International Journal of Accounting and Economics Studies, 12 (6) (2025) 203-219

International Journal of Accounting and Economics Studies

Website: www.sciencepubco.com/index.php/IJAES
https://doi.org/10.14419/amhmng68
Research paper

An Integrated Two-Echelon Supply Chain Inventory Model with Trade Credit, Product Deterioration and Carbon Emissions

Anamika Sharma, Geetanjali Sharma *

Banasthali Vidyapith, Rajasthan-304022, India
**Corresponding author E-mail: roma.kaushik@gmail.com*

Received: August 13, 2025, Accepted: August 22, 2025, Published: October 7, 2025

Abstract

This research aims to develop a sustainable supply chain framework that integrates energy-efficient practices with environmental responsibility. We investigate a two-echelon green supply chain, considering factors such as carbon emissions from waste disposal, product deterioration, transportation, and manufacturing processes. By implementing preservation technologies, product deterioration rates can be effectively mitigated. The supply chain model incorporates a credit period offered by the manufacturer to the supplier, who in turn offers a credit period to the retailer, serving as a competitive advantage to drive sales growth. This study extends the traditional two-echelon supply chain model by incorporating controllable deterioration, waste, and carbon emissions. An analytical solution is derived to optimize timing and preservation technology cost, yielding optimal results. A comprehensive numerical analysis and sensitivity analysis validate the proposed model, demonstrating its convexity through analytical and graphical representations. The primary objectives are to minimize overall carbon emissions and total cost. The proposed model offers a practical framework for optimizing supply chain integration, reducing costs, and minimizing carbon footprint.

Keywords: *Two Levels of Trade Credit; Supply Chain; Deterioration; Carbon Emission.*

1. Introduction

Collaboration and integration between supply chain participants have demonstrated significant potential in supply chain management for more than two decades. Global awareness of environmental sustainability motivates many researchers to find ways to meet environmental demands without compromising the environment's ability to support human well-being both now and in the future. Economic, social, and environmental growth, often called people, profit, and planet, respectively, are the main foci of sustainability. Numerous studies have included carbon emissions in inventory management. The ability to sustain is the most crucial component of becoming environmentally friendly. The overall integrated cost in this study provides carbon emissions and emission regulation costs. The analysis mentioned above also takes carbon taxation into account. Because demand and deterioration work together to reduce inventory flow, deterioration is one of the essential aspects that shouldn't be ignored. Evaporation, obsolescence, decay, spoiling, damage, dryness, and other processes that lower a product's quality and quantity are typically called degradation. Traditional inventory models made the premise that objects will maintain their physical attributes well throughout storage. However, this isn't always the case. Therefore, investing in preservation technology is crucial for preventing item deterioration, lowering financial losses, enhancing customer service, and boosting competition in the market. After this discovery, many researchers used preservation technology to create inventory models for decaying goods.
An inventory model depends on the idea that the supplier must reimburse the producer for the goods as soon as they are delivered. Today, however, it is normal to see that the producer will set over a certain amount of time, known as the trade credit period (TCP), to pay the suppliers for the goods at the full price that the producer pays them. If the money is repaid within the time frame the producer specifies, interest is typically not imposed. The study's contribution. It is evident from the literature review that no studies have been conducted by combining the following factors simultaneously: Controllable deterioration, trade credit period, and carbon emissions (manufacturing process, warehousing, transportation, storing, and waste activities) are the four components of the two-echelon inventory model. Thus, we aim to integrate these elements to create a more realistic representation. This model determines the best time, cost of preservation technology, and overall cost. This model evaluates the optimal time, the cost of preservation technology, and the total price. Furthermore, the mathematical software Mathematica 12 is used for numerical and sensitive analysis.

2. Literature Review

The increasing environmental concerns, particularly regarding carbon emissions, have significantly influenced supply chain management decisions. In two-echelon supply chains, where goods are distributed from manufacturers to retailers and eventually to customers, inventory management becomes even more complex due to factors such as product deterioration, credit periods, and demand variability. This literature review highlights key research contributions related to carbon emissions, inventory decisions, product deterioration, trade-credit policies, and demand dependencies. Carbon emissions are critical in modern supply chain management due to environmental regulations and increasing consumer awareness.

Numerous studies, such as the analysis of Chen et al. (2013) suggested that operational adjustment alone can potentially lead to substantial emissions reductions without incurring significant additional costs. Furthermore, they explored the applicability of our findings to various regulatory frameworks, including stringent carbon caps, carbon taxation, cap-and-offsets, and cap-and-price systems.

Chopra and Meindl (2016) emphasise the complexity of managing inventory in such systems due to the interdependencies between echelons. Furthermore, the impact of transportation emissions and inventory holding emissions has been studied extensively, with authors like Govindan et al. (2014) suggesting an integrated model for sustainable inventory management. Product deterioration, particularly in industries like pharmaceuticals, food, and chemicals, presents unique challenges for inventory control.

