Fuzzy pre-continuous and fuzzy pre*-continuous function of fuzzy pre-compact space in fuzzy topological space

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Abstract

The aim of this paper is to introduce and study the notion of a fuzzy pre-continuous function, fuzzy pre*- continuous function, fuzzy pre-compact space and some properties, remarks related to them.

Keywords: Fuzzy Pre-Continuous; Fuzzy Pre*-Continuous; Fuzzy Compact Space in Fuzzy Topological Space.

1. Introduction


2. Fuzzy pre-compact space

In this section we study some definitions, remarks, Propositions and theorems about fuzzy Pre- compact space in fuzzy topological spaces.

Definition 2.1: [1] [2] A collection $\mathcal{T}$ of a sub set of $\tilde{A}$ that is $\tilde{A} \subseteq p(\tilde{A})$ is said to be fuzzy topology on $\tilde{A}$ if satisfies the following condition:

1) $\emptyset, \tilde{A} \in \mathcal{T}$

2) If $\tilde{G}, \tilde{H} \in \mathcal{T}$, Then $\tilde{G} \cap \tilde{H} \in \mathcal{T}$

3) If $\tilde{G}_i \in \mathcal{T}$, Then $\bigcup \tilde{G}_i \in \mathcal{T}$

The pair $(\tilde{A}, \mathcal{T})$ is said to be fuzzy topological, every member of $\mathcal{T}$ is called fuzzy open ($\tilde{T}$-open) set in $\tilde{A}$ and the complement is called fuzzy ($\tilde{T}$-closed) set.

Definition 2.2: [8] Let $(\tilde{A}, \mathcal{T})$ be a fuzzy topological spaces a family $W$ of fuzzy sets is pre - open cover of a fuzzy set $\tilde{B}$ if and only if $\tilde{B} \subseteq \bigcup \{\tilde{C} : \tilde{C} \in \mathcal{W}\}$ and each member of $W$ is a fuzzy pre- open set. A sub cover of $W$ is a sub family which is also cover.

Definition 2.3: [8] A fuzzy topological spaces $(\tilde{A}, \mathcal{T})$ is fuzzy pre- compact if and only if every fuzzy pre- open cover of $\tilde{A}$ has a finite sub cover.
Definition 2.4: [4] Let $\tilde{B}$ is fuzzy sub set of a fuzzy topological space $(\tilde{A}, \tilde{T})$. $\tilde{B}$ is said to be fuzzy pre-open relative to $\tilde{A}$ if for every fuzzy pre-open cover $\{\tilde{N}_\lambda : \lambda \in \Lambda \}$ such that $\tilde{N}_\lambda$ is fuzzy pre-open sets in $(\tilde{A}, \tilde{T})$ having finite subcover.

Definition 2.5: [10] A fuzzy topological space $(\tilde{A}, \tilde{T})$ is said to be fuzzy ((pre-$T_2$) i.e. (fuzzy pre-Housdorff) if for each pair of distinct point $\tilde{x}$, $\tilde{y}$ of $(\tilde{A}, \tilde{T})$, there exists disjoint fuzzy pre-open sets $\tilde{U}$ and $\tilde{V}$ such that $\tilde{x} \in \tilde{U}$ and $\tilde{y} \in \tilde{V}$

Remark 2.6: [8] Every fuzzy open cover is a fuzzy pre-open cover.

Proposition 2.7: [4] Every fuzzy pre-compact is fuzzy compact space.

Proof:

Let $(\tilde{A}, \tilde{T})$ is fuzzy pre-compact space
And $\{\tilde{N}_\lambda : \lambda \in \Lambda \}$ is open cover to $\tilde{A}$
$\therefore \{\tilde{N}_\lambda : \lambda \in \Lambda \}$ is pre-open cover to $\tilde{A}$
$\therefore \tilde{A}$ is fuzzy pre-compact space.
Such that $\mu_\lambda (x) = \max \{ \mu_{N_\lambda_i} (x) : \lambda \in \Lambda \}$, $i=1, 2, 3... \infty$
$\therefore \tilde{A}$ is fuzzy compact space.

Remark 2.8: [4] Fuzzy compact space need not to be fuzzy pre-compact space Proposition (2.9) [7]
If every fuzzy pre-open subset of a fuzzy topological space $(\tilde{A}, \tilde{T})$ is a fuzzy pre-compact, then every subset of $\tilde{A}$ is a fuzzy pre-compact.

