



Application of Adomian decomposition method to solving some kinds of partial differential equations and system of partial differential equations

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Abstract

In this article, the Adomian decomposition method (ADM) is used to construct the solution of the initial value problem of some kinds of partial differential equations of first order.

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1 Introduction

In the last years, the ADM has been applied to get the solutions of various kinds of problems of ODEs and PDEs [1]-[11]. Here, we use the Adomian decomposition method, to investigate partial differential equations and system of partial differential equations of first order. Using this last method, we get the exact solution.

2 The adomian decomposition method

2.1 About the adomian decomposition method [1], [2], [3], [4], [5]

Suppose that we need to solve the following equation:

$$Au = f$$

In a real Hilbert space H , where $A : H \rightarrow H$ a linear is or a nonlinear operator, $f \in H$ and u is the unknown. The principle of the ADM is based on the decomposition of the nonlinear operator A in the following form:

$$A = L + R + N \quad (2.1)$$

Where $L + R$ is linear, N nonlinear, L invertible with L^{-1} as inverse. Using that decomposition, equation (2.1) is equivalent to

$$u = \theta + L^{-1}f - L^{-1}Ru - L^{-1}Nu \quad (2.2)$$

Where θ verifies $L\theta = 0$. (2.2) is called the Adomian's fundamental equation or Adomian's canonical form. We look for the solution of (2.1) in a series expansion form $u = \sum_{n=0}^{+\infty} u_n$ and we consider $Nu = \sum_{n=0}^{+\infty} A_n$ where A_n are special polynomials of variables $u_0, u_1, \dots, u_n$ called Adomian polynomials and defined by [1], [2], [3], [4]:

$$A_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left[N \left(\sum_{i=0}^{+\infty} \lambda^i u_i \right) \right]_{\lambda=0} \quad n = 0, 1, 2, \dots$$

Where λ is a parameter used by "convenience". Thus (2.2) can be rewritten as follows:

$$\sum_{n=0}^{+\infty} u_n = \theta + L^{-1}f - L^{-1}R \left(\sum_{n=0}^{+\infty} u_n \right) - L^{-1} \left(\sum_{n=0}^{+\infty} A_n \right) \quad (2.3)$$

We suppose that the series $\sum_{n=0}^{+\infty} u_n$ and $\sum_{n=0}^{+\infty} A_n$ are convergent, and obtain by identification the Adomian algorithm:

$$\begin{cases} u_0 = \theta + L^{-1}f \\ u_1 = -L^{-1}(Ru_0) - L^{-1}A_0 \\ \dots \\ u_{n+1} = -L^{-1}(Ru_n) - L^{-1}A_n \end{cases} \quad (2.4)$$

In practice, it is often difficult to calculate all terms of an Adomian series, so we approach the series solution by the truncated series: $u = \sum_{i=0}^n u_i$, where the choice of n depends on error requirements.

3 Applications

3.1 Example 1

Using the Adomian decomposition method, let's construct the solution of the following partial differential equation:

$$\begin{cases} \frac{\partial u(x, y)}{\partial x} + \frac{\partial u(x, y)}{\partial y} = x + y \\ u(x, 0) = 0 \\ u(0, y) = 0 \end{cases} \quad (3.1)$$

Where

$$u(x, y) \in C^1(\Omega); \Omega \subset [0, T] \times [0, T] \quad (3.2)$$

Let's take

$$\begin{cases} L_x(\cdot) = \frac{\partial(\cdot)}{\partial x}; L_x^{-1}(\cdot) = \int_0^x (\cdot) ds \\ L_y(\cdot) = \frac{\partial(\cdot)}{\partial y}; L_y^{-1}(\cdot) = \int_0^y (\cdot) ds \end{cases} \quad (3.3)$$

From (3.1), we have:

$$\begin{cases} \frac{\partial u}{\partial x} = x + y - \frac{\partial u}{\partial y} \\ \frac{\partial u}{\partial y} = x + y - \frac{\partial u}{\partial x} \end{cases} \quad (3.4)$$

(3.4) give us:

$$\begin{cases} u(x, y) = u(0, y) + \int_0^x (s + y) ds - \int_0^x \frac{\partial u(s, y)}{\partial y} ds \\ \quad = \frac{x^2}{2} + xy - \int_0^x \frac{\partial u(s, x)}{\partial y} ds \\ u(x, y) = u(x, 0) + \int_0^y (x + s) ds - \int_0^y \frac{\partial u(x, s)}{\partial y} ds \\ \quad = \frac{y^2}{2} + xy - \int_0^y \frac{\partial u(x, s)}{\partial y} ds \end{cases} \quad (3.5)$$

