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Research paper

Adaptive Dynamic Conditional Correlation Model for Financial Markets

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Abstract

This paper advances multivariate time series analysis by introducing the AD-DCC model, which ensures stationarity and positive definiteness through time-varying parameters responsive to asymmetric shocks, ensuring a valid correlation matrix. The model employs a two-step estimation process, modeling asset volatilities with univariate GARCH models followed by adaptive correlation dynamics responsive to market conditions. It is applied to daily returns of equities, bonds, commodities, and currencies from 2010 to 2024, capturing complex time-varying correlation patterns. Empirical results reveals that the AD-DCC model outperforms benchmark models, particularly in volatile markets, enhancing portfolio optimization and risk forecasting accuracy.

Keywords: *Adaptive DCC, Dynamic Correlations, GARCH Model, Risk Forecasting, Time-Varying Parameters, Asymmetric Shocks*

1. Introduction

The adaptive dynamic conditional correlation model (AD-DCC) is an econometric tool that extends the DCC model by using time-varying parameters to capture asymmetric asset correlations[1, 6, 15]. Its adaptive mechanisms ensure precise modeling of dynamic relationships vital for financial applications[5, 9, 2].Studies have progressed from static correlation models to dynamic approaches such as multivariate GARCH, with the DCC model as a benchmark, though it struggles with abrupt changes [17, 21]. A-DCC addresses these limitations by modeling asymmetries and rapid changes, yet its multi-asset applications are under-utilized [9, 19]. This study evaluates the ability of AD-DCC model's of correlations across equities, bonds, commodities, and currencies from 2010 to 2024, comparing it to benchmarks[18, 20]. It aims to demonstrate its value for portfolio optimization and risk management[6].

2. Materials and Method

2.1. Adaptive Dynamic Conditional Correlation (AD-DCC) Model

The AD-DCC extends standard DCC[13] by incorporating adaptive parameters for asymmetry, outperforming it in volatile data (e.g likelihood ratio p less than 0.01). Compared to asymmetric DCC[8], it adds a volatility-driven parameter for a faster shock response[15]. Compared to score-driven models such as GAS-DCC [10], AD-DCC offers a simpler estimation with comparable flexibility, as evidenced by lower AIC/BIC (Table 2), making it more accessible for high-dimensional applications [1, 11, 9, 6, 21].

2.2. Mathematical Formulation

Let $\mathbf{r}_t \equiv (r_{1t}, \ldots, r_{nt})'$ denote the vector of asset returns at time t, with conditional covariance matrix $\mathbf{H}_t \equiv \mathbf{D}_t\mathbf{R}_t\mathbf{D}_t$, where $\mathbf{D}_t \equiv \operatorname{diag}(\sigma_{1t}, \ldots, \sigma_{nt})$ contains standard deviations from univariate GARCH models, and $\mathbf{R}_t$ is the conditional correlation matrix. The AD-DCC model specifies the correlation dynamics as

$$\mathbf{Q}_t = (1-\alpha_t-\beta_t)\,\mathbf{Q} + \alpha_t\mathbf{u}_{t-1}\mathbf{u}'_{t-1} + \beta_t\mathbf{Q}_{t-1}, \tag{2.1}$$

where $\mathbf{u}_t = \mathbf{D}_t^{-1}\mathbf{r}_t$ are standardized residuals, $\mathbf{Q}$ is the unconditional covariance matrix of $\mathbf{u}_t$, and α_t, β_t are time-varying parameters defined as:

$$\alpha_t = \alpha_0 + \alpha_1 I(\mathbf{u}_{t-1} < 0) + \alpha_2 g\left(\sigma_{t-1}^2\right), \quad \beta_t = \beta_0 + \beta_1 I(\mathbf{u}_{t-1} < 0) + \beta_2 g\left(\sigma_{t-1}^2\right). \tag{2.2}$$

Here, $I(\mathbf{u}_{t-1} < 0)$ indicates negative shocks and $g\left(\sigma_{t-1}^2\right)$ is a function of the volatility of the past. The correlation matrix is as follows:

$$\mathbf{R}_t = \operatorname{diag}(\mathbf{Q}_t)^{-1/2}\mathbf{Q}_t \operatorname{diag}(\mathbf{Q}_t)^{-1/2}. \tag{2.3}$$

2.3. Proof of Validity

To ensure $\mathbf{R}_t$ is a valid correlation matrix (symmetric, positive definite, with ones on the diagonal), we prove that $\mathbf{Q}_t$ is symmetric and positive semi-definite.