Proof:

Obvious $\blacksquare$

Proposition (2.10) [4]
A fuzzy pre-closed subset of fuzzy pre-compact space is fuzzy pre-compact

Proof:

Suppose that $(\tilde{A}, \tilde{T})$ be a fuzzy pre-compact space
And $\tilde{B}$ be a fuzzy pre-closed subset of $(\tilde{A}, \tilde{T})$
And $\{\mu_{\tilde{N}} (x) : \lambda \in \Lambda \}$ is pre-open cover to $\tilde{B}$
Since $\tilde{B}$ is a fuzzy pre-open cover.
Such that $\max \{ \{\mu_{\tilde{N}_\lambda} (x) : \lambda \in \Lambda \}, \mu_{\tilde{B}} (x) \}$ is fuzzy pre-open cover of $\tilde{B}$.
Since $(\tilde{A}, \tilde{T})$ is fuzzy pre-compact space.
Hence $\tilde{B}$ is fuzzy pre-compact space.$\blacksquare$

Corollary (2.11) [4]
A fuzzy closed subset of a fuzzy pre-compact is fuzzy compact.

Proof:

Suppose that $\mu_\tilde{C} (x) = \max \{ \mu_{\tilde{N}_\lambda} (x) : \lambda \in \Lambda \}$ be fuzzy pre-open cover to $\tilde{B}$
Suppose that $\mu_{\tilde{C}_1} (x) = \max \{ \mu_{\tilde{C}_1} (x), \mu_{\tilde{B}_1} (x) \}$
$\therefore \tilde{C}_1$ is fuzzy open cover to $(\tilde{A}, \tilde{T})$
$\therefore \tilde{A}$ is a fuzzy pre-compact.
Such that $\mu_\lambda (x) \leq \max \{ \mu_{\tilde{N}_\lambda} (x), \mu_{\tilde{B}_1} (x) \}$
$\therefore \mu_{\tilde{B}} (x) \leq \max \{ \mu_{\tilde{N}_\lambda} (x) \}$
Then $\tilde{B}$ is fuzzy pre-compact.$\blacksquare$

Remark 2.12: [6] A fuzzy pre-closed subset of a fuzzy compact space is needed not to be fuzzy compact.

Theorem 2.13: [8] Let $(\tilde{A}, \tilde{T})$ be a fuzzy topological space if $\tilde{B}$ and $\tilde{C}$ are two Fuzzy pre-compact subsets of $\tilde{A}$, then $\tilde{B} \cup \tilde{C}$ is also fuzzy pre-compact.

Proof:

Suppose that $\{\tilde{N}_\lambda : \lambda \in \Lambda \}$ be a fuzzy pre-open cover of $\tilde{B} \cup \tilde{C}$
Then $\max \{ \mu_{\tilde{B}} (x), \mu_{\tilde{C}} (x) \} \leq \max \{ \mu_{\tilde{N}_\lambda} (x) : \lambda \in \Lambda \}$
Since $\mu_{\tilde{B}} (x) \leq \max \{ \mu_{\tilde{B}} (x), \mu_{\tilde{C}} (x) \}$
And $\mu_{\tilde{C}} (x) \leq \max \{ \mu_{\tilde{B}} (x), \mu_{\tilde{C}} (x) \}$
Also $\{\tilde{N}_\lambda : \lambda \in \Lambda \}$ is fuzzy pre-open cover of $\tilde{B}$ and a fuzzy pre-open cover of $\tilde{C}$
Since, $\tilde{B}$ and $\tilde{C}$ are two pre-compact sets, then there exists a finite sub cover $(\tilde{N}_1, \tilde{N}_2, \ldots, \tilde{N}_m)$ and $\mu_{\tilde{B}}(x) \leq \max \{ \mu_{\tilde{N}_i}(x) \}, i = 1, 2, 3, \ldots, n$

Hence $\mu_{\tilde{B}}(x) \leq \max \{ \mu_{\tilde{N}_i}(x) \}$

And $\mu_{\tilde{B}}(x), \mu_{\tilde{C}}(x) \leq \max \{ \mu_{\tilde{N}_i}(x) \}, k = 1, 2, 3, \ldots, n+m$

Thus, $\tilde{A} \cup \tilde{B}$ is fuzzy pre compact.

Remark 2.14: [8] If $\tilde{B}$ and $\tilde{C}$ are a fuzzy pre-compact subsets of a fuzzy topological space $(\tilde{A}, \tilde{T})$ then $\tilde{B} \cap \tilde{C}$ is need not to be fuzzy pre-compact space.

