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Research paper

The Randic Spectra of variants of corona of two regular graphs

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Abstract

Let G be a graph with vertex set $V = \{v_1, v_2, v_3, \ldots, v_n\}$. Let d_i represent the degree of v_i in G. We compute the Randic spectrum of neighborhood corona, extended corona, extended neighborhood corona, Identity extended corona, Identity extended neighborhood corona of two graphs G_1 and G_2 in terms of the adjacency spectrum of G_1 and G_2.

Keywords: *Randic matrix; Adjacency Matrix; Spectrum; neighborhood corona; extended corona; extended neighborhood corona; Identity extended neighborhood corona; Identity extended corona.*

1. Introduction

In this work we exclusively consider simple graph $G = (V, E)$ with n vertices and m edges refer [1, 2, 3, 4, 5]. Let $V = \{v_1, v_2, v_3, \ldots, v_n\}$ be the vertex set of G. The adjacency matrix of G, denoted by $A(G)$, is defined as $A(G) = (a_{ij})_{n\times n}$ where

$$a_{ij} = \begin{cases} 1 \text{ if } v_i v_j \in E(G) \\ 0 \text{ otherwise} \end{cases}$$

For a given matrix M of size n, the characteristic polynomial of M is given by

$$\phi(M; x) = det(xI_n - M).$$

where I_n is the identity matrix of size n. The values $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the roots of $\phi(A(G); x) = det(xI_n - A(G)) = 0$ which are usually referred as the Eigen values of the graph G.
For every vertex $v_i \in V$, let d_i represent the degree of v_i in G. If the vertices v_i and v_j are adjacent we denote $v_i v_j \in E(G)$.
The molecular structure-descriptor, Randic index, [12, 13, 14, 15, 16, 17, 18, 19] was put forward by Milan Randic in 1975 and is defined as

$$R = R(G) = \sum_{i\sim j} \frac{1}{\sqrt{d_i d_j}}$$

$\sum_{i\sim j}$ indicates summation over all pairs of adjacent vertices $v_i\ v_j$.
The concept of Randić matrix [22] was introduced by Rodríguez in 2005. It is a matrix representation of a graph that encodes information about the graph's structure and connectivity. The Randic matrix $R(G) = (r_{ij})$ is defined by

$$r_{ij} = \begin{cases} \frac{1}{\sqrt{d_i d_j}} \text{ if } v_i v_j \in E(G) \\ 0 \text{ otherwise} \end{cases}$$

In [22] Rodríguez extended this idea to spectral graph theory, giving rise to the Randić characteristic polynomial and its corresponding Randić Eigen value. The Randić spectrum is the set of eigenvalues of the Randic matrix. Studying these eigenvalues help to understand the structure, symmetry, and behavior of graphs. The Randić spectra provides a unique perspective on graph structure, enabling researchers to extract valuable information about the graph's properties and behavior. Mathematicians later applied these ideas in the field of research and many results were developed [12, 13, 14, 15, 16, 17, 18, 19]. Its applications in various fields prove it to be an important area of research in graph theory even today [18, 19, 20, 21].

The corona of two graphs is defined in [7, 8] and the variation of corona in [9, 10], along with its spectrum is discussed. Here we compute the Randic spectrum of neighborhood corona, extended corona, extended neighborhood corona, Identity extended corona, Identity extended neighborhood corona of two graphs G_1 and G_2 in terms of the adjacency spectrum of G_1 and G_2. It helps us to understand properties of these new graphs, which is useful in more complex chemical compounds and complex networks.

2. Preliminaries

In this section, the definitions and the lemma that are useful to prove the main results are stated.

We use the following definitions.

Definition 2.1. *[9] "The neighborhood corona of G_1 and G_2 denoted by $G_1 \star G_2$ is the graph obtained by taking one copy of G_1 and n_1 copies of G_2 and joining every neighbor of the i^{th} vertex of G_1 to every vertex in the i^{th} copy of G_2."*

Definition 2.2. *[10] "The extended corona $G_1 \bullet G_2$ of two graphs G_1 and G_2 is a graph obtained by taking the corona $G_1 \circ G_2$ and joining each vertex of i^{th} copy of G_2 to every vertex of j^{th} copy of G_2 provided the vertices v_i and v_j are adjacent in G_1."*

Definition 2.3. *[10] "The extended neighborhood corona $G_1 * G_2$ of two graphs G_1 and G_2 is a graph obtained by taking the neighborhood corona $G_1 \star G_2$ and joining each vertex of i^{th} copy of G_2 to every vertices of j^{th} copy of G_2 provided the vertices v_i and v_j are adjacent in G_1."*

Definition 2.4. *[11] "The identity extended corona $I_{ex}(G_1 \circ G_2)$ of two graphs G_1 and G_2 is the graph obtained by taking the corona $G_1 \circ G_2$ and joining the vertex v_{ik} of i^{th} copy of G_2 to the vertex v_{jk} of j^{th} copy of G_2 provided the vertices v_i and v_j are adjacent in G_1."*

Definition 2.5. *[11] "The identity extended neighborhood corona $I_{ex}(G_1 \star G_2)$ of two graphs G_1 and G_2 is the graph obtained by taking the neighborhood corona $G_1 \star G_2$ and joining the vertex v_{ik} of i^{th} copy of G_2 to the vertex v_{jk} of j^{th} copy of G_2 provided the vertices v_i and v_j are adjacent in G_1."*