Santanu (2017) developed a supply chain inventory model for a deteriorating item, incorporating a two-level trade credit policy. The policy includes credit offered by suppliers to retailers and by retailers to end customers, with consideration for default risk. The proposed model assumes that demand is linearly dependent on both time and the credit facility offered by retailers.

More recent work by Taleizadeh et al. (2018) addresses controllable deterioration, where storage conditions and technologies can reduce spoilage rates, thus influencing inventory policies and carbon emissions. Trade-credit periods are commonly used as a financial incentive in supply chain transactions. Jaggi et al. (2008) investigated single-level trade-credit policies. These studies highlight how credit terms influence order quantities, cash flow, and the overall cost-effectiveness of supply chains. Incorporating environmental considerations, such as emissions, into trade-credit models is a growing area of interest. Time, selling price, and inventory visibility often influence supply chain demand patterns. Recent advancements in supply chain modelling have focused on integrating factors such as carbon emissions, deterioration, trade-credit terms, and dynamic demand.

Jaggi et al. (2020) considered optimal replenishing strategies for constant demand in various financial scenarios by considering a non-instantaneous deteriorating item under the progressive trade credit policy. This work aims to develop a cost function for various situations depending on the trade credit period in an economic environment. In the same year, Priyan et al. (2020) introduced a novel mathematical model that integrates transportation and storage costs with emissions to determine optimal operational adjustments for deteriorating products within a two-echelon inventory system framework. Notably, the deterioration rate is modelled as a triangular fuzzy number. The authors employed the extension principle to derive the membership function of the fuzzy total cost and subsequently applied the centroid defuzzification method to estimate the total cost in a fuzzy environment. An optimal dynamic decision-making problem is investigated by Mahata and Mahata (2021) for a retailer offering a single deteriorating product. They considered a demand rate that varies dynamically with both the on-hand inventory level and the length of the credit period offered to customers. Vandana et al. (2021) examined a two-echelon supply chain comprising a producer and supplier, incorporating a two-level trade-credit policy to optimize consumption. To minimize the integrated total inventory cost, we assume that demand is influenced by both the credit period and the selling price. Our analysis explores the cost function under two scenarios: credit period-dependent demand rate and selling price-dependent demand rate. Also in the same year, Dong et al. (2021) developed a two-echelon supply chain model comprising a low-carbon product manufacturer and a retailer. The study demonstrated that supply chain members can alleviate financing constraints by leveraging a portfolio financing approach, combining bank loans, trade credit, and asset-based securitization. The findings revealed that under a financing mode consisting of bank loans and trade credit, tax incentives can only motivate the manufacturer to reduce carbon emissions when consumers are highly price-sensitive to low-carbon products.

Handa et al. (2022) considered an integrated two-echelon green supply chain model with carbon emission from production, warehousing, transporting, deterioration of items, as well as disposing of waste. The deterioration rate is controlled by utilizing preservation technology investment and obtained the optimal solution in a quasi-closed form solution. In the same year, Mishra (2022) proposed a supplier retailer inventory model for deterioration with carbon emission-dependent demand, and considered three payment options: Preliminary, cash, and post-payment. A unified green supply chain inventory model is developed by Das et al. (2023) with capacity constraints, incorporating order-size dependent credit, all-units discount, and partial backordering. The model also accounts for carbon emissions during production, storage, and transportation, with the retailer bearing the costs of emission-related expenses. In the same year, a sustainable production inventory model for sweet products with preservation is developed by Shah et al. (2023) for diabetic individuals. Notably, investing in green technology yields significant benefits, including substantial reductions in carbon emissions and total costs. Additionally, investing in preservation measures effectively decreases deterioration costs, further optimizing supply chain efficiency.

