left(X_t = S_m^{is} \middle| X_{t-1} = S_l^{is}\right) \quad (4)$$

Where $l, m \in \{1,2,3,4,5\}$

- The emission (state-to-grade) probability matrix (B): The probability that a member in latent class l of grade $d_{is}$ moves to an observable grade $d_{is}$ in the next period is given by:

$$b_{ljr}^{is} = P\left(Y_{t+1} = d_{jr} \middle| X_t = S_l^{is}\right) \quad (5)$$

This matrix B captures the “propensity to move” inherent in the latent classes (e.g., movers having a higher. $b_{ijr}$ for $r > s$ than stayers).

- Initial state probability distribution ($\pi$): The probability that a member belongs to a specific latent class at the start of the observation is:

$$\pi_l^{is} = P\left(X_0 = S_l^{is}\right) \quad (6)$$

Let $n_{ij(sr)}(t)$ be the observed number of members in state $d_{is}$ at time t who move to state $d_{jr}$ at $t+1$. The total manpower stock in state $d_{is}$ at time t, denoted as $N_{i(s)}(t)$, is the sum of all members who remain, move to other grades/departments, or leave the system:

$$N_{i(s)}(t) = \sum_{j=1}^{k+1} \sum_{r=1}^{u} n_{ij(sr)}(t) \quad (7)$$

As established by [9], the likelihood of these observed flows follows a multinomial distribution. The maximum likelihood estimator (MLE) often referred to as the expectation-maximization algorithm for the transition probabilities assuming homogeneity, would be:

$$\hat{p}_{ij(sr)} = \frac{\sum_{i=1}^{T} n_{ij(sr)}(t)}{\sum_{i=1}^{T} N_{i(s)}(t)} \quad (8)$$

However, in this generalized framework, these probabilities are decomposed into the latent components defined in equation (3) to provide a more granular forecast of the personnel structure.

## 3. Departmentalized Parameter Estimation and Forecast Structure

In a multi-departmental system, the total personnel stock vector $\underline{n}(t)$ is a concatenated vector of all k departments. Each departmental vector is further composed of u grades, and each grade is decomposed into $k' = 5$ latent classes:

$$\underline{n}(t) = \left[\underline{n}_1(t), \underline{n}_2(t), \ldots, \underline{n}_k(t)\right] \quad (9)$$

Where for each department i, the grade-specific latent sub-vector is:

$$\underline{n}_{is}^{(l)}(t) = \left(\underline{n}_{is}^{(1)}(t), \underline{n}_{is}^{(2)}(t), \underline{n}_{is}^{(3)}(t), \underline{n}_{is}^{(4)}(t), \underline{n}_{is}^{(5)}(t)\right) \quad (10)$$

Using the concept from [10], the distribution of personnel into these latent classes for grade s in department i is calculated using the grade-specific hidden transition matrix $A^{is}$:

$$\underline{n}_{is}^{(l)}(t) = \left[\pi^{is}\left(A^{is}\right)^t\right] n_{is}(t) \quad (11)$$

Where:

- $\underline{n}_{is}^{(l)}(t)$ is a $1 \times 5$ latent personnel stock sub-vector representing the number of personnel in each of the five latent classes for grade s in department i at time t. It is shown in equation (10), where each element corresponds to a latent class.

- $\pi^{is}$ is the initial latent state distribution, which is a $1 \times 5$ probability vector representing the starting distribution of personnel across the latent classes within that specific grade and department at the beginning of the observation period ($t = 0$). It is expressed as $\pi^{is} = (\pi_1^{is}, \pi_2^{is}, \pi_3^{is}, \pi_4^{is}, \pi_5^{is})$. Each $\pi_l^{is}$ is the probability that a member belongs to latent class l at the initial time.
- $A^{is}$ is the hidden transition matrix; it is a $5 \times 5$ stochastic matrix representing the probabilities of members moving between latent classes within the same grade s and department i. Its structure is given as:

$$A^{is} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix} \tag{12}$$

Each entry $a_{lm}^{is}$ is the probability that a person in latent class l at time t will switch to latent class m by time $t + 1$.

- $(A^{is})^t$ represents the t-step hidden transition probabilities over t sessions. It determines how the initial distribution $\pi^{is}$ evolves into the current distribution at time t. The product $\{ \pi^{is}(A^{is})^t]$ yields the probability distribution vector $\underline{\psi}_{is}(t)$ which tells the percentage of the grade currently in the 5 latent subgroups.
- $n_{is}(t)$ is a scalar value representing the total number of personnel physically present in grade s of department i at time t.