**Theorem 2.15:** [4] Every fuzzy pre-closed off $(\tilde{A}, \tilde{T})$ is fuzzy pre-compact if and only if $(\tilde{A}, \tilde{T})$ is fuzzy pre-compact.

Proof:

Let $\{ \tilde{N}_\lambda : \lambda \in \Lambda \}$ is fuzzy pre-open cover in $\tilde{A}$

Then $\mu_{\tilde{A}}(x) = \max \{ \mu_{\tilde{N}_\lambda}(x) : \lambda \in \Lambda \}$

Suppose that $\mu_{\tilde{G}}(x) = \max \{ \mu_{\tilde{N}_\lambda}(x) \cap \tilde{G} : \lambda \in \Lambda \}$

Then $\tilde{K}$ is fuzzy closed.

And $\mu_{\tilde{G}}(x) \leq \max \{ \mu_{\tilde{N}_\lambda}(x) : \lambda \in \Lambda \}$

Then there exist $\Lambda$ fuzzy subset finite on $\Lambda - \{ \lambda_0 \}$

Such that $\mu_{\tilde{G}}(x) \leq \max \{ \mu_{\tilde{N}_\lambda}(x) : \lambda \in \Lambda - \{ \lambda_0 \} \}$

Then $\tilde{A}$ is fuzzy pre-compact.

Conversely $\leftarrow$

Let $\tilde{B}$ is fuzzy pre-compact

Suppose that $\tilde{K}$ is fuzzy closed set in $\tilde{A}$

Then $\tilde{K}$ is fuzzy pre-compact.

**Theorem 2.16:** [3] A fuzzy topological space $(\tilde{A}, \tilde{T})$ is fuzzy pre-compact If and only if for every collection $\{ \tilde{N}_\lambda : \lambda \in \Lambda \}$ of fuzzy pre-closed set of $(\tilde{A}, \tilde{T})$ having the finite intersection property

$$\text{Min} \{ \mu_{\tilde{R}_A}(x) \} \neq \mu_{\tilde{B}}(x)$$

Proof:

Suppose that $\{ \tilde{N}_\lambda : \lambda \in \Lambda \}$ be a collection of fuzzy pre-closed with the finite intersection property

Let $\mu_{\tilde{R}_A}(x) = \tilde{B}(x)$ and $\mu_{\tilde{N}_\lambda}(x) \neq \mu_{\tilde{B}}(x)$

Since $\{ \tilde{N}_\lambda : \lambda \in \Lambda \}$ is a collection of fuzzy pre-open set cover of $\tilde{A}$ it follows that there exists a finite subset $\mu \subseteq \Lambda$

Such that $\max \{ \mu_{\tilde{R}_A}(x) : \mu \in \Lambda \}$

Then min $\{ \mu_{\tilde{R}_A}(x) \} = \mu_{\tilde{B}}(x)$ where $\Lambda \in \mu$

Which the contradiction

And therefore min $\{ \mu_{\tilde{R}_A}(x) \} \neq \mu_{\tilde{B}}(x)$

Conversely

Obvious

**Theorem 2.16:** [4] Let $\tilde{B}$ is fuzzy open subset of $(\tilde{A}, \tilde{T})$, then $\tilde{B}$ is fuzzy pre-compact if and only if $\tilde{B}$ sub space to $\tilde{A}$.

Proof:

Suppose that $\tilde{B}$ is fuzzy pre-compact to $\tilde{A}$

Suppose that $\{ \tilde{N}_\lambda(x) : \lambda \in \Lambda \}$ is covering to $\tilde{B}$

Such that $\tilde{N}_\lambda$ is fuzzy pre-open set in $\tilde{A}$

Thus min $\{ \mu_{\tilde{B}}(x), \mu_{\tilde{R}_A}(x) \}$ is fuzzy pre-open set in $\tilde{A}$

Then $\{ \tilde{N}_\lambda(x) : \lambda \in \Lambda \}$ is fuzzy pre-open cover to $\tilde{B}$

$\cap \tilde{B}$ is fuzzy pre-compact

Then $\mu_{\tilde{B}}(x) < \max \{ \min \{ \mu_{\tilde{B}}(x), \mu_{\tilde{R}_A}(x) : \lambda \in \Lambda \} \}$ such that $\lambda \subseteq \Lambda$