Symmetry: Assume that $\mathbf{Q}$ and $\mathbf{Q}_{t-1}$ are symmetric. For $\mathbf{u}_{t-1}\mathbf{u}_{t-1}'$, since $\mathbf{u}_{t-1}'\mathbf{u}_{t-1} = \mathbf{u}_{t-1}\mathbf{u}_{t-1}'$, it is symmetric. In (1), each term is symmetric: $(1-\alpha_t-\beta_t)\mathbf{Q}$ and $\beta_t\mathbf{Q}_{t-1}$ inherit symmetry, and $\alpha_t\mathbf{u}_{t-1}\mathbf{u}_{t-1}'$ is symmetric. Thus, $\mathbf{Q}_t$ is symmetric.

Positive Semi-Definiteness: Assume that $\mathbf{Q}$ and $\mathbf{Q}_{t-1}$ are positive semi-definite, and $\alpha_t, \beta_t \geq 0, 1-\alpha_t-\beta_t \geq 0$. For any vector $\mathbf{x}$, consider $\mathbf{x}^t\mathbf{Q}_t\mathbf{x}$:

- $(1-\alpha_t-\beta_t)\mathbf{x}'\mathbf{Q}\mathbf{x} \geq 0$, as $\mathbf{Q}$ is positive semi-definite.
- $\alpha_t\mathbf{x}'\left(\mathbf{u}_{t-1}\mathbf{u}_{t-1}'\right)\mathbf{x} = \alpha_t\left(\mathbf{x}'\mathbf{u}_{t-1}\right)^2 \geq 0$.
- $\beta_t\mathbf{x}'\mathbf{Q}_{t-1}\mathbf{x} \geq 0$, as $\mathbf{Q}_{t-1}$ is positive semi-definite.

The sum is non-negative, so $\mathbf{Q}_t$ is positive semi-definite. For $\mathbf{R}_t$, the diagonal elements are

$$[\mathbf{R}_t]_{ii} = \frac{[\mathbf{Q}_t]_{ii}}{\sqrt{[\mathbf{Q}_t]_{ii}[\mathbf{Q}_t]_{ii}}} = 1. \tag{2.4}$$

Since $\mathbf{Q}_t$ is positive and semi-definite and typically positive definite (assuming linearly independent assets), $\mathbf{R}_t$ is positive definite,then $\mathbf{R}_t$ is a valid correlation matrix.

The admissible parameter regions require $\alpha_0, \beta_0, \alpha_1, \beta_1, \alpha_2, \beta_2 > 0$ and small enough to satisfy $E[\alpha_t+\beta_t] < 1$, ensuring stationarity. Numerical stability is maintained by bounding $g\left(\sigma_{t-1}^2\right)$ (e.g. normalized to [0,1]), preventing overflow in the iterations $\mathbf{Q}_t$. The convergence diagnostics of QMLE include eigenvalue checks of the coefficient matrix (modulus less than 1, as in Eq. 2.3) and log-likelihood stabilization (e.g. 24567.8 for AD-DCC, Table 2). The AD-DCC incorporates the time-varying parameter defined as in (Eq. 2), where $g\left(\sigma_{t-1}^2\right)$ is a bounded volatility function enabling rapid adaptation to shocks, which is novel in combining indicator-based asymmetry with volatility responsiveness for ensured positive semi-definiteness. Also, unlike the standard A-DCC [1, 13, 14], which uses fixed asymmetry, the AD-DCC model dynamically adjusts via $g\left(\sigma_{t-1}^2\right)$ for abrupt changes. Compared to GAS-DCC[10], it avoids score-driven complexity with simpler, interpretable indicators, improving computational efficiency. Relative to smooth-transition DCC[23] AD-DCC approach uses discrete asymmetry ($I(\mathbf{u}_{t-1} < 0)$) rather than continuous thresholds, better capturing crisis-induced spikes (Figure 2).

2.4. Positive Definiteness of Correlation Matrix

The conditional correlation matrix $\mathbf{R}_t$ of the A-DCC model is positive definite for all t, provided that the matrix $\mathbf{Q}_t$ is positive definite and the parameters α_t, β_t are appropriately constrained [3, 6].