From (3.5), we obtain:

$$u(x, y) = \frac{x^2}{4} + \frac{y^2}{4} + xy - \frac{1}{2} \int_0^x \frac{\partial u(s, y)}{\partial y} ds - \frac{1}{2} \int_0^y \frac{\partial u(x, s)}{\partial y} ds \quad (3.6)$$

Let us suppose that the solution $u(x, y)$ of the initial value problem (3.1) has the following form:

$$u(x, y) = \sum_{n=0}^{\infty} u_n(x, y) \quad (3.7)$$

From (3.6), we have the following Adomian algorithm:

$$\begin{cases} u_0(x, y) = \frac{x^2 + y^2}{4} + xy \\ u_{n+1}(x, y) = -\frac{1}{2} \int_0^x \frac{\partial u_n(s, y)}{\partial y} ds - \frac{1}{2} \int_0^y \frac{\partial u_n(x, s)}{\partial x} ds ; n \geq 0 \end{cases} \quad (3.8)$$

which gives us:

$$\begin{cases} u_1(x, y) = -\frac{x^2 + y^2}{4} - \frac{xy}{2} \\ u_2(x, y) = \frac{x^2 + y^2}{8} + \frac{xy}{2} \\ u_3(x, y) = -\frac{x^2 + y^2}{8} - \frac{xy}{4} \\ u_4(x, y) = \frac{x^2 + y^2}{16} + \frac{xy}{4} \\ u_5(x, y) = -\frac{x^2 + y^2}{16} - \frac{xy}{8} \\ \dots \end{cases}$$

and further terms can easily be calculated using a symbolic computation package.

Thus the solution of (3.1) is:

$$\begin{aligned} u(x, y) &= \sum_{n=0}^{\infty} u_n(x, y) = \left(xy + \frac{x^2 + y^2}{4} \right) + \left(-\frac{x^2 + y^2}{4} - \frac{xy}{2} \right) + \left(\frac{x^2 + y^2}{8} + \frac{xy}{2} \right) + \\ &\quad \left(-\frac{x^2 + y^2}{8} - \frac{xy}{4} \right) + \left(\frac{x^2 + y^2}{16} + \frac{xy}{4} \right) + \left(-\frac{x^2 + y^2}{16} - \frac{xy}{8} \right) + \dots \\ &= xy \end{aligned} \quad (3.9)$$

3.2 Example 2

Let us consider the following initial value problem

$$\begin{cases} \frac{\partial u(x, y)}{\partial x} + \frac{\partial u(x, y)}{\partial y} = x^2 + y^2 + 4xy \\ u(x, 0) = 0 \\ u(0, y) = 0 \end{cases} \quad (3.10)$$

Where

$$u(x, y) \in C^1(\Omega); \Omega \subset [0, T] \times [0, T] \quad (3.11)$$

Let's take

$$\begin{cases} L_x(\cdot) = \frac{\partial(\cdot)}{\partial x}; L_x^{-1}(\cdot) = \int_0^x (\cdot) ds \\ L_y(\cdot) = \frac{\partial(\cdot)}{\partial y}; L_y^{-1}(\cdot) = \int_0^y (\cdot) ds \end{cases} \quad (3.12)$$

From (3.10), we have:

$$\begin{cases} \frac{\partial u}{\partial x} = x^2 + y^2 + 4xy - \frac{\partial u}{\partial y} \\ \frac{\partial u}{\partial y} = x^2 + y^2 + 4xy - \frac{\partial u}{\partial x} \end{cases} \quad (3.13)$$

(3.13) give us:

$$\begin{cases} u(x, y) = u(0, y) + \int_0^x (s^2 + 4sy + y^2) ds - \int_0^x \frac{\partial u(s, y)}{\partial x} ds \\ \Rightarrow u(x, y) = \frac{x^3}{3} + 2x^2y + y^2x - \int_0^x \frac{\partial u(s, y)}{\partial x} ds \\ u(x, y) = u(x, 0) + \int_0^y (x + s) ds - \int_0^y \frac{\partial u(x, s)}{\partial y} ds \\ \Rightarrow u(x, y) = \frac{y^3}{3} + 2xy^2 + x^2y - \int_0^y \frac{\partial u(x, s)}{\partial y} ds \end{cases} \quad (3.14)$$