In this paper, suppose that J denotes the $a \times b$ matrix all entries equal to one and J' is obtained by replacing every entry of $J_{a\times b}$ by zero except the first element of the first row.
For a matrix M of order n we denote 1_n and $J_{m\times n}$ the column vector of size n and matrix of size $m \times n$ with all the entries equal to one . M- coronal [4, 6] $\Gamma_M(\lambda)$ is defined to be the sum of entries of matrix $(\lambda I - M)^{-1}$ ie

$$\Gamma_M(\lambda) = 1_n^T (\lambda I - M)^{-1} 1_n.$$

If M has constant row sum t then $\Gamma_M(\lambda) = \frac{n}{\lambda - t}$

The kronecker product [6] $A \otimes B$ of two matrices $A = (a_{ij})_{n\times m}$ and $B = (b_{ij})_{p\times q}$ is the $np \times mq$ matrix obtained from A by repalcing each element a_{ij} by $a_{ij}B$. Then

1. $\otimes$ is an associative operation.
2. $(A \otimes B)^T = A^T \otimes B^T$
3. $(A \otimes B)^{-1} = A^{-1} \otimes B^{-1}$
4. If A is an $n \times n$ and B is an $p \times p$, then $det(A \otimes B) = (detA)^p (detB)^n$

Lemma 2.1. *[4] Let M_1, M_2, M_3, M_4 be respectively $p \times p$, $p \times q$, $q \times p$, $q \times q$ matrices with M_1 and M_4 invertible. Then*

$$det \begin{pmatrix} M_1 & M_2 \\ M_3 & M_4 \end{pmatrix} = det(M_4) \cdot det\left(M_1 - M_2 {M_4}^{-1} M_3\right)$$
$$= det(M_1) \cdot det\left(M_4 - M_3 {M_1}^{-1} M_2\right)$$

where $M_1 - M_2{M_4}^{-1}M_3$ and $M_4 - M_3{M_1}^{-1}M_2$ are called Schur complement of M_4 and M_1 respectively.

All graphs considered in this paper are simple.

3. Randic Spectra of neighborhood corona

If the Adjacency spectra of two graphs G_1 and G_2 are known then we calculate the Randic spectra of their neighbourhood corona $G_1 \star G_2$.

Theorem 3.1. *Let G_1 be a r_1 regular graph on n_1 vertices and G_2 be a r_2 -regular graph on n_2 vertices. Then the characteristic polynomial of Randic matrix of neighbourhood corona of G_1 and G_2 $G_1 \star G_2$ is*

$$\phi\left(R(G_1 \star G_2);x\right) = \prod_{i=1}^{n_2}\left(\phi\left(\frac{A(G_2)}{r_1+r_2};x\right)\right)^{n_1}\prod_{i=1}^{n_1}\left(x - \frac{\lambda_i(G_1)}{r_1(1+n_2)} - \frac{\lambda_i(G_1)^2}{(r_2+r_1)r_1(1+n_2)}\frac{n_2(r_1+r_2)}{(x(r_1+r_2)-r_2)}\right)$$

Proof. In the neighborhood corona the vertices of G_1 will have degree $r_1(1+n_2)$ and G_2 will have degree r_1+r_2. Then by the suitable labeling of the vertices $G_1 \star G_2$ the Randic matrix $R(G)$ can be written as follows:

$$R(G) = R(G_1 \star G_2) = \begin{pmatrix} \frac{A(G_1)}{r_1(1+n_2)} & \frac{A(G_1)}{\sqrt{(r_2+r_1)r_1(1+n_2)}} \otimes J_{1\times n_2} \\ \frac{A(G_1)}{\sqrt{(r_2+r_1)r_1(1+n_2)}} \otimes J_{n_2\times 1} & I_{n_1} \otimes \frac{A(G_2)}{r_1+r_2} \end{pmatrix}$$

$$det(xI_n - R(G)) = det\begin{pmatrix} xI_{n_1} - \frac{A(G_1)}{r_1(1+n_2)} & -\frac{A(G_1)}{\sqrt{(r_2+r_1)r_1(1+n_2)}} \otimes J_{1\times n_2} \\ -\frac{A(G_1)}{\sqrt{(r_2+r_1)r_1(1+n_2)}} \otimes J_{n_2\times 1} & I_{n_1} \otimes \left(xI_{n_2} - \frac{A(G_2)}{r_1+r_2}\right) \end{pmatrix}$$

$$= det\left(I_{n_1} \otimes \left(xI_{n_2} - \frac{A(G_2)}{r_1+r_2}\right)\right).det\left(\left(xI_{n_1} - \frac{A(G_1)}{r_1(1+n_2)}\right) - \frac{A(G_1)}{\sqrt{(r_2+r_1)r_1(1+n_2)}} \otimes J_{1\times n_2}\left(xI_{n_1} \otimes \left(xI_{n_2} - \frac{A(G_2)}{r_1+r_2}\right)\right)^{-1}. \frac{A(G_1)}{\sqrt{(r_2+r_1)r_1(1+n_2)}} \otimes J_{n_2\times 1}\right)$$

$$= \left(det\left(xI_{n_2} - \frac{A(G_2)}{r_1+r_2}\right)\right)^{n_1} det\left(xI_{n_1} - \frac{A(G_1)}{r_1(1+n_2)} - \frac{A(G_1)^2}{(r_2+r_1)r_1(1+n_2)}\Gamma_{\frac{A(G_2)}{r_1+r_2}}(x)\right)$$

$$= \prod_{i=1}^{n_2}\left(\phi\left(\frac{A(G_2)}{r_1+r_2};x\right)\right)^{n_1}\prod_{i=1}^{n_1}\left(x - \frac{\lambda_i(G_1)}{r_1(1+n_2)} - \frac{\lambda_i(G_1)^2}{(r_2+r_1)r_1(1+n_2)}\frac{n_2(r_1+r_2)}{(x(r_1+r_2)-r_2)}\right)$$

□

4. Randic Spectra of extended corona

In this section, if the Adjacency spectra of two graphs are known then we calculate the Randic spectra of the extended corona of these graphs.