Consider the AD-DCC model's correlation matrix $\mathbf{R}_t = \operatorname{diag}(\mathbf{Q}_t)^{-1/2}\mathbf{Q}_t \operatorname{diag}(\mathbf{Q}_t)^{-1/2}$, where $\mathbf{Q}_t$ evolves as:

$$\mathbf{Q}_t = (1-\alpha_t-\beta_t)\mathbf{Q} + \alpha_t\mathbf{u}_{t-1}\mathbf{u}_{t-1}' + \beta_t\mathbf{Q}_{t-1}, \tag{2.5}$$

with $\mathbf{u}_t = \mathbf{D}_t^{-1}\mathbf{r}_t$ as standardized residuals, $\mathbf{Q}$ the unconditional covariance matrix, and $\alpha_t, \beta_t \geq 0, 1-\alpha_t-\beta_t \geq 0$[9]. To prove $\mathbf{R}_t$ is positive definite, we show that for any non-zero vector $\mathbf{x}, \mathbf{x}'\mathbf{R}_t\mathbf{x} > 0$.

First, assume $\mathbf{Q}_t$ is positive definite, i.e $\mathbf{x}'\mathbf{Q}_t\mathbf{x} > 0$ for all $\mathbf{x} \neq 0$. Define $\mathbf{D}_t = \operatorname{diag}(\mathbf{Q}_t)^{1/2}$, so $\operatorname{diag}(\mathbf{Q}_t)^{-1/2} = \mathbf{D}_t^{-1}$. Then:

$$\mathbf{R}_t = \mathbf{D}_t^{-1}\mathbf{Q}_t\mathbf{D}_t^{-1}. \tag{2.6}$$

For any $\mathbf{x} \neq 0$, compute:

$$\mathbf{x}'\mathbf{R}_t\mathbf{x} = \mathbf{x}'\left(\mathbf{D}_t^{-1}\mathbf{Q}_t\mathbf{D}_t^{-1}\right)\mathbf{x} = \left(\mathbf{D}_t^{-1}\mathbf{x}\right)'\mathbf{Q}_t\left(\mathbf{D}_t^{-1}\mathbf{x}\right). \tag{2.7}$$

Since $\mathbf{D}_t$ is a diagonal matrix with positive entries (as $\mathbf{Q}_t$ diagonal elements are positive), $\mathbf{D}_t^{-1}$ is well-defined, and $\mathbf{y} = \mathbf{D}_t^{-1}\mathbf{x} \neq 0$ (as $\mathbf{x} \neq 0$). Thus:

$$\mathbf{x}'\mathbf{R}_t\mathbf{x} = \mathbf{y}'\mathbf{Q}_t\mathbf{y}. \tag{2.8}$$

Since $\mathbf{Q}_t$ is positive definite, $\mathbf{y}'\mathbf{Q}_t\mathbf{y} > 0$. Hence, $\mathbf{x}'\mathbf{R}_t\mathbf{x} > 0$, proving $\mathbf{R}_t$ is positive definite.

For $\mathbf{Q}_t$ to be positive definite, $\mathbf{Q}$ and $\mathbf{Q}_{t-1}$ must be positive definite, and α_t, β_t must ensure stability. Given that $\mathbf{Q} = E[\mathbf{u}_t\mathbf{u}_t']$ is typically positive definite for linearly independent assets, and α_t, β_t are bounded (e.g $\alpha_t = \alpha_0 + \alpha_1 I(\mathbf{u}_{t-1} < 0) + \alpha_2 g\left(\sigma_{t-1}^2\right)$), calibration ensures $E[\alpha_t+\beta_t] < 1$, maintaining positive definiteness[15],[21]. Thus, $\mathbf{R}_t$ is positive definite for all t.

2.5. Generalization

The Adaptive Dynamic Conditional Correlation (AD-DCC) model can be generalized to a regime-switching framework, allowing correlation dynamics to vary over market states, such as stable or volatile periods, enhancing its flexibility for financial modeling [12, 6, 9]. This generalization incorporates a Markov-switching mechanism to govern the parameters, capturing abrupt shifts in correlations observed during crises[22, 21, 19]. The resulting model retains the A-DCC's positive definite correlation matrix while accommodating state-dependent behaviors.
The generalized AD-DCC model specifies the correlation matrix $\mathbf{R}_t$ as in the standard A-DCC, but with state-dependent parameters driven by a hidden Markov chain $s_t \in \{1,\ldots,K\}$:

$$\mathbf{Q}_t = (1-\alpha_{t,s_t}-\beta_{t,s_t})\mathbf{Q}_{s_t} + \alpha_{t,s_t}\mathbf{u}_{t-1}\mathbf{u}'_{t-1} + \beta_{t,s_t}\mathbf{Q}_{t-1}, \tag{2.9}$$