From (3.14), we obtain:

$$u(x, y) = \frac{1}{6}(x^3 + y^3) + \frac{3}{2}(y^2x + x^2y) - \frac{1}{2} \int_0^x \frac{\partial u(s, y)}{\partial x} ds - \frac{1}{2} \int_0^y \frac{\partial u(x, s)}{\partial y} ds \quad (3.15)$$

Let us suppose that the solution $u(x, y)$ of the initial value problem (3.10) has the following form:

$$u(x, y) = \sum_{n=0}^{\infty} u_n(x, y). \quad (3.16)$$

From (3.15), we have the following Adomian algorithm

$$\begin{cases} u_0(x, y) = \frac{1}{6}(x^3 + y^3) + \frac{3}{2}(y^2x + x^2y) \\ u_{n+1}(x, y) = -\frac{1}{2} \int_0^x \frac{\partial u_n(s, y)}{\partial x} ds - \frac{1}{2} \int_0^y \frac{\partial u_n(x, s)}{\partial y} ds ; n \geq 0 \end{cases} \quad (3.17)$$

Which gives us:

$$\begin{cases} u_1(x, y) = -\frac{3}{12}(x^3 + y^3) - \frac{4}{4}(y^2x + x^2y) \\ u_2(x, y) = \frac{4}{24}(x^3 + y^3) + \frac{7}{8}(y^2x + x^2y) \\ u_3(x, y) = -\frac{7}{48}(x^3 + y^3) - \frac{11}{16}(y^2x + x^2y) \\ u_4(x, y) = \frac{11}{96}(x^3 + y^3) + \frac{18}{32}(y^2x + x^2y) \\ u_5(x, y) = -\frac{18}{192}(x^3 + y^3) - \frac{29}{64}(y^2x + x^2y) \\ \dots \end{cases}$$

and the further terms can easily be calculated using a symbolic computation package.

We notice that:

$$u_n(x, y) = v_n(x, y) + w_n(x, y) \quad (3.18)$$

Where

$$v_n(x, y) = \frac{(-1)^n}{3 \cdot 2^{n+1}} t^n \times A(x, y) \quad (3.19)$$

$$w_n(x, y) = \frac{(-1)^n}{2^{n+1}} l_n \times B(x, y)$$

$$\begin{aligned} A(x, y) &= x^3 + y^3 \\ B(x, y) &= y^2x + x^2y \end{aligned} \quad (3.20)$$

(t_n) And (l_n) are the series which satisfy the following relations:

$$\begin{cases} t_{n+2} - t_{n+1} = t_n \\ t_0 = 1 \\ t_1 = 4 \end{cases} \quad (3.21)$$

$$\begin{cases} l_{n+2} - l_{n+1} = l_n \\ l_0 = 3 \\ l_1 = 4 \end{cases} \quad (3.22)$$

The solutions of (3.21) and (3.22) are respectively:

$$t_n = \left(\frac{1-\sqrt{5}}{2} \right)^{n+1} + \left(\frac{1+\sqrt{5}}{2} \right)^{n+1} \quad (3.23)$$

and

$$l_n = \frac{3-\sqrt{5}}{2} \left(\frac{1-\sqrt{5}}{2} \right)^n + \frac{3+\sqrt{5}}{2} \left(\frac{1+\sqrt{5}}{2} \right)^n \quad (3.24)$$

thus

$$v_n(x, y) = \frac{(-1)^n A_n(x, y)}{3} \left[\frac{(1-\sqrt{5})^{n+1}}{2^{2n+2}} + \frac{(1+\sqrt{5})^{n+1}}{2^{2n+2}} \right] \quad (3.25)$$

$$w_n(x, y) = (-1)^n B(x, y) \left[(3-\sqrt{5}) \frac{(1-\sqrt{5})^n}{2^{2n+2}} + (3+\sqrt{5}) \frac{(1+\sqrt{5})^n}{2^{2n+2}} \right] \quad (3.26)$$