Theorem 4.1. *Let G_1 be a r_1 regular graph on n_1 vertices and G_2 be a r_2 -regular graph on n_2 vertices. Then the characteristic polynomial of Randic matrix of G_1 extended corona G_2, $G_1 \bullet G_2$ is*

$$\phi\left(R(G_1 \bullet G_2);x\right) = \prod_{i=2}^{n_2}\left(x - \frac{\lambda_i(G_2)}{r_2+n_2r_1+1}\right)\prod_{i=1}^{n_1}\left(x^2 - x\left(\frac{\lambda_i(G_1)}{r_1+n_2} + \frac{r_2+n_2\lambda_i(G_1)}{r_2+n_2r_1+1}\right) + \frac{r_2\lambda_i(G_1)+n_2\left(\lambda_i(G_1)^2-1\right)}{(r_1+n_2)(r_2+n_2r_1+1)}\right)$$

Proof. In the graph $G_1 \circ G_2$, the vertices of G_1 will have degree r_1+n_2 and G_2 will have degree r_2+1. Hence in the graph G_1 extended corona G_2, $G_1 \bullet G_2$, G_1 has degree r_1+n_2 and G_2 has degree $r_1n_2+r_2+1$. Then by the suitable labeling of the vertices $G_1 \bullet G_2$ the Randic matrix $R(G)$ can be written as follows:

$$R(G) = R(G_1 \bullet G_2) = \begin{pmatrix} \frac{A(G_1)}{r_1+n_2} & \frac{I_{n_1}}{\sqrt{(r_1+n_2)(r_2+n_2r_1+1)}} \otimes J_{1\times n_2} \\ \frac{I_{n_1}}{\sqrt{(r_1+n_2)(r_2+n_2r_1+1)}} \otimes J_{n_2\times 1} & I_{n_1} \otimes \frac{A(G_2)}{r_2+n_2r_1+1} + \frac{A(G_1)}{r_2+n_2r_1+1} \otimes J_{n_2\times n_2} \end{pmatrix}$$

Since G_2 is a r_2 -regular graph $\frac{A(G_2)}{r_1+r_2+n_2r_1}$ is diagonalizable, $R(G) = R(G_1 \bullet G_2)$ is similar to

$$B = \begin{pmatrix} \frac{A(G_1)}{r_1+n_2} & -\frac{I_{n_1}}{\sqrt{(r_1+n_2)(r_2+n_2r_1+1)}} \otimes \sqrt{n_2}J'_{1\times n_2} \\ -\frac{I_{n_1}}{\sqrt{(r_1+n_2)(r_2+n_2r_1+1)}} \otimes \sqrt{n_2}J'_{1\times n_2} & I_{n_1} \otimes \frac{D(G_2)}{r_2+n_2r_1+1} + \frac{A(G_1)}{r_2+n_2r_1+1} \otimes n_2J'_{n_2\times n_2} \end{pmatrix}$$

So,

$$detx(xI - R(G)) = det(xI - B)$$

Expanding $|xI-B|$ by Laplace's method of expansion, we get

$$\left|I_{n_1}\otimes diag\left(x-\frac{\lambda_2(G_2)}{r_2+n_2r_1+1},\ldots,x-\frac{\lambda_{n_2}(G_2)}{r_2+n_2r_1+1}\right)\right|\begin{vmatrix} xI_{n_1}-\frac{A(G_1)}{r_1+n_2} & \frac{\sqrt{n_2}I_{n_1}}{\sqrt{(r_1+n_2)(r_2+n_2r_1+1)}} \\ \frac{\sqrt{n_2}I_{n_1}}{\sqrt{(r_1+n_2)(r_2+n_2r_1+1)}} & (x-\frac{r_2}{r_2+n_2r_1+1})I_{n_1}-\frac{n_2A(G_1)}{r_2+n_2r_1+1}\end{vmatrix}$$

Simialrly $A(G_1)$ is diagonalizable, we get

$$\begin{vmatrix} xI_{n_1}-\frac{A(G_1)}{r_1+n_2} & \frac{\sqrt{n_2}I_{n_1}}{\sqrt{(r_1+n_2)(r_2+n_2r_1+1)}} \\ \frac{\sqrt{n_2}I_{n_1}}{\sqrt{(r_1+n_2)(r_2+n_2r_1+1)}} & (x-\frac{r_2}{r_2+n_2r_1+1})I_{n_1}-\frac{n_2A(G_1)}{r_2+n_2r_1+1}\end{vmatrix}=\begin{vmatrix} xI_{n_1}-\frac{D(G_1)}{r_1+n_2} & \frac{\sqrt{n_2}I_{n_1}}{\sqrt{(r_1+n_2)(r_2+n_2r_1+1)}} \\ \frac{\sqrt{n_2}I_{n_1}}{\sqrt{(r_1+n_2)(r_2+n_2r_1+1)}} & (x-\frac{r_2}{r_2+n_2r_1+1})I_{n_1}-\frac{n_2D(G_1)}{r_2+n_2r_1+1}\end{vmatrix}$$