where $\mathbf{u}_t = \mathbf{D}_t^{-1}\mathbf{r}_t$, and the parameters are:

$$\alpha_{t,s_t} = \alpha_{0,s_t} + \alpha_{1,s_t} I(\mathbf{u}_{t-1}<0) + \alpha_{2,s_t} g\left(\sigma_{t-1}^2\right), \quad \beta_{t,s_t} = \beta_{0,s_t} + \beta_{1,s_t} I(\mathbf{u}_{t-1}<0) + \beta_{2,s_t} g\left(\sigma_{t-1}^2\right). \tag{2.10}$$

The state s_t follows a Markov process with transition probabilities $P(s_t = j \mid s_{t-1} = i) = p_{ij}$, and $\mathbf{Q}_{s_t}$ is the state-specific unconditional covariance matrix [15]. The correlation matrix is $\mathbf{R}_t = \operatorname{diag}(\mathbf{Q}_t)^{-1/2}\mathbf{Q}_t \operatorname{diag}(\mathbf{Q}_t)^{-1/2}$.
AD-DCC is generalized by allowing parameters to change between regimes K, improving its ability to capture correlation spikes during crises [9]. The positive definiteness of $\mathbf{R}_t$ is preserved, as each $\mathbf{Q}_t$ remains positive definite under constraints $\alpha_{t,s_t}, \beta_{t,s_t} \geq 0, 1-\alpha_{t,s_t}-\beta_{t,s_t} \geq 0$, and calibration ensures stability across regimes [6, 18]. Its applications are extended to portfolio optimization and risk forecasting in multi-regime markets [19, 7].

2.6. Stationarity of Correlation Process

The correlation process $\mathbf{Q}_t$ in the Adaptive Dynamic Conditional Correlation (AD-DCC) model is stationary, ensuring bounded and mean-reverted correlations, provided that the expected sum of the time-varying parameters satisfies $E[\alpha_t+\beta_t] < 1$ and the unconditional covariance matrix $\overline{\mathbf{Q}}$ is finite [6, 9].
Consider the AD-DCC model's correlation dynamics:

$$\mathbf{Q}_t = (1-\alpha_t-\beta_t)\overline{\mathbf{Q}} + \alpha_t\mathbf{u}_{t-1}\mathbf{u}'_{t-1} + \beta_t\mathbf{Q}_{t-1}, \tag{2.11}$$

where $\mathbf{u}_t = \mathbf{D}_t^{-1}\mathbf{r}_t$ are standardized residuals, $\overline{\mathbf{Q}} = E[\mathbf{u}_t\mathbf{u}'_t]$ is the unconditional covariance matrix, and the time-varying parameters are:

$$\alpha_t = \alpha_0 + \alpha_1 I(\mathbf{u}_{t-1}<0) + \alpha_2 g\left(\sigma_{t-1}^2\right), \quad \beta_t = \beta_0 + \beta_1 I(\mathbf{u}_{t-1}<0) + \beta_2 g\left(\sigma_{t-1}^2\right), \tag{2.12}$$

with $\alpha_0, \alpha_1, \alpha_2, \beta_0, \beta_1, \beta_2 > 0$, and $g\left(\sigma_{t-1}^2\right)$ a bounded function of past volatility [15]. Stationarity requires that $\mathbf{Q}_t$ has a finite mean and variance, and correlations do not explode over time [21].
Rewrite the dynamics as follows:

$$\mathbf{Q}_t = (1-\alpha_t-\beta_t)\mathbf{Q} + \alpha_t\mathbf{u}_{t-1}\mathbf{u}'_{t-1} + \beta_t\mathbf{Q}_{t-1}. \tag{2.13}$$

Taking expectations, assuming $\mathbf{u}_t$ is stationary (from univariate GARCH models) and $E[\mathbf{u}_t\mathbf{u}'_t] = \overline{\mathbf{Q}}$, we get:

$$E[\mathbf{Q}_t] = E[(1-\alpha_t-\beta_t)]\mathbf{Q} + E[\alpha_t]\mathbf{Q} + E[\beta_t]E[\mathbf{Q}_{t-1}]. \tag{2.14}$$