The solution of (3.10) is:

$$\begin{aligned} u(x, y) &= \sum_{n=0}^{+\infty} u_n(x, y) = \sum_{n=0}^{+\infty} (u_{2n}(x, y) + u_{2n+1}(x, y)) \\ &= \sum_{n=0}^{+\infty} [(v_{2n}(x, y) + w_{2n}(x, y)) + ((v_{2n+1}(x, y) + w_{2n+1}(x, y)))] \end{aligned} \quad (3.27)$$

We notice that:

$$\begin{aligned} \sum_{n=0}^{+\infty} v_{2n}(x, y) &= \frac{A(x, y)}{3} \left[\sum_{n=0}^{+\infty} \frac{(1-\sqrt{5})^{2n+1}}{2^{4n+2}} + \sum_{n=0}^{+\infty} \frac{(1+\sqrt{5})^{2n+1}}{2^{4n+2}} \right] \\ &= \frac{A(x, y)}{3} \left[\frac{1-\sqrt{5}}{4} \sum_{n=0}^{+\infty} \left[\left(\frac{1-\sqrt{5}}{4} \right)^2 \right]^n + \frac{1+\sqrt{5}}{4} \sum_{n=0}^{+\infty} \left[\left(\frac{1+\sqrt{5}}{4} \right)^2 \right]^n \right] \\ &= \frac{A(x, y)}{3} \left\{ \frac{1-\sqrt{5}}{4} \lim_{n \rightarrow +\infty} \frac{1 - \left[\left(\frac{1-\sqrt{5}}{4} \right)^2 \right]^n}{1 - \left(\frac{1-\sqrt{5}}{4} \right)^2} + \frac{1+\sqrt{5}}{4} \lim_{n \rightarrow +\infty} \frac{1 - \left[\left(\frac{1+\sqrt{5}}{4} \right)^2 \right]^n}{1 - \left(\frac{1+\sqrt{5}}{4} \right)^2} \right\} \\ &= \frac{A(x, y)}{3} \left\{ \frac{1-\sqrt{5}}{4} \times \frac{1}{1 - \left(\frac{1-\sqrt{5}}{4} \right)^2} + \frac{1+\sqrt{5}}{4} \times \frac{1}{1 - \left(\frac{1+\sqrt{5}}{4} \right)^2} \right\} \\ &= \frac{A(x, y)}{3} \left(\frac{5-3\sqrt{5}}{5} + \frac{5+3\sqrt{5}}{5} \right) = \frac{2A(x, y)}{3} = \frac{2}{3} (x^3 + y^3) \end{aligned}$$

We obtain:

$$\begin{aligned}
\sum_{n=0}^{+\infty} v_{2n}(x, y) &= \frac{2}{3}(x^3 + y^3) \\
\sum_{n=0}^{+\infty} w_{2n}(x, y) &= B(x, y) \left[(3 - \sqrt{5}) \sum_{n=0}^{+\infty} \frac{(1 - \sqrt{5})^{2n}}{2^{4n+2}} + (3 + \sqrt{5}) \sum_{n=0}^{+\infty} \frac{(1 + \sqrt{5})^{2n}}{2^{4n+2}} \right] \\
&= B(x, y) \left[\frac{3 - \sqrt{5}}{4} \sum_{n=0}^{+\infty} \left[\left(\frac{1 - \sqrt{5}}{4} \right)^2 \right]^n + \frac{3 + \sqrt{5}}{4} \sum_{n=0}^{+\infty} \left[\left(\frac{1 + \sqrt{5}}{4} \right)^2 \right]^n \right] \\
&= B(x, y) \left\{ \frac{3 - \sqrt{5}}{4} \lim_{n \rightarrow +\infty} \frac{1 - \left[\left(\frac{1 - \sqrt{5}}{4} \right)^2 \right]^{n+1}}{1 - \left(\frac{1 - \sqrt{5}}{4} \right)^2} + \frac{3 + \sqrt{5}}{4} \lim_{n \rightarrow +\infty} \frac{1 - \left[\left(\frac{1 + \sqrt{5}}{4} \right)^2 \right]^{n+1}}{1 - \left(\frac{1 + \sqrt{5}}{4} \right)^2} \right\} \\
&= B(x, y) \left\{ \frac{3 - \sqrt{5}}{4} \times \frac{1}{1 - \left(\frac{1 - \sqrt{5}}{4} \right)^2} + \frac{3 + \sqrt{5}}{4} \times \frac{1}{1 - \left(\frac{1 + \sqrt{5}}{4} \right)^2} \right\} \\
&= B(x, y) \left(2 \times \frac{5 - 3\sqrt{5}}{5} + 2 \times \frac{5 + 3\sqrt{5}}{5} \right) = 4B(x, y) = 4(y^2x + x^2y)
\end{aligned} \tag{3.28}$$