Where $D(G_1)=diag\left(\lambda_1(G_1),\lambda_2(G_1),\ldots,\lambda_n(G_2)\right)$
Thus

$$\begin{vmatrix} xI_{n_1}-\frac{A(G_1)}{r_1+n_2} & \frac{\sqrt{n_2}I_{n_1}}{\sqrt{(r_1+n_2)(r_2+n_2r_1+1)}} \\ \frac{\sqrt{n_2}I_{n_1}}{\sqrt{(r_1+n_2)(r_2+n_2r_1+1)}} & (x-\frac{r_2}{r_2+n_2r_1+1})I_{n_1}-\frac{n_2A(G_1)}{r_2+n_2r_1+1}\end{vmatrix}=det\left(xI_{n_1}-\frac{D(G_1)}{r_1+n_2}\right).$$

$$det\left((x-\frac{r_2}{r_2+n_2r_1+1})I_{n_1}-\frac{n_2D(G_1)}{r_2+n_2r_1+1}-\frac{\sqrt{n_2}I_{n_1}}{\sqrt{(r_1+n_2)(r_2+n_2r_1+1)}}\left(xI_{n_1}-\frac{D(G_1)}{r_1+n_2}\right)^{-1}\frac{\sqrt{n_2}I_{n_1}}{\sqrt{(r_1+n_2)(r_2+n_2r_1+1)}}\right)$$

$$=x^2-x\left(\frac{\lambda_i(G_1)}{r_1+n_2}+\frac{r_2}{r_2+n_2r_1+1}+\frac{n_2\lambda_i(G_1)}{r_2+n_2r_1+1}\right)+\frac{r_2\lambda_i(G_1)}{(r_1+n_2)(r_2+n_2r_1+1)}+\frac{n_2\left(\lambda_i(G_1)^2-1\right)}{(r_1+n_2)(r_2+n_2r_1+1)}$$

$$=x^2-x\left(\frac{\lambda_i(G_1)}{r_1+n_2}+\frac{r_2+n_2\lambda_i(G_1)}{r_2+n_2r_1+1}\right)+\frac{r_2\lambda_i(G_1)+n_2\left(\lambda_i(G_1)^2-1\right)}{(r_1+n_2)(r_2+n_2r_1+1)}$$

Thus the Randic characteristic polynomial of G_1 extended corona G_2, $G_1\bullet G_2$ is

$$\phi\left(R(G_1\bullet G_2);x\right)=\prod_{i=2}^{n_2}\left(x-\frac{\lambda_i(G_2)}{r_2+n_2r_1+1}\right)\prod_{i=1}^{n_1}\left(x^2-x\left(\frac{\lambda_i(G_1)}{r_1+n_2}+\frac{r_2+n_2\lambda_i(G_1)}{r_2+n_2r_1+1}\right)+\frac{r_2\lambda_i(G_1)+n_2\left(\lambda_i(G_1)^2-1\right)}{(r_1+n_2)(r_2+n_2r_1+1)}\right)$$

□

5. Randic Spectra of extended neighborhood corona

In this section, we determine the Randic spectra of extended neighborhood corona of two graphs if the adjacency spectra of the graph is known.

Theorem 5.1. *Let G_1 be a r_1 regular graph on n_1 vertices and G_2 be a r_2 -regular graph on n_2 vertices. Then the characteristic polynomial of Randic matrix of $G_1\star G_2$ is*

$$\phi\left(R(G_1*G_2);x\right)==\prod_{1\le i\le n_1}\prod_{2\le j\le n_2}\left(x-\frac{\lambda_j(G_2)}{(r_1+r_2+r_1n_2)}-\frac{\lambda_i(G_1)}{(r_1+r_1n_2)}\right)$$
$$\cdot\prod_{1\le i\le n_1}\left(x^2-\left(2\frac{\lambda_i(G_1)}{(r_1+r_1n_2)}+\frac{r_2}{r_1+r_2+r_1n_2}\right)x+r_2\frac{\lambda_i(G_1)}{(r_1+r_1n_2)}+(1-n_2)\frac{\lambda_i^2(G_1)}{(r_1+r_1n_2)^2}\right)$$

Proof. In the extended neighborhood corona the vertices of G_1 will have degree $r_1+r_1n_2$ and G_2 will have degree $r_1+r_2+r_1n_2$.Then by suitable labeling of the vertices G_1*G_2 the Randic matrix $R(G)$ can be written as follows:

$$R(G)=R(G_1*G_2)=R(I_{ex}(G_1\circ G_2))=\begin{pmatrix}\frac{A(G_1)}{r_1+r_1n_2} & \frac{A(G_1)}{\sqrt{(r_1+r_1n_2)(r_1+r_2+r_1n_2)}}\otimes\sqrt{n_2}J'_{1\times n_2} \\ \frac{A(G_1)}{\sqrt{(r_1+r_1n_2)(r_1+r_2+r_1n_2)}}\otimes\sqrt{n_2}J'_{n_2\times 1} & I_{n_1}\otimes\frac{A(G_2)}{r_1+r_2+r_1n_2}+\frac{A(G_1)}{r_1+r_1n_2}\otimes n_2J'_{n_2\times n_2}\end{pmatrix}$$