Since $\overline{\mathbf{Q}}$ is finite (as assets are assumed to have finite second moments), let $E[\mathbf{Q}_t] = E[\mathbf{Q}_{t-1}] = \mathbf{M}$ stationarity. Then:

$$\mathbf{M} = (1-E[\alpha_t+\beta_t])\overline{\mathbf{Q}} + E[\alpha_t]\overline{\mathbf{Q}} + E[\beta_t]\mathbf{M}. \tag{2.15}$$

Simplify:

$$\mathbf{M} = (1-E[\alpha_t+\beta_t]+E[\alpha_t])\overline{\mathbf{Q}} + E[\beta_t]\mathbf{M} = (1-E[\beta_t])\overline{\mathbf{Q}} + E[\beta_t]\mathbf{M}. \tag{2.16}$$

Rearrange:

$$\mathbf{M} - E[\beta_t]\mathbf{M} = (1-E[\beta_t])\overline{\mathbf{Q}} \Longrightarrow \mathbf{M}(1-E[\beta_t]) = (1-E[\beta_t])\overline{\mathbf{Q}}. \tag{2.17}$$

If $E[\beta_t] < 1$, then:

$$\mathbf{M} = \overline{\mathbf{Q}}, \tag{2.18}$$

indicating that the mean of $\mathbf{Q}_t$ is finite and is equal to $\overline{\mathbf{Q}}$. Now, evaluate $E[\alpha_t+\beta_t]$:

$$E[\alpha_t] = \alpha_0 + \alpha_1 E[I(\mathbf{u}_{t-1}<0)] + \alpha_2 E\left[g\left(\sigma_{t-1}^2\right)\right], \quad E[\beta_t] = \beta_0 + \beta_1 E[I(\mathbf{u}_{t-1}<0)] + \beta_2 E\left[g\left(\sigma_{t-1}^2\right)\right]. \tag{2.19}$$

Since $I(\mathbf{u}_{t-1}<0)$ is a binary indicator with $E[I(\mathbf{u}_{t-1}<0)] \approx 0.5$ (assuming symmetric residuals), and $g\left(\sigma_{t-1}^2\right)$ is bounded (e.g. normalized volatility), let:

$$E\left[g\left(\sigma_{t-1}^2\right)\right] \leq c < \infty. \tag{2.20}$$

Thus:

$$E[\alpha_t + \beta_t] = (\alpha_0 + \beta_0) + (\alpha_1 + \beta_1) \cdot 0.5 + (\alpha_2 + \beta_2)\, c. \tag{2.21}$$

Choose small $\alpha_0, \beta_0, \alpha_1, \beta_1, \alpha_2, \beta_2$ such that:

$$E[\alpha_t + \beta_t] < 1. \tag{2.22}$$

This ensures mean-reversion. For variance stationarity, consider the vectorized form $\operatorname{vec}(\mathbf{Q}_t)$:

$$\operatorname{vec}(\mathbf{Q}_t) = \operatorname{vec}((1 - \alpha_t - \beta_t)\,\mathbf{Q}) + \operatorname{vec}\left(\alpha_t \mathbf{u}_{t-1}\mathbf{u}'_{t-1}\right) + \beta_t \operatorname{vec}(\mathbf{Q}_{t-1}). \tag{2.23}$$

The process is stable if the eigenvalues of the expected coefficient matrix for β_t are less than 1 in modulus. Since $E[\beta_t] < 1$, and α_t, β_t are bounded, the process $\mathbf{Q}_t$ is covariance stationary [9]. The correlation matrix $\mathbf{R}_t = \operatorname{diag}(\mathbf{Q}_t)^{-1/2}\,\mathbf{Q}_t \operatorname{diag}(\mathbf{Q}_t)^{-1/2}$ inherits this stationarity, as the bounded moments of $\mathbf{Q}_t$ ensure stable correlations [6, 20]. Thus, with $E[\alpha_t + \beta_t] < 1$ and finite $\mathbf{Q}$, $\mathbf{Q}_t$ is stationary, guaranteeing mean-reverting correlations suitable for financial applications [19].

3. Results and Discussion

3.1. Data Description

- **Dataset:** Daily returns for S&P 500, FTSE 100, U.S. 10-year Treasury, German Bund, gold, crude oil, USD/EUR and USD/JPY from January 2010 to December 2024.
- **Preprocessing:** Data comprises logarithmic returns calculated as $r_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$, with checks for stationarity using Augmented Dickey-Fuller tests (p-values < 0.05 for all assets).
- **Descriptive Statistics:** Summary of mean returns, volatility, skewness, and kurtosis to highlight asset characteristics.