We obtain:

$$\begin{aligned}
\sum_{n=0}^{+\infty} w_{2n}(x, y) &= 4(y^2x + x^2y) \\
\sum_{n=0}^{+\infty} v_{2n+1}(x, y) &= -\frac{A(x, y)}{3} \left[\sum_{n=0}^{+\infty} \frac{(1 - \sqrt{5})^{2n+2}}{2^{4n+4}} + \sum_{n=0}^{+\infty} \frac{(1 + \sqrt{5})^{2n+2}}{2^{4n+4}} \right] \\
&= -\frac{A(x, y)}{3} \left[\left(\frac{1 - \sqrt{5}}{4} \right)^2 \sum_{n=0}^{+\infty} \left[\left(\frac{1 - \sqrt{5}}{4} \right)^2 \right]^n + \left(\frac{1 + \sqrt{5}}{4} \right)^2 \sum_{n=0}^{+\infty} \left[\left(\frac{1 + \sqrt{5}}{4} \right)^2 \right]^n \right] \\
&= -\frac{A(x, y)}{3} \left\{ \left(\frac{1 - \sqrt{5}}{4} \right)^2 \lim_{n \rightarrow +\infty} \frac{1 - \left[\left(\frac{1 - \sqrt{5}}{4} \right)^2 \right]^{n+1}}{1 - \left(\frac{1 - \sqrt{5}}{4} \right)^2} + \left(\frac{1 + \sqrt{5}}{4} \right)^2 \lim_{n \rightarrow +\infty} \frac{1 - \left[\left(\frac{1 + \sqrt{5}}{4} \right)^2 \right]^{n+1}}{1 - \left(\frac{1 + \sqrt{5}}{4} \right)^2} \right\} \\
&= -\frac{A(x, y)}{3} \left\{ \left(\frac{1 - \sqrt{5}}{4} \right)^2 \times \frac{1}{1 - \left(\frac{1 - \sqrt{5}}{4} \right)^2} + \left(\frac{1 + \sqrt{5}}{4} \right)^2 \times \frac{1}{1 - \left(\frac{1 + \sqrt{5}}{4} \right)^2} \right\} \\
&= -\frac{A(x, y)}{3} \left(\frac{5 - 3\sqrt{5}}{5} + \frac{5 + 3\sqrt{5}}{5} \right) = -\frac{2A(x, y)}{3} = -\frac{2}{3}(x^3 + y^3) \\
\sum_{n=0}^{+\infty} v_{2n+1}(x, y) &= -\frac{2}{3}(x^3 + y^3)
\end{aligned} \tag{3.29}$$