The matrix in equation (3) is decomposed into two sub-stochastic matrices. We define these as the latent internal mobility matrix ($P_{int}^{is}$) and latent attrition vector ($w^{is}$).

$$P_{int}^{is} = \begin{bmatrix} p_{1,11}^{is} & p_{1,12}^{is} & \cdots & p_{1,ku}^{is} \\ p_{2,11}^{is} & p_{2,12}^{is} & \cdots & p_{2,ku}^{is} \\ \vdots & \vdots & \ddots & \vdots \\ p_{k',11}^{is} & p_{k',12}^{is} & \cdots & p_{k',ku}^{is} \end{bmatrix} \tag{13}$$

$$w^{is} = \begin{bmatrix} p_{1,k+1}^{is} \\ p_{2,k+1}^{is} \\ \vdots \\ p_{k',k+1}^{is} \end{bmatrix} = \begin{bmatrix} p_{10}^{is} \\ p_{20}^{is} \\ \vdots \\ \vdots \\ p_{k',0}^{is} \end{bmatrix} \tag{14}$$

Both matrices in equations (13) and (14) are sub-stochastic because they account for only a portion of the total transitions.
Note that $P^{is}$ in equation (3) is a pooled matrix of the emission probabilities for the k departments and u grades, $w^{is}$ is a pooled vector of probability of wastages from $d_{is}$ to the external environment $D_{k+1}$ with 0 conditional on the latent subclasses, and $P_{int}^{is}$ is a pooled submatrix of probabilities of internal transition of the members within the system. It reflects rules associated with transition (intra-departmental promotion or inter-departmental transfer) of members between observable states. $d_{jr}$ ($j = 1, \dots, k; r = 1, \dots u$) and latent subclasses $S_l^{is}$ ($l = 1, \dots, k'$) of the manpower system, denoted by $p_{ijr}^{is}$ in equation (2).
Moreover, $p_0$ which denotes the conditional probability of allocating the newly recruited members, $R(t)$, to state $d_{jr}$, as estimated by the EM re-estimation algorithm (see [9]; [13]) is given as:

$$p_0 = \begin{bmatrix} b_{0(1,1)}^{(1)} & \cdots & b_{0(1,u)}^{(1)} & \cdots & b_{0(k,1)}^{(1)} & \cdots & b_{0(k,u)}^{(1)} \\ b_{0(1,1)}^{(2)} & \cdots & b_{0(1,u)}^{(2)} & \cdots & b_{0(k,1)}^{(2)} & \cdots & b_{0(k,u)}^{(2)} \\ b_{0(1,1)}^{(3)} & \cdots & b_{0(1,u)}^{(3)} & \cdots & b_{0(k,1)}^{(3)} & \cdots & b_{0(k,u)}^{(3)} \\ b_{0(1,1)}^{(4)} & \cdots & b_{0(1,u)}^{(4)} & \cdots & b_{0(k,1)}^{(4)} & \cdots & b_{0(k,u)}^{(4)} \\ b_{0(1,1)}^{(5)} & \cdots & b_{0(1,u)}^{(5)} & \cdots & b_{0(k,1)}^{(5)} & \cdots & b_{0(k,u)}^{(5)} \end{bmatrix} \tag{15}$$

According to [1], the basic equation for a manpower model is given by:

$$\underline{n}(t+1) = \underline{n}(t)P + R(t+1)r \tag{16}$$

This equation establishes that the structure at time. $t + 1$ is derived from the survivors of the previous stock plus the new recruits.
This study adopts the modified form by [22] and [10], expressed as $\underline{n}(t+1) = \underline{n}(t)Q$, where Q is a composite matrix accounting for both transitions and replacement. In this generalized framework with k departments, u grades, and $k' = 5$ latent classes, the forecast structure is expressed in terms of the 5uk dimensional latent sub-vector $\underline{n}^{(l)}(t)$ and the $5uk \times uk$ transition-replacement matrix Q.
The forecast equation is given as:

$$\underline{n}(t+1) = \underline{n}^{(l)}(t)Q \tag{17}$$

Where:

- $\underline{n}(t+1)$ is the predicted observable stock vector over the uk grades of the system at time $t + 1$.