Since G_2 is a r_2 -regular graph $\frac{A(G_2)}{r_1+r_2+r_1n_2}$ is diagonalizable, $R(G)=R(G_1*G_2)$ is similar to

$$B=\begin{pmatrix}\frac{A(G_1)}{r_1+r_1n_2} & \frac{A(G_1)}{\sqrt{(r_1+r_1n_2)(r_1+r_2+r_1n_2)}}\otimes\sqrt{n_2}J'_{1\times n_2} \\ \frac{A(G_1)}{\sqrt{(r_1+r_1n_2)(r_1+r_2+r_1n_2)}}\otimes\sqrt{n_2}J'_{n_2\times 1} & I_{n_1}\otimes\frac{D(G_2)}{r_1+r_2+r_1n_2}+\frac{A(G_1)}{r_1+r_1n_2}\otimes n_2J'_{n_2\times n_2}\end{pmatrix}$$

So,

$$detx(xI-R(G))=det(xI-B)$$

Expanding $|xI-B|$ by Laplace's method of expansion we get

$$\prod_{1\le i\le n_1}\prod_{2\le j\le n_2}\left(x-\frac{\lambda_j(G_2)}{(r_1+r_2+r_1n_2)}-\frac{\lambda_i(G_1)}{(r_1+r_1n_2)}\right)\begin{vmatrix} xI_{n_1}-\frac{A(G_1)}{r_1+r_1n_2} & \frac{\sqrt{n_2}A(G_1)}{\sqrt{(r_1+r_1n_2)(r_1+r_2+r_1n_2)}} \\ \frac{\sqrt{n_2}A(G_1)}{\sqrt{(r_1+r_1n_2)(r_1+r_2+r_1n_2)}} & (x-\frac{r_2}{r_1+r_2+r_1n_2})I_{n_1}-\frac{n_2A(G_1)}{r_1+r_1n_2}\end{vmatrix}$$

Similarly $A(G_1)$ is diagonalizable, we get

$$det\,(xI - R(G_1 * G_2)) = \prod_{1\le i\le n_1}\prod_{2\le j\le n_2}\left(x - \frac{\lambda_j(G_2)}{(r_1+r_2+r_1n_2)} - \frac{\lambda_i(G_1)}{(r_1+r_1n_2)}\right)$$
$$\cdot\begin{vmatrix} xI_{n_1} - \frac{D(G_1)}{r_1+r_1n_2} & \frac{\sqrt{n_2}D(G_1)}{\sqrt{(r_1+r_1n_2)(r_1+r_2+r_1n_2)}} \\ \frac{\sqrt{n_2}D(G_1)}{\sqrt{(r_1+r_1n_2)(r_1+r_2+r_1n_2)}} & (x - \frac{r_2}{r_1+r_2+r_1n_2})I_{n_1} - \frac{n_2D(G_1)}{r_1+r_1n_2}\end{vmatrix}$$
$$= \prod_{1\le i\le n_1}\prod_{2\le j\le n_2}\left(x - \frac{\lambda_j(G_2)}{(r_1+r_2+r_1n_2)} - \frac{\lambda_i(G_1)}{(r_1+r_1n_2)}\right)$$
$$\cdot\prod_{1\le i\le n_1}\left(x^2 - \left(2\frac{\lambda_i(G_1)}{(r_1+r_1n_2)} + \frac{r_2}{r_1+r_2+r_1n_2}\right)x + r_2\frac{\lambda_i(G_1)}{(r_1+r_1n_2)} + (1-n_2)\frac{\lambda_i^2(G_1)}{(r_1+r_1n_2)^2}\right)$$

□

6. Randic Spectra of Identity extended corona

In this section we determine the Randic spectra of Identity extended corona of two graphs if the adjacency spectra of the graphs are known.

Theorem 6.1. *Let G_1 be a r_1 regular graph on n_1 vertices and G_2 be a r_2 -regular graph on n_2 vertices. Then the characteristic polynomial of Randic matrix of $I_{ex}(G_1 \circ G_2)$ is*

$$\phi\,(R(I_{ex}(G_1\circ G_2));x) = \prod_{j=1}^{n_1}\prod_{i=2}^{n_2}\left(x - \frac{\lambda_i(G_1)+\lambda_j(G_2)}{r_1+r_2+1}\right)$$
$$\prod_{i=1}^{n_1}\left(x^2 - x\left(\frac{\lambda_i(G_1)}{r_1+r_2+1} + \frac{\lambda_i(G_1)}{r_1+n_2} + \frac{r_2}{r_1+r_2+1}\right) + \frac{r_2\lambda_i(G_1)+\lambda_i(G_1)^2 - n_2}{(r_1+r_2+1)(r_1+n_2)}\right)$$

Proof. In the Identity extended corona, the vertices of G_1 will have degree r_1+n_2 and G_2 will have degree r_1+r_2+1. The Randic matrix of $I_{ex}(G_1\circ G_2)$ can be expressed in the form:

$$R(G) = R(I_{ex}(G_1\circ G_2) = \begin{pmatrix} \frac{A(G_1)}{r_1+n_2} & \frac{I_{n_1}}{\sqrt{(r_1+n_2)(r_1+r_2+1)}}\otimes J_{1\times n_2} \\ \frac{I_{n_1}}{\sqrt{(r_1+n_2)(r_1+r_2+1)}}\otimes J_{n_2\times 1} & I_{n_1}\otimes\frac{A(G_2)}{r_1+r_2+1} + \frac{A(G_1)}{r_1+r_2+1}\otimes I_{n_2}\end{pmatrix}$$

Since $A(G_2)$ is a real Hermitian matrix, it is orthogonally diagonalizable and Since G_2 is r_2-regular, $A(G_2) = PD(G_2)P^T$ where P is a square matrix of order n_2 and $PP^T = I_{n_2}$ and $D(G_2) = diag(r_2, \lambda_2(G_2), \lambda_3(G_2), \ldots, \lambda_{n_2}(G_2))$.