And $\mu_{\tilde{B}}(x) < \max \{ \mu_{\tilde{R}_A}(x) : \lambda \in \Lambda \}$
\[\therefore \tilde{B} \text{ is fuzzy pre-compact sub space to } \tilde{A} \]  

**Theorem 2.17:** [4] *Every fuzzy pre-compact space in fuzzy Hausdorff space is fuzzy pre-closed*

**Proof:**

Suppose that \((\tilde{B}, \tilde{T})\) is fuzzy pre-compact in fuzzy Hausdorff space.
\[\therefore \tilde{B} \text{ is fuzzy compact space} \]
\[\therefore \tilde{A} \text{ is fuzzy Hausdorff space} \]
\[\therefore \tilde{B} \text{ is fuzzy closed} \]
\[\therefore \tilde{B} \text{ is fuzzy pre-closed in } \tilde{A} \]

**Proposition 2.18:** [4] Let \(\tilde{B}, \tilde{C}\) be two fuzzy subset of \((\tilde{A}, \tilde{T}), \tilde{B}\subseteq \tilde{C}\) and \(\tilde{C}\) is fuzzy open set of \(\tilde{A}\), then \(\tilde{B}\) is fuzzy pre-compact relative to subspace \(\tilde{C}\) if and only if \(\tilde{B}\) is fuzzy pre-compact relative to \(\tilde{A}\).

**Proof:**

Suppose that \(\tilde{B}\) is fuzzy pre-compact subspace in \(\tilde{C}\)
\[\therefore \tilde{B} \text{ is fuzzy pre-compact relative to } \tilde{C} \]
And \(\tilde{B}\) is fuzzy pre-compact relative to \(\tilde{A}\)
Hence \(\tilde{B}\) is fuzzy pre-compact in \(\tilde{A}\)

Conversely

Let \(\tilde{B}\) is fuzzy pre-compact in \(\tilde{A}\)
\[\therefore \tilde{B} \text{ is fuzzy pre-compact relative to } \tilde{A} \]
Then \(\tilde{B}\) is fuzzy pre-compact relative to \(\tilde{C}\)
Hence \(\tilde{B}\) is fuzzy pre-compact in \(\tilde{C}\)

### 3. Fuzzy pre-continuous and fuzzy pre*-continuous

**Definition 3.1:** [3] A function \(f: (\tilde{A}, \tilde{T}) \rightarrow (\tilde{B}, \tilde{T})\) is fuzzy continuous (f-continuous) if and only if the inverse image of any fuzzy open set in \(\tilde{T}\) is fuzzy open set in \(\tilde{T}\).

**Definition 3.2:** A function \(f: (\tilde{A}, \tilde{T}) \rightarrow (\tilde{B}, \tilde{T})\) is said to be a fuzzy pre-continuous if and only if the inverse image of any fuzzy open set in \(\tilde{T}\) is fuzzy pre-open set in \(\tilde{T}\).

**Definition 3.3:** A function \(f: (\tilde{A}, \tilde{T}) \rightarrow (\tilde{B}, \tilde{T})\) is said to be a fuzzy pre*-continuous if and only if the inverse image of any fuzzy pre-open set in \(\tilde{T}\) is fuzzy pre-open set in \(\tilde{T}\).

**Proposition 3.5:** [3] If \(f: (\tilde{A}, \tilde{T}) \rightarrow (\tilde{B}, \tilde{T})\) is fuzzy continuous function, then \(f\) is fuzzy pre*-continuous

**Proof:**

Suppose that \((\tilde{C})\) is a fuzzy pre-open fuzzy in \(\tilde{B}\)

Then \(\mu_{\tilde{C}}(x) \leq \mu_{int\tilde{C}}(x)\) and \(\mu_{f^{-1}\tilde{C}}(x) \leq \mu_{f^{-1}int\tilde{C}}(x) \leq \mu_{f^{-1}int\tilde{C}}(x)\)

Since \(f\) is fuzzy continuous

Then \(\mu_{f^{-1}int\tilde{C}}(x) \leq \mu_{int\tilde{f^{-1}\tilde{C}}}(x) \leq \mu_{int\tilde{f^{-1}\tilde{C}}}(x)\)

Thus \(\mu_{f^{-1}\tilde{C}}(x) \leq \mu_{int\tilde{f^{-1}\tilde{C}}}(x) \leq \mu_{int\tilde{f^{-1}\tilde{C}}}(x)\)

Then \(f\) is fuzzy pre*-continuous

**Theorem 3.6:** [3] Let \(f: (\tilde{A}, \tilde{T}) \rightarrow (\tilde{B}, \tilde{T})\) be a function, and then the following are equivalent