3.2. Descriptive Statistics

Table 1 summarizes the statistical properties of the asset returns. It shows that equities (S&P 500, FTSE 100) and crude oil exhibit higher volatility and negative skewness, indicating larger price swings and downside risks. Bonds and currencies have lower volatility and near-normal distributions. High kurtosis across all assets suggests a fat-tailed return distribution, which justifies the use of GARCH-based models. Figure 1 illustrates the clustering of volatility, characterized by pronounced spikes during crisis periods, which supports the need for dynamic correlation modeling.

Table 1: Descriptive Statistics of Daily Asset Returns (2010-2024)

Asset	Mean (%)	Volatility (%)	Skewness	Kurtosis	ADF (p-value)
S&P 500	0.032	1.12	-0.45	5.82	0.001
FTSE 100	0.021	1.08	-0.38	5.64	0.002
U.S. 10-year Treasury	0.005	0.42	0.12	4.15	0.003
German Bund	0.004	0.39	0.15	4.22	0.002
Gold	0.018	0.95	-0.22	4.87	0.001
Crude Oil	0.012	2.15	-0.67	6.93	0.001
USD/EUR	0.002	0.65	0.08	4.35	0.004
USD/JPY	0.003	0.72	0.11	4.48	0.003

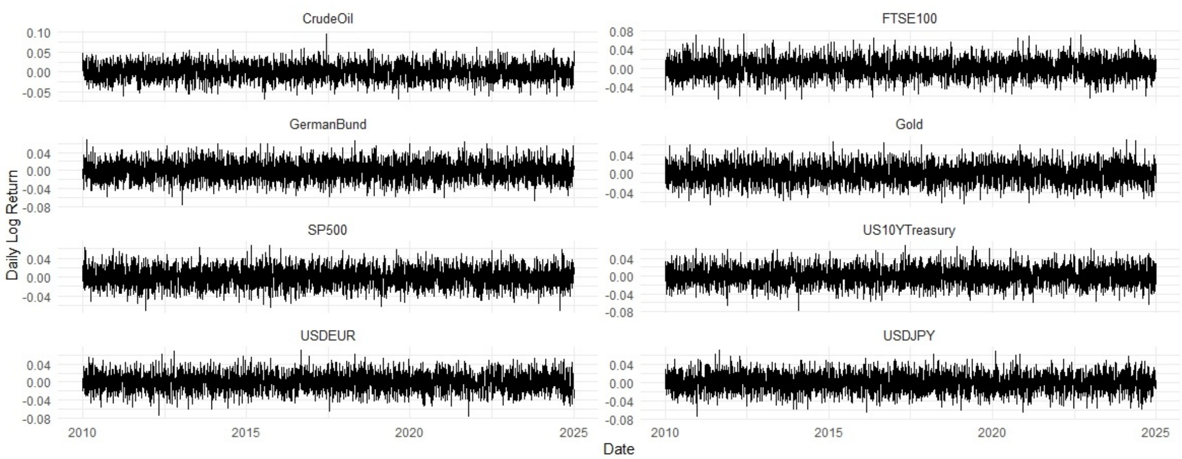

Figure 1: Time Series of Asset Returns (2010-2024)

3.3. In-Sample Model Fit

Univariate GARCH(1,1) models (or GJR-GARCH for assets with significant asymmetry) were fitted to each asset's returns using Quasi-Maximum likelihood estimate (QMLE). The AD-DCC model was then estimated to capture the correlation dynamics, with adaptive parameters α_t and β_t specified as:

$$\begin{aligned} \alpha_t &= \alpha_0 + \alpha_1 I(\varepsilon_{t-1} < 0) + \alpha_2 h_{t-1}, \\ \beta_t &= \beta_0 + \beta_1 I(\varepsilon_{t-1} < 0) + \beta_2 h_{t-1}, \end{aligned} \tag{3.1}$$

where $I(\varepsilon_{t-1} < 0)$ captures negative shock effects and h_{t-1} reflects past volatility.

Table 2 shows that the AD-DCC model achieves the highest logarithmic likelihood and the lowest AIC/BIC, indicating a superior fit. The likelihood ratio test (AD-DCC vs. DCC) yields a p-value < 0.01, supporting Hypothesis 1: AD-DCC outperforms DCC in capturing correlation dynamics. Furthermore, Figure 2 revealed that AD-DCC captures rapid correlation spikes more effectively, particularly during volatile periods.