$$R(G) = R(I_{ex}(G_1\circ G_2) = \begin{pmatrix} I_{n_1}\otimes P & 0 \\ 0 & I_{n_1}\end{pmatrix}\begin{pmatrix} \frac{A(G_1)}{r_1+n_2} & \frac{I_{n_1}}{\sqrt{(r_1+n_2)(r_1+r_2+1)}}\otimes J_{1\times n_2} \\ \frac{I_{n_1}}{\sqrt{(r_1+n_2)(r_1+r_2+1)}}\otimes J_{n_2\times 1} & I_{n_1}\otimes\frac{D(G_2)}{r_1+r_2+1} + \frac{A(G_1)}{r_1+r_2+1}\otimes I_{n_2}\end{pmatrix}\begin{pmatrix} I_{n_1}\otimes P^T & 0 \\ 0 & I_{n_1}\end{pmatrix}$$

Thus $R(I_{ex}(G_1\circ G_2)$ is similar to

$$B = \begin{pmatrix} \frac{A(G_1)}{r_1+n_2} & \frac{I_{n_1}}{\sqrt{(r_1+n_2)(r_1+r_2+1)}}\otimes J_{1\times n_2} \\ \frac{I_{n_1}}{\sqrt{(r_1+n_2)(r_1+r_2+1)}}\otimes J_{n_2\times 1} & I_{n_1}\otimes\frac{D(G_2)}{r_1+r_2+1} + \frac{A(G_1)}{r_1+r_2+1}\otimes I_{n_2}\end{pmatrix}$$

. Thus

$$det\,((xI - R(I_{ex}(G_1\circ G_2)) = det\,((xI - B))$$

$$det(xI-B) = \begin{pmatrix} xI_{n_1} - \frac{A(G_1)}{r_1+n_2} & \frac{-I_{n_1}}{\sqrt{(r_1+n_2)(r_1+r_2+1)}}\otimes\sqrt{n_2}J'_{1\times n_2} \\ \frac{-I_{n_1}}{\sqrt{(r_1+n_2)(r_1+r_2+1)}}\otimes\sqrt{n_2}J'_{n_2\times 1} & I_{n_1}\otimes\left(xI_{n_2} - \frac{D(G_2)}{r_1+r_2+1}\right) - \frac{A(G_1)}{r_1+r_2+1}\otimes I_{n_2}\end{pmatrix}$$

By using Laplace expansion

$$det(xI-B) = det\left(I_{n_1}\otimes diag\left(x - \frac{\lambda_2(G_2)}{r_1+r_2+1}, x - \frac{\lambda_3(G_2)}{r_1+r_2+1}, \ldots x - \frac{\lambda_{n_2}(G_2)}{r_1+r_2+1}\right) - \frac{A(G_1)}{r_1+r_2+1}\otimes I_{n_2-1}\right)$$
$$\begin{vmatrix} xI_{n_1} - \frac{A(G_1)}{r_1+n_2} & \frac{-\sqrt{n_2}I_{n_1}}{\sqrt{(r_1+n_2)(r_1+r_2+1)}} \\ \frac{-\sqrt{n_2}I_{n_1}}{\sqrt{(r_1+n_2)(r_1+r_2+1)}} & \left(x - \frac{r_2}{r_1+r_2+1}\right)I_{n_1} - \frac{A(G_1)}{r_1+r_2+1}\end{vmatrix}$$

Since $A(G_1)$ is diaganalizable, we get

$$det(xI-B)=\prod_{i=2}^{n_2} det\left(\left(x-\frac{\lambda_i(G_2)}{r_1+r_2+1}\right)-\frac{A(G_1)}{r_1+r_2+1}\right)$$

$$\begin{vmatrix} xI_{n_1}-\frac{D(G_1)}{r_1+n_2} & \frac{-\sqrt{n_2}I_{n_1}}{\sqrt{(r_1+n_2)(r_1+r_2+1)}} \\ \frac{-\sqrt{n_2}I_{n_1}}{\sqrt{(r_1+n_2)(r_1+r_2+1)}} & \left(x-\frac{r_2}{r_1+r_2+1}\right)I_{n_1}-\frac{D(G_1)}{r_1+r_2+1} \end{vmatrix}$$

$$=\prod_{i=2}^{n_2} det\left(\left(x-\frac{\lambda_i(G_2)}{r_1+r_2+1}\right)-\frac{A(G_1)}{r_1+r_2+1}\right)$$

$$det\left(xI_{n_1}-\frac{D(G_1)}{r_1+n_2}\right)\prod_{i=1}^{n_1}\left(x-\frac{r_2}{r_1+r_2+1}-\frac{\lambda_i(G_1)}{r_1+r_2+1}-\frac{n_2}{(r_1+n_2)(r_1+r_2+1)}\frac{1}{\left(x-\frac{\lambda_i(G_1)}{r_1+n_2}\right)}\right)$$

$$=\prod_{i=2}^{n_2}\prod_{j=1}^{n_1}\left(x-\frac{\lambda_i(G_2)}{r_1+r_2+1}-\frac{\lambda_j(G_1)}{r_1+r_2+1}\right).$$

$$\prod_{i=1}^{n_1}\left(x-\frac{\lambda_i(G_1)}{r_1+n_2}\right)\prod_{i=2}^{n_1}\left(x-\frac{r_2}{r_1+r_2+1}-\frac{\lambda_i(G_1)}{r_1+r_2+1}-\frac{n_2}{(r_1+n_2)(r_1+r_2+1)}\frac{1}{\left(x-\frac{\lambda_i(G_1)}{r_1+n_2}\right)}\right)$$

$$=\prod_{i=2}^{n_2}\prod_{j=1}^{n_1}\left(x-\frac{\lambda_i(G_2)}{r_1+r_2+1}-\frac{\lambda_j(G_1)}{r_1+r_2+1}\right).$$

$$\prod_{i=1}^{n_1}\left(x^2-x\left(\frac{\lambda_i(G_1)}{r_1+r_2+1}+\frac{\lambda_i(G_1)}{r_1+n_2}+\frac{r_2}{r_1+r_2+1}\right)+\frac{r_2\lambda_i(G_1)+\lambda_i(G_1)^2-n_2}{(r_1+r_2+1)(r_1+n_2)}\right)$$