$$\underline{n}(t+1) = [n_{11}(t+1), \dots, n_{ku}(t+1)] \tag{18}$$

$\underline{n}^{(l)}(t)$ is the latent stock vector derived from the decomposition in equation (10).
The matrix $Q$ serves as the operator that maps the unobservable latent states at time $t$ to the observable states at time $t+1$. It is defined as the sum of the latent internal mobility matrix in equation (13) and the recruitment matrix in equation (15).

$$Q = P_{int}^{is} + (p_0 \times w^{is}) \tag{19}$$

Given the forecasted structure for $\underline{n}(t+1)$ in equation (17), the prediction is performed iteratively. Since the latent composition of the workforce evolves, the observable stock forecasted for $t+1$ must be re-decomposed before predicting $t+2$.
The procedure is as follows:

1) Compute the observable stock $\underline{n}(t+1)$ using the latent vector $\underline{n}^{(l)}(t)$ and matrix $Q$
2) For the next step, decompose the new stock $\underline{n}(t+1)$ into its latent components $\underline{n}^{(l)}(t+1)$ using the updated hidden transition probabilities:

$$\underline{n}_{is}^{(l)}(t+1) = \{\pi^{is}(A^{is})^{t-1}]n_{is}(t) \tag{20}$$

3) Use $\underline{n}^{(l)}(t+1)$ to predict $\underline{n}(t+2)$, and so on.

## 4. Illustration

To demonstrate the practical application of the generalized hidden Markov framework, the study utilizes a real-world dataset analyzed by [19]. The data is derived from the non-academic manpower system of a university, which is structurally divided into two distinct functional departments: the academic department (AD) and the non-academic department (NAD).
Within this departmentalized structure, personnel are stratified into two observable hierarchical grades: junior staff (JS) and senior staff (SS). The manpower flows are recorded over four consecutive time periods. $(t = 1,2,3,4)$.
To account for latent heterogeneity, the extended Markov-switching manpower model with $k' = 5$ hidden states was fitted to the data. This model assumes that within each observable grade, personnel fall into one of five latent categories: high movers, movers, above mediocre, mediocres, and stayers.
The estimation of the model parameters, specifically the transition probability matrix in equation (3) and the recruitment matrix in equation (15), was conducted using the depmixS4 package in the R programming environment. This package utilizes the expectation-maximization (EM) algorithm to iteratively optimize the parameters of the hidden Markov models.
$t = 4$ $(k = 2)$ $(u = 2)k' = 5$ We therefore report standard errors for the estimated transition probabilities in $P_{EMSMM5}$ obtained from the Hessian of the fitted depmixS4 model. For example, the emission probabilities for states within the academic department were estimated as 0.883 (SE = 0.017), 0.044 (SE = 0.011), 0.029 (SE = 0.009), 0.006(SE = 0.004) and 0.038(SE = 0.010) for State 1, and 0.906(SE = 0.011), 0.022(SE = 0.006), 0.045(SE = 0.008), 0.007(SE = 0.003), and 0.019(SE = 0.005) for State 2. By contrast, a substantial number of the hidden state transition probabilities were estimated exactly at the boundary of the parameter space, for which the standard error obtained from the Hessian is undefined. Two further parameters, both estimated at exactly 0.5 and therefore not at a boundary, also returned undefined standard errors, indicating a local problem of identifiability in the current parameterization. Taken together, these results confirm that several latent state transitions in $k' = 5$ are effectively fixed by limited data rather than freely and stably estimated, consistent with the constrained sample size relative to the number of free parameters. Following the model-comparison approach of [19], $k' = 5$ $(k' = 1)$ $k' = 5$ The comparison between HMMM1 ($k' = 1$, the classical non-hidden departmentalized Markov model) and HMMM5 ($k' = 5$) yields $L_r = 180.2416$ with 160 degrees of freedom, which exceeds the critical value $\chi^2_{0.05}(160) = 124.3$. This indicates that HMMM5 fits the data significantly better than the more parsimonious HMMM1, supporting the use of the five-state specification for this illustration. $k' = 3k' = 4$From equation (3), when $k' = 5$ , the estimated transition probability matrix as obtained by [19] is presented below. This matrix captures the specific probabilities of movement for each latent class across the departments and grades.