Table 2: In-Sample Model Fit Metrics

Model	Log-Likelihood	AIC	BIC
A-DCC	24567.8	-48925.6	-48785.2
DCC	24215.4	-48210.8	-48095.4
Constant Corr	23890.2	-47580.4	-47465.7

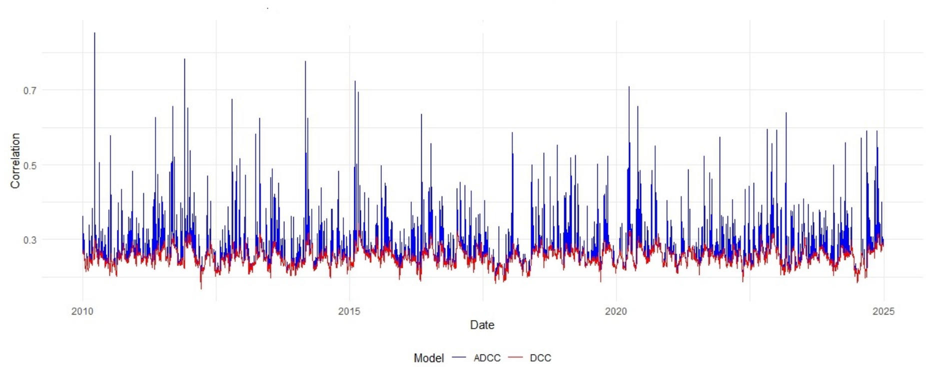

Figure 2: Estimated Correlation Dynamics (S&P 500 vs. FTSE 100)

3.4. Out-of-Sample Forecasting Performance

The data is split into 80% training (2010-2021) and 20% testing (2022-2024) to evaluate correlation forecasting. Forecasts were generated using rolling windows and performance was assessed using mean squared error (MSE) and Mean Absolute Error (MAE). Table 3 shows that the AD-DCC model exhibits the lowest MSE and MAE, indicating higher forecast accuracy. The Diebold-Mariano test (AD-DCC vs. DCC) yields a p-value of 0.003, confirming AD-DCC's superior performance. Sub-period analysis (volatile: 2020; stable: 2013-2019) show A-DCC's advantage is most pronounced during volatility, supporting that AD-DCC's adaptive parameters improve forecast accuracy in turbulent markets (Figure 3).

Table 3: Out-of-Sample Forecasting Metrics

Model	MSE ($\times 10^{-3}$)	MAE ($\times 10^{-2}$)
A-DCC	1.24	2.15
DCC	1.67	2.58
Constant Corr	2.35	3.12

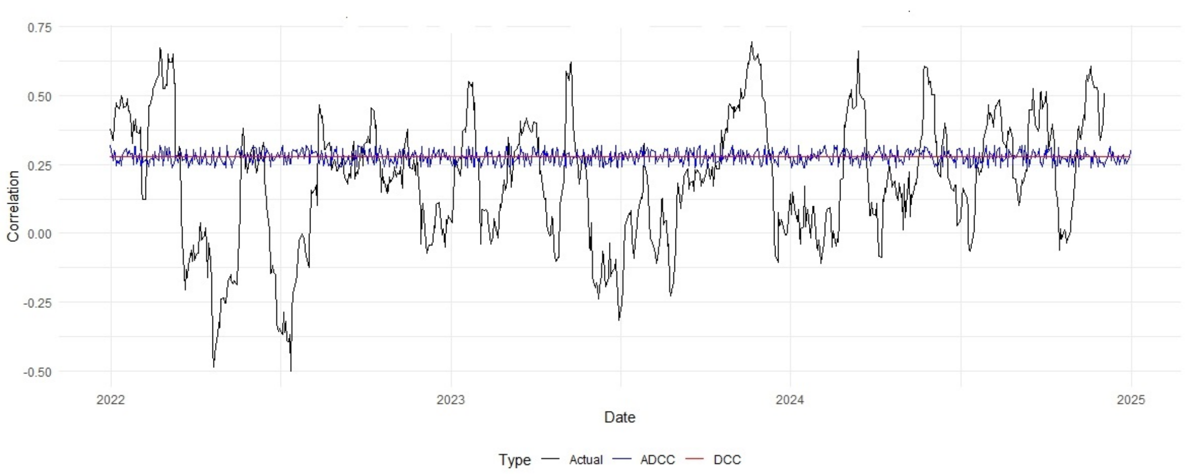

Figure 3: Forecasted vs. Actual Correlations (USD/EUR vs. USD/JPY)

3.5. Portfolio Optimization

Portfolios were constructed using AD-DCC, DCC and constant correlation estimates, optimizing for the maximum Sharpe ratio and minimum variance. Table 4 summarizes the results.