1. \(f\) is fuzzy pre*-continuous
2. \(f(p-cl\tilde{C}) \subseteq p(cl\tilde{f\tilde{C}})\) for every fuzzy set \(\tilde{C}\) in \(\tilde{A}\)

**Proof:**

2 → 1. Suppose that \(\tilde{C}\) be a fuzzy set of \(\tilde{A}\)
Then \(p-cl\tilde{f\tilde{C}}\) is fuzzy pre-closed
By the (1) $f^{-1}$ (p-cl ($\tilde{C}$)) is pre-closed
And ($\mu_f^{-1}$ p-cl($f(\tilde{C})$)($x$) = ($\mu_{p-cl(f^{-1}(p-cl(f(\tilde{C}))))}$)$x$)
Since $\mu_{f(\tilde{C})}(x) \leq \mu_{f^{-1}(\tilde{C})}(x)$
And $\mu_{p-cl(\tilde{C})}(x) \leq (\mu_{p-cl(f^{-1}(\tilde{C})))}(x) = \mu_{p-cl(f^{-1}(p-cl(f(\tilde{C}))))}(x)$
Hence $\mu_{f^{-1}(\tilde{C})}(x) \leq \mu_{p-cl(f(\tilde{C}))}(x)$, $f^{-1}$

2←1 Suppose that $\tilde{D}$ be a fuzzy pre-closed set in $\tilde{B}$
And If $\mu_{f^{-1}(\tilde{D})}(x) \geq \mu_{f^{-1}(\tilde{D})}(x)$
Then $\mu_{p-cl(\tilde{D})}(x) \leq \mu_{f^{-1}(p-cl(f(\tilde{D})))}(x)$
$\leq \mu_{f^{-1}(p-cl(f(\tilde{D})))}(x) = \mu_{f^{-1}(\tilde{D})}(x)$
Since $\mu_{f^{-1}(\tilde{D})}(x) \leq \mu_{p-cl(f^{-1}(\tilde{D}))}(x)$
Then $\mu_{f^{-1}(\tilde{D})}(x) = \mu_{p-cl(f^{-1}(\tilde{D}))}(x)$

Hence $f^{-1}(\tilde{D})$ is fuzzy pre-closed set in $\tilde{B}$ and $f$ is fuzzy pre*-continuous

Corollary 3.7: [3] Let $f$: ($\tilde{A}$, $\tilde{T}$) $\rightarrow$ ($\tilde{B}$, $\tilde{T}'$) be a function, then the following are equivalent
1) $f$ is fuzzy pre-continuous
2) $f$ (p-cl ($\tilde{C}$) $\subseteq$ cl(ƒ ($\tilde{C}$)), for every fuzzy set $\tilde{C}$ in $\tilde{A}$

Proof:
Obvious

Proposition 3.8: [3] If $f$: ($\tilde{A}$, $\tilde{T}$) $\rightarrow$ ($\tilde{B}$, $\tilde{T}'$) is fuzzy open and fuzzy continuous function and $\tilde{A}$ is a fuzzy pre-compact
Then $f$ ($\tilde{A}$) is fuzzy pre-compact space.

Proof:
Obvious

**Theorem 3.9:** [8] The fuzzy pre-continuous image of a fuzzy pre-compact space is fuzzy compact space.

Proof:

Suppose that ($\tilde{A}$, $\tilde{T}$) be a fuzzy pre-compact space
And $f$: ($\tilde{A}$, $\tilde{T}$) $\rightarrow$ ($\tilde{B}$, $\tilde{T}'$) be a fuzzy pre-continuous function
To prove ($\tilde{B}$, $\tilde{T}'$) is a fuzzy compact space
Let {$\tilde{N}_i$: $\lambda \in A$} is a fuzzy open cover of $\tilde{B}$.
Then { $f^{-1}(\tilde{N}_i)$ ($x$): $\lambda \in A$} is fuzzy pre-open cover of $\tilde{A}$
Since $f$ is fuzzy pre-continuous function a finite sub cover {$f^{-1}(\tilde{N}_{i1}$) ($x$), $i = 1, 2, 3... n$} which covering $\tilde{A}$
Then $\tilde{B}$ is a fuzzy compact space

Remark 3.10: [4] The fuzzy continuous image of fuzzy pre-compact need not be a fuzzy pre-compact space.