$$P_{EMSMM5} = \begin{array}{cc} & \begin{array}{cccccc} d_{11} & d_{12} & d_{21} & d_{22} & d_{31} & d_{32} \end{array} \\ \begin{array}{c} \\ d_{11} \\ \\ \\ \\ \\ d_{12} \\ \\ \\ \\ \\ \\ d_{21} \\ \\ \\ \\ \\ d_{22} \\ \\ \\ \end{array} & \begin{bmatrix} 0.883 & 0.044 & 0.029 & 0.006 & 0.038 & 0 \\ 0.906 & 0.022 & 0.045 & 0.007 & 0.019 & 0 \\ 0.945 & 0.012 & 0.008 & 0.027 & 0.008 & 0 \\ 0.909 & 0.027 & 0.030 & 0.024 & 0.009 & 0 \\ 0.883 & 0.044 & 0.029 & 0.006 & 0.038 & 0 \\ 0 & 0.924 & 0 & 0.023 & 0 & 0.052 \\ 0 & 0.921 & 0 & 0.032 & 0 & 0.047 \\ 0 & 0.883 & 0 & 0.077 & 0 & 0.039 \\ 0 & 0.922 & 0 & 0.046 & 0 & 0.032 \\ 0 & 0.924 & 0 & 0.023 & 0 & 0.052 \\ 0.012 & 0.012 & 0.945 & 0.009 & 0.023 & 0 \\ 0.008 & 0.009 & 0.936 & 0.019 & 0.029 & 0 \\ 0.022 & 0.006 & 0.927 & 0.012 & 0.033 & 0 \\ 0.034 & 0.014 & 0.908 & 0.023 & 0.021 & 0 \\ 0.012 & 0.012 & 0.945 & 0.009 & 0.023 & 0 \\ 0 & 0.025 & 0 & 0.832 & 0 & 0.143 \\ 0 & 0.039 & 0 & 0.843 & 0 & 0.118 \\ 0 & 0.029 & 0 & 0.854 & 0 & 0.117 \\ 0 & 0.042 & 0 & 0.856 & 0 & 0.102 \\ 0 & 0.025 & 0 & 0.832 & 0 & 0.143 \end{bmatrix} \end{array}$$

**Table 2:** Recruitment into the System

| | AD (j = 1) | | NAD (j = 2) | |
|---|---|---|---|---|
| Time Period (t) | JS ($d_{11}$) | SS ($d_{12}$) | JS ($d_{21}$) | SS ($d_{22}$) |
| t = 1 | 68 | 14 | 162 | 39 |
| t = 2 | 74 | 11 | 171 | 42 |
| t = 3 | 61 | 17 | 158 | 36 |
| t = 4 | 79 | 13 | 176 | 41 |

From equation (15), the recruitment matrix from the data provided in Table 2 above is estimated using the depmixS4 package based on the EM algorithm re-estimation procedures, as given below.

$$p_0 = \begin{bmatrix} 0.256 & 0.042 & 0.570 & 0.133 \\ 0.248 & 0.037 & 0.574 & 0.141 \\ 0.224 & 0.062 & 0.581 & 0.132 \\ 0.240 & 0.049 & 0.572 & 0.138 \\ 0.256 & 0.042 & 0.570 & 0.133 \end{bmatrix}$$

By decomposing the matrix $P_{EMSMM5}$ matrix into two sub-stochastic components: the latent mobility matrix and the latent attrition vector shown in equations (13) and (14) respectively. We have

$$w^{is} = \begin{bmatrix} 0.038 & 0 \\ 0.019 & 0 \\ 0.008 & 0 \\ 0.009 & 0 \\ 0.038 & 0 \\ 0 & 0.052 \\ 0 & 0.047 \\ 0 & 0.039 \\ 0 & 0.032 \\ 0 & 0.052 \\ 0.023 & 0 \\ 0.029 & 0 \\ 0.033 & 0 \\ 0.021 & 0 \\ 0.023 & 0 \\ 0 & 0.143 \\ 0 & 0.118 \\ 0 & 0.117 \\ 0 & 0.102 \\ 0 & 0.143 \end{bmatrix}$$