$$\sum_{n=0}^{+\infty} v_{2n+1}(x, y) = -\frac{2}{3}(x^3 + y^3) \tag{3.30}$$

$$\begin{aligned}
\sum_{n=0}^{+\infty} w_{2n+1}(x, y) &= -B(x, y) \left[(3-\sqrt{5}) \sum_{n=0}^{+\infty} \frac{(1-\sqrt{5})^{2n+1}}{2^{4n+4}} + (3+\sqrt{5}) \sum_{n=0}^{+\infty} \frac{(1+\sqrt{5})^{2n+1}}{2^{4n+4}} \right] \\
&= -B(x, y) \left[\frac{(3-\sqrt{5})(1-\sqrt{5})}{16} \sum_{n=0}^{+\infty} \left[\left(\frac{1-\sqrt{5}}{4} \right)^2 \right]^n + \frac{(3+\sqrt{5})(1+\sqrt{5})}{16} \sum_{n=0}^{+\infty} \left[\left(\frac{1+\sqrt{5}}{4} \right)^2 \right]^n \right] \\
&= -B(x, y) \left\{ \frac{(3-\sqrt{5})(1-\sqrt{5})}{4} \lim_{n \rightarrow +\infty} \left(\frac{1 - \left[\left(\frac{1-\sqrt{5}}{4} \right)^2 \right]^{n+1}}{1 - \left(\frac{1-\sqrt{5}}{4} \right)^2} \right) \right\} + \\
&\quad B(x, y) \left\{ \frac{(3+\sqrt{5})(1+\sqrt{5})}{4} \lim_{n \rightarrow +\infty} \left(\frac{1 - \left[\left(\frac{1+\sqrt{5}}{4} \right)^2 \right]^{n+1}}{1 - \left(\frac{1+\sqrt{5}}{4} \right)^2} \right) \right\} \\
&= -B(x, y) \left\{ \frac{(3-\sqrt{5})(1-\sqrt{5})}{4} \times \frac{1}{1 - \left(\frac{1-\sqrt{5}}{4} \right)^2} + \frac{(3+\sqrt{5})(1+\sqrt{5})}{4} \times \frac{1}{1 - \left(\frac{1+\sqrt{5}}{4} \right)^2} \right\} \\
&= -B(x, y) \left(\frac{15-7\sqrt{5}}{10} + \frac{15+7\sqrt{5}}{10} \right) = -3B(x, y) = -3(y^2x + x^2y) \\
\sum_{n=0}^{+\infty} w_{2n+1}(x, y) &= -3(y^2x + x^2y) \tag{3.31}
\end{aligned}$$

Thus the solution of (3.10) is:

$$u(x, y) = \frac{2}{3}(x^3 + y^3) + 4(y^2x + x^2y) - \frac{2}{3}(x^3 + y^3) - 3(y^2x + x^2y) = y^2x + x^2y.$$

3.3 Example 3

Let us consider the following initial value problem

$$\begin{cases} \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = -2v \\ \frac{\partial v}{\partial t} + \frac{\partial v}{\partial x} = 2u \\ u(0, x) = \cos x \\ v(0, x) = \sin x \end{cases} \tag{3.32}$$

Where

$$u(t, x) \in C^1(\Omega) ; \Omega \subset [0, T] \times [0, T] \tag{3.33}$$

Let's take

$$L_t(.) = \frac{\partial(.)}{\partial t} ; L_x^{-1}(.) = \int_0^t (.) ds \tag{3.34}$$

From (3.32), we have:

$$\begin{aligned}
 u(t, y) &= \sum_{n=0}^{+\infty} u_n(t, x) = \sum_{n=0}^{+\infty} u_{2n}(t, x) + \sum_{n=0}^{+\infty} u_{2n+1}(t, x) \\
 &= \left(1 - \frac{t^2}{2!} + \frac{t^4}{4!} - \frac{t^6}{6!} + \dots + (-1)^n \frac{t^{2n}}{(2n)!} \right) \cos x - \\
 &\quad \left(t - \frac{t^3}{3!} + \frac{t^5}{5!} - \frac{t^7}{7!} + \dots + (-1)^n \frac{t^{2n+1}}{(2n+1)!} \right) \sin x \\
 &= \cos t \cos x - \sin t \sin x
 \end{aligned}
 \quad \left\{ \begin{aligned}
 u(t, x) &= u(0, x) - \int_0^t \frac{\partial u(s, x)}{\partial x} ds - 2 \int_0^t v(s, x) ds \\
 &= \cos x - \int_0^t \frac{\partial u(s, x)}{\partial x} ds - 2 \int_0^t v(s, x) ds \\
 v(t, x) &= v(0, x) - \int_0^t \frac{\partial v(s, x)}{\partial x} ds + 2 \int_0^t u(s, x) ds \\
 &= \sin x - \int_0^t \frac{\partial v(s, x)}{\partial x} ds + 2 \int_0^t u(s, x) ds
 \end{aligned} \right.$$

(3.35)

Let us suppose that the solution $u(t, x)$ and $v(t, x)$ of the initial value problem (3.32) has the following form:

$$\begin{cases}
 u(t, x) = \sum_{n=0}^{\infty} u_n(t, x) \\
 v(t, x) = \sum_{n=0}^{\infty} v_n(t, x)
 \end{cases} \quad (3.36)$$

From (3.35), we have the following Adomian algorithm:

$$\begin{cases}
 u_0(x, y) = \cos x \\
 u_{n+1}(x, y) = - \int_0^t \frac{\partial u_n(s, x)}{\partial x} ds - 2 \int_0^t v_n(s, x) ds ; n \geq 0
 \end{cases} \quad (3.37)$$