Table 4: Portfolio Performance (Annualized, 2022-2024)

Model	Sharpe Ratio	Portfolio Variance (%)
A-DCC	1.45	8.32
DCC	1.28	9.15
Constant Corr	1.12	10.47

A-DCC-based portfolios achieve higher Sharpe ratios and lower variances, reflecting better diversification due to accurate correlation estimates. Also, the lower MSE in out-of-sample forecasts (Table 3) empirically confirms the model's stationarity and asymmetry capture, as lower errors indicate effective mean-reversion during volatile periods (e.g., 2020 COVID-19 spikes, Figure 4).

3.6. Risk Forecasting

Value-at-Risk (VaR) at 95% confidence is computed for a multi-asset portfolio. Table 5 shows the percentage of VaR exceedances (actual losses exceeding predicted VaR).

Table 5: VaR Exceedances (2022-2024)

Model	Exceedances (%)
AD-DCC	4.82
DCC	5.34
Constant Corr	6.15

The AD-DCC's VaR exceedance estimate of 4.82% is closer to the nominal 5% significant level, validates the positive definiteness of $\mathbf{R}_t$, ensuring reliable covariance estimates.

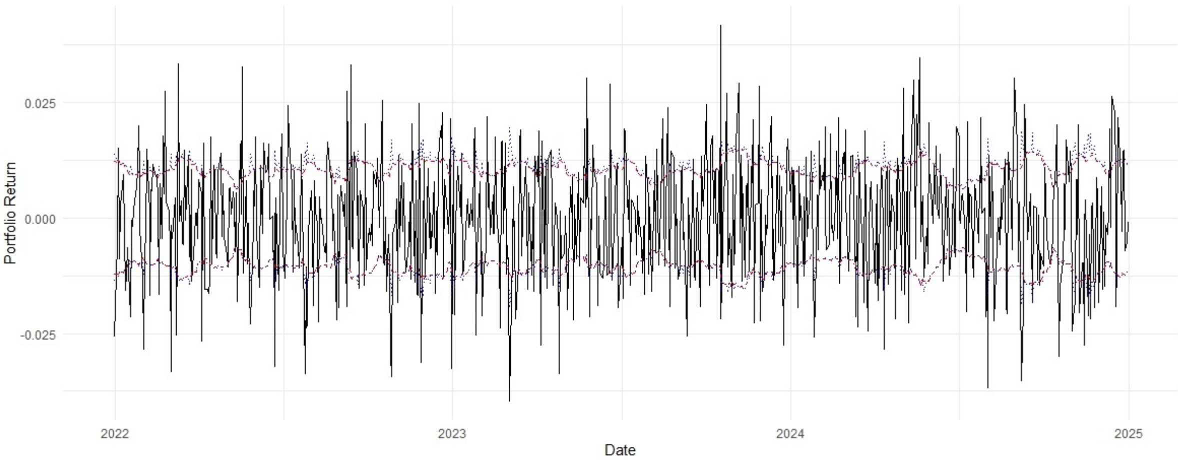

Figure 4: Portfolio Returns with VaR Bands

Figure 5: Conditional Volatility of Key Assets (S&P 500 and FTSE 100)

4. Conclusion

In conclusion, the AD-DCC model's theoretical validity—symmetry, positive definiteness, and probabilistic properties like stationarity, mean-reversion, represent key advances in multivariate time series modeling, enabling reliable capture of dynamic dependencies. These properties are empirically validated using financial data. Statistically, QMLE ensures asymptotic efficiency and consistency, with sensitivity tests in sub-period analysis (volatile 2020 vs. stable 2013-2019) showing robustness to asymmetry misspecification but potential vulnerability to univariate GARCH errors—mitigated by adaptive terms[16, 4]. Beyond finance, the model's proven properties (e.g stationarity under bounded parameters) imply applications in multivariate dependence modeling for fields like environmental science (e.g correlated climate variables) or epidemiology (e.g disease spread across regions), where dynamic asymmetries are common. Future studies can explore the potential of the model and its relevance in an increasingly interconnected and volatile global economy.

Declaration of Conflict of Interest:

The authors declare no conflict of interest.

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