$$P_{int}^{is} = \begin{bmatrix} 0.883 & 0.044 & 0.029 & 0.006 \\ 0.906 & 0.022 & 0.045 & 0.007 \\ 0.945 & 0.012 & 0.008 & 0.027 \\ 0.909 & 0.027 & 0.030 & 0.024 \\ 0.883 & 0.044 & 0.029 & 0.006 \\ 0 & 0.924 & 0 & 0.023 \\ 0 & 0.921 & 0 & 0.032 \\ 0 & 0.883 & 0 & 0.077 \\ 0 & 0.922 & 0 & 0.046 \\ 0 & 0.924 & 0 & 0.023 \\ 0.012 & 0.012 & 0.945 & 0.009 \\ 0.008 & 0.009 & 0.936 & 0.019 \\ 0.022 & 0.006 & 0.927 & 0.012 \\ 0.034 & 0.014 & 0.908 & 0.023 \\ 0.012 & 0.012 & 0.945 & 0.009 \\ 0 & 0.025 & 0 & 0.832 \\ 0 & 0.039 & 0 & 0.843 \\ 0 & 0.029 & 0 & 0.854 \\ 0 & 0.042 & 0 & 0.856 \\ 0 & 0.025 & 0 & 0.832 \end{bmatrix}$$

From equation (19), the matrix Q is calculated and represented in the form:

$$Q = \begin{bmatrix} 0.89273 & 0.04560 & 0.05066 & 0.01105 \\ 0.91071 & 0.02270 & 0.05591 & 0.00968 \\ 0.94679 & 0.01250 & 0.01265 & 0.02806 \\ 0.91116 & 0.02744 & 0.03515 & 0.02524 \\ 0.89273 & 0.04560 & 0.05066 & 0.01105 \\ 0.01331 & 0.92618 & 0.02964 & 0.02992 \\ 0.01166 & 0.92274 & 0.02698 & 0.03863 \\ 0.00874 & 0.88542 & 0.02266 & 0.08215 \\ 0.00768 & 0.92357 & 0.01830 & 0.05042 \\ 0.01331 & 0.92618 & 0.02964 & 0.02992 \\ 0.01789 & 0.01297 & 0.95811 & 0.01206 \\ 0.01519 & 0.00907 & 0.95265 & 0.02309 \\ 0.02939 & 0.00805 & 0.94617 & 0.01636 \\ 0.03904 & 0.01503 & 0.92001 & 0.02590 \\ 0.01789 & 0.01297 & 0.95811 & 0.01206 \\ 0.03661 & 0.03102 & 0.08151 & 0.85102 \\ 0.02926 & 0.04337 & 0.06773 & 0.85964 \\ 0.02621 & 0.03625 & 0.06798 & 0.86944 \\ 0.02448 & 0.04700 & 0.05834 & 0.87008 \\ 0.03661 & 0.03102 & 0.08151 & 0.85102 \end{bmatrix}$$

Using the relationships established in equation (20), the stock sub-vector corresponding to 20 (5uk) Staff subcategories are first obtained for each time period. t. On this basis, the manpower structure $n_{is}(t)$ for every departmental-grade category $d_{is}$ is then forecast recursively for the subsequent four periods ($t = 5,6,7,8$by applying the forecasting equation given in (17). This projection is carried out under the assumption that the existing promotion and recruitment policies remain unchanged over the forecasting horizon. The resulting forecasted staff distributions are presented in Table 3.

**Table 3:** Forecasted Departmental Manpower Structure for Periods $(t = 5 - 8)$

| | AD | | NAD | |
|---|---|---|---|---|
| Time Period (t) | JS | SS | JS | SS |
| $t = 5$ | 677 | 695 | 1794 | 1029 |
| $t = 6$ | 737 | 670 | 1806 | 981 |
| $t = 7$ | 724 | 713 | 1845 | 903 |
| $t = 8$ | 829 | 726 | 1845 | 878 |

## 5. Discussion

The estimated emission (transition) probability matrix $P_{EMSMM5}$ exhibits a clear structural pattern in which the dominant probability mass is concentrated along the main diagonal. At the same time t, the upper off-diagonal elements corresponding to upward movements across grades and departments remain relatively small. This pattern indicates that, on average, only a small proportion of personnel experience promotion or upward mobility within each time period, while the majority tend to remain in their current departmental–grade categories. A similar dominance of diagonal elements has been reported by [10], which they interpreted as evidence of slow progression dynamics in manpower systems characterized by latent heterogeneity. In the present study, this behaviour may be attributed to stringent promotion requirements, limited availability of higher-grade positions, or differential engagement of personnel in activities that facilitate advancement, such as research output, professional development, or administrative responsibilities. It may also reflect institutional policies that prioritise workforce stability over rapid promotion.