Finally

$$\phi\left(R(I_{ex}(G_1\circ G_2));x\right)=\prod_{i=2}^{n_2}\prod_{j=1}^{n_1}\left(x-\frac{\lambda_i(G_1)+\lambda_i(G_2)}{r_1+r_2+1}\right)$$
$$\prod_{i=1}^{n_1}\left(x^2-x\left(\frac{\lambda_i(G_1)}{r_1+r_2+1}+\frac{\lambda_i(G_1)}{r_1+n_2}+\frac{r_2}{r_1+r_2+1}\right)+\frac{r_2\lambda_i(G_1)+\lambda_i(G_1)^2-n_2}{(r_1+r_2+1)(r_1+n_2)}\right)$$

□

7. Randic Spectra of Identity extended neighborhood corona

In this section we determine the Randic spectra of Identity extended corona of two graphs if the adjacency spectra of the graphs are known.

Theorem 7.1. *Let G_1 be a r_1 regular graph on n_1 vertices and G_2 be a r_2 -regular graph on n_2 vertices. Then the characteristic polynomial of Randic matrix of $I_{ex}(G_1\circ G_2)$ is*

$$\phi\left(R(I_{ex}(G_1\circ G_2));x\right)=\prod_{i=2}^{n_2}\prod_{j=1}^{n_1}\left(x-\frac{\lambda_i(G_2)}{2r_1+r_2}-\frac{\lambda_j(G_1)}{2r_1+r_2}\right)$$
$$\prod_{i=1}^{n_1}\left(x^2-x\left(\frac{r_2+\lambda_i(G_1)}{2r_1+r_2}-\frac{\lambda_i(G_1)}{r_1(1+n_2)}+\frac{r_2\lambda_i(G_1)+\lambda_i^2(G_1)}{2r_1+r_2}\right)-\frac{n_2}{r_1(1+n_2)(2r_1+r_2)}\right)$$

Proof. In the Identity extended neighbourhood corona, the vertices of G_1 will have degree $r_1(1+n_2)$ and G_2 will have degree $2r_1+r_2$. The Randic matrix of $I_{ex}(G_1\circ G_2)$ can be expressed in the form:

$$R(G)=R(I_{ex}(G_1\circ G_2)=\begin{pmatrix} \frac{A(G_1)}{r_1(1+n_2)} & \frac{I_{n_1}}{\sqrt{r_1(1+n_2)(2r_1+r_2)}}\otimes J_{1\times n_2} \\ \frac{I_{n_1}}{\sqrt{r_1(1+n_2)(2r_1+r_2)}}\otimes J_{n_2\times 1} & I_{n_1}\otimes\frac{A(G_2)}{2r_1+r_2}+\frac{A(G_1)}{2r_1+r_2}\otimes I_{n_2} \end{pmatrix}$$

Since $A(G_2)$ is a real Hermitian matrix, it is orthogonally diagonalizable and Since G_2 is r_2-regular, $A(G_2)=PD(G_2)P^T$ where P is a square matrix of order n_2.

$$R(G)=\begin{pmatrix} \frac{A(G_1)}{r_1(1+n_2)} & \frac{I_{n_1}}{\sqrt{r_1(1+n_2)(2r_1+r_2)}}\otimes J_{1\times n_2} \\ \frac{I_{n_1}}{\sqrt{r_1(1+n_2)(2r_1+r_2)}}\otimes J_{n_2\times 1} & I_{n_1}\otimes\frac{A(G_2)}{2r_1+r_2}+\frac{A(G_1)}{2r_1+r_2}\otimes I_{n_2} \end{pmatrix}$$

$$=\begin{pmatrix} I_{n_1}\otimes P & 0 \\ 0 & I_{n_1}\end{pmatrix}\begin{pmatrix} \frac{A(G_1)}{r_1(1+n_2)} & \frac{I_{n_1}}{\sqrt{r_1(1+n_2)(2r_1+r_2)}}\otimes J_{1\times n_2} \\ \frac{I_{n_1}}{\sqrt{r_1(1+n_2)(2r_1+r_2)}}\otimes J_{n_2\times 1} & I_{n_1}\otimes\frac{D(G_2)}{2r_1+r_2}+\frac{A(G_1)}{2r_1+r_2}\otimes I_{n_2} \end{pmatrix}\begin{pmatrix} I_{n_1}\otimes P^T & 0 \\ 0 & I_{n_1}\end{pmatrix}$$