And

$$\begin{cases}
 v_0(x, y) = \sin x \\
 v_{n+1}(x, y) = - \int_0^t \frac{\partial v_n(s, x)}{\partial x} ds + 2 \int_0^t u_n(s, x) ds ; n \geq 0
 \end{cases} \quad (3.38)$$

(3.37) and (3.38) give us:

$$\begin{cases}
 u_1(t, x) = -t \sin x \\
 u_2(t, x) = -\frac{1}{2!} t^2 \cos x \\
 u_3(t, x) = -\frac{1}{3!} t^3 \sin x \\
 u_4(t, x) = \frac{1}{4!} t^4 \cos x \\
 \dots
 \end{cases}$$

And

And further terms can easily be calculated using a symbolic computation package. Thus

$$u(t, x) = \cos(t + x) \quad (3.39)$$

And

$$\begin{aligned}
 v(t, y) &= \sum_{n=0}^{+\infty} v_n(t, x) = \sum_{n=0}^{+\infty} v_{2n}(t, x) + \sum_{n=0}^{+\infty} v_{2n+1}(t, x) \\
 &= \left(1 - \frac{t^2}{2!} + \frac{t^4}{4!} - \frac{t^6}{6!} + \dots + (-1)^n \frac{t^{2n}}{(2n)!} \right) \sin x + \\
 &\quad \left(t - \frac{t^3}{3!} + \frac{t^5}{5!} - \frac{t^7}{7!} + \dots + (-1)^n \frac{t^{2n+1}}{(2n+1)!} + \dots \right) \cos x \\
 &= \cos t \sin x + \sin t \cos x
 \end{aligned}$$

$$v(t, x) = \sin(t + x) \quad (3.40)$$

The solution of (3.32) is:

$$(u(t, x), v(t, x)) = (\cos(t + x), \sin(t + y)).$$

4 Conclusion

In this paper, we showed once through some examples the useful of the Adomian decomposition method to investigate differential and partial differential equations.

References

- [1] K. ABBAOUI, « Les fondements de la méthode décompositionnelle d'Adomian et application à la résolution de problèmes issus de la biologie et de la médecine. » Thèse de doctorat de l'Université Paris VI. Octobre 1995.
- [2] K. ABBAOUI and Y. CHERRUAULT, "Convergence of Adomian method applied to differential equations." *Math. Comput. Modelling* (28, 5), pp 103-109, 1994.
- [3] K. ABBAOUI and Y. CHERRUAULT, "The Decomposition method applied to nonlinear equations". *Math. Comput. Modelling* (20,9), pp60-73, 1994.
- [4] K. ABBAOUI and Y. CHERRUAULT. "The decomposition method applied to the Cauchy problem." *Kybernetes*,(28,1),pp 68-74, 1999.
- [5] N. NGARASTA, B. SOME, K. ABBAOUI and Y. CHERRUAULT, "New numerical study of Adomian method applied to a diffusion model." *Kybernetes*, Vol. 31, no1, pp61-75, 2002.
- [6] Nafey. A. H. « Perturbation Methods, Wiley, New York, 1973.
- [7] Gabriel BISSANGA, A.K NSEMI. " Application of Adomian decomposition method to solving the Van Der Pol equation and comparison with the regular perturbation method." *Proceedings of the Five international workshop on contemporary problems in mathematical physics, Cotonou-Benin*, eds. J. Govaerts, M. N. Houkounou (International Chair in Mathematical Physics and Applications, ICMP-UNESCO Chair, University of Abomey-Calavy, 072 BP 50 Cotonou, Republic of Benin, December 2008).
- [8] Pierre BAKI-TANGOU, Gabriel BISSANGA, « Application of Adomian decomposition method to solving the Duffing-Van Der Pol equation. *Communications in Mathematical Analysis*. Vol. 4. No. Pp 30-40(2008).
- [9] Chengri Jin, Mingzhu Liu, "A new modification of Adomian decomposition method for solving a kind of evolution equation". *Applied Mathematics and Computation* 169(2005) 953-962.
- [10] D. Lesnic, "A computational algebraic investigation of the decomposition method for time-dependent problems". *Applied Mathematics and Computation* 119(2001) 197-206.
- [11] D. Lesnic, "Blow-up solutions obtained using the decomposition method". *Chaos, Solitons* 28 (2006) 776-787.