The forecasting results in Table 3 further reinforce these observations. The predicted manpower structure shows a persistent decline in personnel across lower-grade categories over the forecast horizon, accompanied by stagnation or marginal changes in the highest-grade categories. This pattern aligns with [10], who noted that when promotion probabilities are low and attrition is non-negligible, lower grades tend to experience gradual depletion over time. The relative stability observed in the top-grade categories suggests limited upward inflow into these grades, implying that vacancies created through retirement or wastage may not be fully offset by internal promotions. Within the departmentalized framework adopted in this study, these dynamics also reflect the interaction between intra-departmental promotion structures and inter-departmental mobility constraints, which jointly shape long-run manpower evolution.

## 6. Conclusion

Manpower systems in real organisations are inherently heterogeneous, comprising personnel with diverse behavioural characteristics that make effective manpower planning and forecasting challenging. Classical Markov manpower models attempt to address this challenge by partitioning aggregated personnel into categories that are homogeneous with respect to observable characteristics such as department and grade. However, such models fail to account for heterogeneity arising from latent, unobservable factors, thereby rendering these "homogeneous" categories internally heterogeneous and potentially leading to biased or unreliable forecasts.

This study developed and applied a generalized hidden Markov manpower modelling framework within a departmentalized system to address both observable and latent sources of heterogeneity simultaneously. By extending the Markov-switching and hidden Markov manpower modelling approaches to a multi-departmental setting, the proposed EMSMM5 framework decomposes each departmental–grade category into five latent subclasses ($k' = 5$) —high movers, movers, above mediocres, mediocres, and stayers—each characterized by distinct transition propensities. Model parameters, including transition and recruitment probabilities, were estimated using the expectation–maximization algorithm, exploiting observed personnel flows and recruitment data.

The forecasting structure derived from the estimated transition probability matrix $P_{EMSMM5}$ enabled iterative prediction of future manpower structures under the assumption of constant promotion and recruitment policies. The resulting forecasts revealed a gradual decline in lower-grade manpower stocks and relative stability in higher-grade categories, reflecting slow promotion dynamics and limited upward mobility.

These findings are consistent with earlier results reported by [10], and further demonstrate how latent heterogeneity plays a decisive role in shaping long-run manpower evolution, even within departmentalized systems.
From a practical standpoint, the forecasting results point to at least three concrete policy levers for institutional administrators. First, where the diagonal dominance of $P_{EMSMM5}$ reflects artificially constrained promotion slots rather than genuine performance differences, administrators could adopt a dynamic recruitment adjustment policy that recalibrates recruitment targets at the grade and department level in response to forecasted stock depletion, which could offset the projected decline in lower grade categories. Second, since the latent decomposition identifies subgroups of personnel with distinct mobility propensities, targeted promotion interventions aimed at the “above mediocre” and “mediocre” latent classes, rather than uniform promotion criteria applied across an entire grade, could raise upward mobility among personnel who are structurally similar to movers but currently transition slowly. Third, periodic updating of the hidden transition matrix as new power flow data become available would allow administrators to detect shifts in the latent composition of the workforce early, supporting proactive rather than reactive adjustment of promotion and recruitment policy. Future research may extend this framework to include optimal control of recruitment and promotion policies, alternative latent state selection criteria, or time-varying transition structures to further improve manpower planning decisions.