Therefore,

$$det(xI-R) = det\begin{pmatrix} xI_{n_1} - \frac{A(G_1)}{r_1(1+n_2)} & -\frac{\sqrt{n_2}I_{n_1}}{\sqrt{r_1(1+n_2)(2r_1+r_2)}} \otimes J'_{1\times n_2} \\ -\frac{\sqrt{n_2}I_{n_1}}{\sqrt{r_1(1+n_2)(2r_1+r_2)}} \otimes J'_{n_2\times 1} & I_{n_1}\otimes\left(xI_{n_2} - \frac{D(G_2)}{2r_1+r_2}\right) - \frac{A(G_1)}{2r_1+r_2}\otimes I_{n_2} \end{pmatrix}$$

$$= det\left(I_{n-1}\otimes diag\left(x-\frac{\lambda_2(G_2)}{2r_1+r_2}, x-\frac{\lambda_3(G_2)}{2r_1+r_2}, \ldots, x-\frac{\lambda_{n-2}(G_2)}{2r_1+r_2}\right) - \frac{A(G_1)}{2r_1+r_2}\otimes I_{n_2}\right)$$

$$det\begin{pmatrix} xI_{n_1} - \frac{A(G_1)}{r_1(1+n_2)} & -\frac{\sqrt{n_2}I_{n_1}}{\sqrt{r_1(1+n_2)(2r_1+r_2)}} \otimes J'_{1\times n_2} \\ -\frac{\sqrt{n_2}I_{n_1}}{\sqrt{r_1(1+n_2)(2r_1+r_2)}} \otimes J'_{n_2\times 1} & \left(x - \frac{r_2)}{2r_1+r_2}\right) I_{n_1} - \frac{A(G_1)}{2r_1+r_2} \end{pmatrix}$$

Since $A(G_1)$ is diagonalizable

$$det(xI-R) = \prod_{i=2}^{n_2}\left(\left(x-\frac{\lambda_i(G_2)}{2r_1+r_2}\right) - \frac{A(G_1)}{2r_1+r_2}\right)$$

$$det\begin{pmatrix} xI_{n_1} - \frac{D(G_1)}{r_1(1+n_2)} & -\frac{\sqrt{n_2}I_{n_1}}{\sqrt{r_1(1+n_2)(2r_1+r_2)}} \\ -\frac{\sqrt{n_2}I_{n_1}}{\sqrt{r_1(1+n_2)(2r_1+r_2)}} & \left(x - \frac{r_2}{2r_1+r_2}\right) I_{n_1} - \frac{D(G_1)}{2r_1+r_2} \end{pmatrix}$$

$$= \prod_{i=2}^{n_2}\left(\left(x-\frac{\lambda_i(G_2)}{2r_1+r_2}\right) - \frac{A(G_1)}{2r_1+r_2}\right)$$

$$det\left(xI_{n_1} - \frac{D(G_1)}{r_1(1+n_2)}\right)\prod_{i=1}^{n_1}\left(x - \frac{r_2+\lambda_i(G_1)}{2r_1+r_2} - \frac{n_2}{r_1(1+n_2)(2r_1+r_2)}\frac{1}{\left(x-\frac{\lambda_(G_1)}{r_1(1+n_2)}\right)}\right)$$

$$= \prod_{i=2}^{n_2}\prod_{j=1}^{n_1}\left(x-\frac{\lambda_i(G_2)}{2r_1+r_2} - \frac{\lambda_j(G_1)}{2r_1+r_2}\right)$$

$$\prod_{i=1}^{n_1}\left(x-\frac{\lambda_i(G_1)}{r_1(1+n_2)}\right)\prod_{i=1}^{n_1}\left(x - \frac{r_2+\lambda_i(G_1)}{2r_1+r_2} - \frac{n_2}{r_1(1+n_2)(2r_1+r_2)}\frac{1}{\left(x-\frac{\lambda_(G_1)}{r_1(1+n_2)}\right)}\right)$$

$$= \prod_{i=2}^{n_2}\prod_{j=1}^{n_1}\left(x-\frac{\lambda_i(G_2)}{2r_1+r_2} - \frac{\lambda_j(G_1)}{2r_1+r_2}\right)$$

$$\prod_{i=1}^{n_1}\left(x^2 - x\left(\frac{r_2+\lambda_i(G_1)}{2r_1+r_2} - \frac{\lambda_i(G_1)}{r_1(1+n_2)} + \frac{r_2\lambda_i(G_1)+\lambda_i^2(G_1)}{2r_1+r_2}\right) - \frac{n_2}{r_1(1+n_2)(2r_1+r_2)}\right)$$

Hence,

$$\phi\left(R(I_{ex}(G_1\circ G_2));x\right) = \prod_{i=2}^{n_2}\prod_{j=1}^{n_1}\left(x-\frac{\lambda_i(G_2)}{2r_1+r_2} - \frac{\lambda_j(G_1)}{2r_1+r_2}\right)$$

$$\prod_{i=1}^{n_1}\left(x^2 - x\left(\frac{r_2+\lambda_i(G_1)}{2r_1+r_2} - \frac{\lambda_i(G_1)}{r_1(1+n_2)} + \frac{r_2\lambda_i(G_1)+\lambda_i^2(G_1)}{2r_1+r_2}\right) - \frac{n_2}{r_1(1+n_2)(2r_1+r_2)}\right)$$

□

Conclusion and Future Scope

The Randić spectrum constitutes an important extension of spectral graph theory, offering a degree-sensitive approach to analyzing graph structure. Its ability to reflect molecular branching, network heterogeneity, and structural irregularity makes it highly valuable in both theoretical investigations and applied research. As interest in graph-based models continues to grow, the Randić spectrum remains a significant analytical tool in chemistry, network theory, and mathematical graph characterization. In future, the study can be extended to new family of graphs which are still open. It helps expand research in various fields as referred above.

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