**Theorem 3.11:** [3] If a function $f$: ($\tilde{A}$, $\tilde{T}$) $\rightarrow$ ($\tilde{B}$, $\tilde{T}'$) is fuzzy pre*-continuous and $\tilde{C}$ is a fuzzy pre-compact relative to $\tilde{A}$ then so is $f$ ($\tilde{C}$) is fuzzy pre-compact.

Proof:

Suppose that {$\tilde{N}_i$: $\lambda \in A$} be a fuzzy pre-open cover of
Since $f$ is fuzzy pre*-continuous and {$\mu_{f^{-1}(\tilde{N}_{i1}$) ($x$): $\lambda \in A$} is a fuzzy pre-open set cover of $S$ ($\tilde{C}$) in $\tilde{A}$
Since $\tilde{C}$ is a fuzzy pre-open compact relative to $\tilde{A}$
There is a finite subfamily {$\mu_{f^{-1}(\tilde{N}_{i1}$) ($x$): $\lambda \in A$}.
Such that $\mu_{f(\tilde{C})}(x) \leq \mu_{f^{-1}(\tilde{N}_{i1}$) ($x$) $f^{-1}$max {$\mu_{f^{-1}(\tilde{N}_{i1}$) ($x$): $\lambda \in A$}} $\leq \mu_{f^{-1}(\tilde{N}_{i1}$) ($x$) $f^{-1}$max {$\mu_{f^{-1}(\tilde{N}_{i1}$) ($x$): $\lambda \in A$}} $\leq \mu_{f^{-1}(\tilde{N}_{i1}$) ($x$) $f^{-1}$max {$\mu_{f^{-1}(\tilde{N}_{i1}$) ($x$): $\lambda \in A$}}
Therefore $f$ ($\tilde{C}$) is a fuzzy pre-compact relative to $\tilde{B}$
Propositions 3.12: [3]

1) If $f$: ($\tilde{A}$, $\tilde{T}$) $\rightarrow$ ($\tilde{B}$, $\tilde{T}'$) is a fuzzy pre*-open and bijective function and $\tilde{B}$ be fuzzy pre-compact then $\tilde{A}$ is a fuzzy pre-compact.
Proof:

Suppose that $\{\tilde{N}_A : \lambda \in \Lambda \}$ be a family of a fuzzy pre-open covering of $A$
Let $\{\mu_f(\tilde{N}_A)(x) : \lambda \in \Lambda \}$ be a fuzzy pre-open set covering of $B$
Since $B$ is fuzzy pre-compact then there exist a finite family $\lambda' \subseteq \Lambda$ covers $B$
Such that $\{\mu_f(\tilde{N}_A)(x) : \lambda \in \lambda' \}$ covers $B$
Since $f$ is bijective
Then $\mu_f^{-1}(\tilde{B})(x) = \mu_f(\tilde{A})(x) = \mu_f(\tilde{A}) \max \{\tilde{N}_A(x) : \lambda \in \Lambda \}$
Hence $\tilde{A}$ is a fuzzy pre-compact $\blacksquare$

2) Let $f : (\tilde{A}, \tilde{T}) \rightarrow (\tilde{B}, \tilde{T}^*)$ be a fuzzy pre-continuous surjective function and $\tilde{A}$ is a fuzzy pre-closed compact
then $\tilde{B}$ is a fuzzy pre-closed compact.

Proof:

Obvious $\blacksquare$

3) Let if $f : (\tilde{A}, \tilde{T}) \rightarrow (\tilde{B}, \tilde{T}^*)$ be a fuzzy pre-continues surjective function of a fuzzy pre-compact a space $\tilde{A}$ onto
a space $\tilde{B}$ then $\tilde{B}$ is fuzzy pre-compact.

Proof:

Obvious $\blacksquare$

4) Let if $f : (\tilde{A}, \tilde{T}) \rightarrow (\tilde{B}, \tilde{T}^*)$ be a fuzzy pre-continues bijective function and $\tilde{B}$ be a fuzzy pre-compact space $\tilde{A}$
then $\tilde{B}$ is fuzzy Pre-compact.

Proof

Obvious $\blacksquare$

Remark 3.13: [7] The following diagram explains the relationships among the different types of fuzzy continuous function.

4. Conclusion

It is an interesting exercise to work on fuzzy, pre-continuous and fuzzy pre*-continuous; function of fuzzy pre-compact space in fuzzy topological space similarly other forms of fuzzy pre-open set can be applied to define different forms of fuzzy pre-compact space.

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