## References


[1] Bartholomew DJ, Forbes AF & McClean SI (1991), Statistical techniques for manpower planning (2nd ed.). Wiley.
[2] Dimitriou VA & Tsantas N (2009), Prospective control in an enhanced manpower planning model. Applied Mathematics and Computation 215(3). https://doi.org/10.1016/j.amc.2009.06.027.
[3] Carette P & Guerry M-A (2022), Markov models for duration-dependent transitions: selecting the states using duration values or duration intervals. Statistical Methods & Applications 31(5), 1203–1223. https://doi.org/10.1007/s10260-022-00637-2.
[4] Ugwuowo FI & McClean SI (2000), Modelling heterogeneity in a manpower system: a review. Applied Stochastic Models in Business and Industry 16(2), 99–110. https://doi.org/10.1002/1526-4025(200004/06)16:2<99::AID-ASMB385>3.0.CO;2-3.
[5] Rabiner LR (1989), A tutorial on hidden Markov models and selected applications in speech recognition. Proceedings of the IEEE 77(2), 257–286. https://doi.org/10.1109/5.18626.
[6] Bartholomew DJ, Hopes RFA & Smith AR (1976), Manpower planning in the face of uncertainty. Personnel Review 5(3), 5–17. https://doi.org/10.1108/eb055311.
[7] De Feyter T (2006), Modelling heterogeneity in manpower planning: dividing the personnel system into more homogeneous subgroups. Applied Stochastic Models in Business and Industry 22(4), 321–334. https://doi.org/10.1002/asmb.619.
[8] Guerry MA (2011), Hidden heterogeneity in manpower systems: a Markov-switching model approach. European Journal of Operational Research 210(1), 106–113. https://doi.org/10.1016/j.ejor.2010.10.039.
[9] Udom AU & Ebedoro UG (2021), On multinomial hidden Markov model for hierarchical manpower systems. Communications in Statistics—Theory and Methods 50(6), 1370–1386. https://doi.org/10.1080/03610926.2019.1650185.
[10] Udom AU, Ebedoro UG & Udoh NS (2019), Structural configuration in a multinomial hierarchical manpower system using hidden Markov model. Taru Journal of Organizational Behaviour & Analytics I(2-4),111-128 https://doi.org/10.47974/2019.TJOBA.014.
[11] Blumen I, Kogan M & McCarthy PJ (1955), The industrial mobility of labour as a probability process. Cornell University Press.
[12] Spilerman S (1972), Extensions of the mover–stayer model. American Journal of Sociology 78(3), 599–626. https://doi.org/10.1086/225366.
[13] Ossai EO, Ezra PN, Ohanuba FO & Eze MN (2022), Homogeneity versus parsimony in Markov manpower models: a hidden Markov chain approach. Asian Journal of Probability and Statistics 20(4), 82–93. https://doi.org/10.9734/ajpas/2022/v20i4441.
[14] Ossai EO & Uche PI (2009), Maintainability of departmentalized manpower structures in Markov chain model. The Pacific Journal of Science and Technology 10(2), 295–302.
[15] Guerry MA & De Feyter T (2012), Optimal recruitment strategies in a multi-level manpower planning model. Journal of the Operational Research Society 63(7), 931–940. https://doi.org/10.1057/jors.2011.99.
[16] Dimitriou VA, Georgiou AC & Tsantas N (2013), The multivariate non-homogeneous Markov manpower system in a departmental mobility framework. European Journal of Operational Research 228(1), 112–121. https://doi.org/10.1016/j.ejor.2012.12.014.
[17] Dimitriou VA, Georgiou AC & Tsantas N (2015), On the equilibrium personnel structure in the presence of vertical and horizontal mobility via multivariate Markov chains. Journal of the Operational Research Society 66(6), 993–1006. https://doi.org/10.1057/jors.2014.66.
[18] Dimitriou VA & Georgiou AC (2021), Introduction, analysis and asymptotic behavior of a multilevel manpower planning model in a continuous time setting under potential department contraction. Communications in Statistics—Theory and Methods 50(5). https://doi.org/10.1080/03610926.2019.1648827.
[19] Ossai EO, Nduka UC, Madukaife MS, Udom AU & Ugwu SO (2024), An extended Markov-switching model approach to latent heterogeneity in departmentalized manpower systems. Communications in Statistics—Theory and Methods 53(19), 6957–6976. https://doi.org/10.1080/03610926.2023.2255322.
[20] Cox DR (1972), Regression models and life-tables. Journal of the Royal Statistical Society: Series B (Methodological) 34(2), 187–202. https://doi.org/10.1111/j.2517-6161.1972.tb00899.x.
[21] Ukaogo OG, Ossai EO, Nduka UC, Ugah TE, Ugwu SO & Nwakobi NM (2026), Selection of optimum number of states for a hidden Markov manpower model in a departmentalized framework. Asian Journal of Probability and Statistics 28(5), 105–122. https://doi.org/10.9734/ajpas/2026/v28i5898.
[22] Ogbogbo GO, Ebuh GU & Aronu CO (2013), Prediction of academic manpower system of a Polytechnic institution in Nigeria. Science Journal of Applied Mathematics and Statistics 1(5), 54–61. https://doi.org/10.11648/j.sjams.20130105.14.