Mahato and Gaur (2024) developed a two-echelon supply chain model, incorporating controllable deterioration rate, production disruption risk, and a Stackelberg game approach, where customer demand is influenced by price, stock level, and product freshness. The model also considers preservation technology investment to reduce deterioration and carbon tax regulation to minimize emissions, aiming to optimize supply chain decisions while considering customer preferences and environmental sustainability. Also in the same year, Jauhari et al. (2024) proposed an integrated supply chain inventory model under a competing retailer scenario. The demand is determined by the two aspects i.e., selling price and service level. This research provides valuable insights to empower decision makers in selecting optimal strategies to enhance the efficiency of the supply chain system, maximize product sales, and prioritize environmental considerations. Sebatjane (2024) developed an integrated inventory model for a cold chain supply network, encompassing warehouse and retail echelons. This innovative model incorporates investments in green technologies to reduce environmental footprint and adheres to carbon emission tax regulations, promoting environmental sustainability in supply chain management. Lu et al. (2024) presented a production-inventory model incorporating a carbon cap-and-trade policy and an advanced cash-credit (ACC) payment scheme. They explored optimal production and replenishment strategies, considering deteriorating raw materials, finished products, and an imperfect production system. The study revealed that under a two-stage trade credit and ACC scheme, a higher credit payment component can increase total profit. Furthermore, the authors demonstrated that lower interest charges, higher earned interest, or delayed payment schedules can lead to reduced carbon emissions. Mahato et al. (2024) developed an inventory model that integrates controllable deterioration, permissible delay in payment, and various carbon emissions regulations. The primary objective of this study was to demonstrate how business partners can benefit from reducing deterioration rates and strengthening their relationships while addressing environmental concerns. According to Chaudhary et al. (2025), an integrated supply chain model incorporating imperfect production and green products with carbon emissions can provide valuable insights into managing inventory and reducing environmental impact. Their study, published in RAIRO-Operations Research, highlights the importance of considering carbon emissions in supply chain decision-making. The authors propose a model that optimizes production and inventory management while minimizing environmental costs. This research contributes to the growing body of literature on sustainable supply chain management and provides a framework for companies to adopt environmentally friendly practices. Chaudhary et al. (2025) presented a sustainable supply chain inventory model that incorporates carbon emissions and investment in preservation technology under inflation. Their research, presented at the International Conference on Advances in Pure & Applied Mathematics (ICAPAM), highlights the importance of considering environmental sustainability and preservation technology in inventory management decisions. The study provides insights into the impact of inflation on sustainable supply chain practices and offers a valuable framework for companies to reduce their environmental footprint.

2.1. Literature review table

References	Deteriorating items	Preservation Technology	Carbon emission	Demand	Trade credit
Vandana et al. (2021)	time-dependent deterioration rate	Not considered	Not considered	Quadratic demand	Considered
Handa et al. (2022)	Constant rate	considered	considered	constant	considered
Shah et al. (2023)	considered	considered	considered	price-stock-dependent demand	Not considered
Mahato and Gaur (2024)	Constant rate	considered	considered	selling price, stock level, and freshness level of the fresh items	Not considered
Jauhari et al. (2024)	Constant rate	considered	considered	Inventory and freshness-dependent demand	Not considered
Sabathani (2024)	Not considered	Not considered	considered	Stock dependent	Not considered
Jauhari et al. (2024)	Not considered	Not considered	considered	Selling price and service level.	Not considered
Rinki Chaudhary et al. (2025)	Consider a constant rate	consider	considered	Price and green degree dependent	Not consider
Dipt Singh et al. (2025)	Constant rate	consider	consider	Deterministic	Not consider
Present paper	Different for different intervals	consider	consider	Price, stock, and time are dependent.	consider

Our present paper distinguishes itself from existing research by embracing a more nuanced and realistic approach to inventory management. Unlike prior studies that often assume a constant deterioration rate, we consider a time-varying rate that differs across intervals, capturing the complexity of real-world systems. Furthermore, our model incorporates preservation technology to mitigate deterioration, accounts for carbon emissions to address environmental concerns, and adopts a price, stock, and time-dependent demand function to reflect market dynamics. Notably, our research also considers trade credit, a crucial aspect often overlooked in previous studies. By integrating these distinctive elements, our paper provides a comprehensive framework that offers valuable insights for both practitioners and academics, setting itself apart as a significant contribution to the field.

3. Assumptions

The following assumptions are taken in this study-

1) Demand is considered as D_1 (p,t,I(t))=a-bp+ct+dI(t)=D+ct+dI(t),a,b,c,d>0
2) The Production rate is more dependent on demand and is more significant than demand, P=βD_1,β>0.
3) The deterioration rate is considered different for different intervals, such that θ_1 for 0≤t≤t_1 and θ_2 for t_1≤t≤t_2.
4) In "n" deliveries, items are transported by truck from one place to another.
5) The preservation technology investment is used to reduce the deterioration rate of items. Let m(μ)=1-e^(-μξ), where ξ is preservation technology investment and μ is a factor representing the percentage increase in m(μ).
6) Shortage is not allowed.
7) Logistic activities, production processes, warehousing of unsold items, storage of deteriorated items, and waste disposal can create carbon emissions.

4. Notations

The following notations are used for this study-

Table1. 2:

Symbol	Description	Symbol	Description
a	Scaling factor	b	Scaling factor
p_1	Selling price for the manufacturer	c	Scaling factor
d	Scaling factor	β	Production rate
O_m	Ordering cost for the manufacturer	C_{pm}	Production cost for the manufacturer
C_{hm}	Holding cost for manufacturer (unit/time)	C_{dm}	Deterioration cost for manufacturer (unit/time)
θ_1	Deterioration rate for $0 \leq t \leq t_1$	θ_2	Deterioration rate for $t_1 \leq t \leq T$
n	Number of shipments	ξ	Preservation technology cost
C_{hS}	Holding cost for suppliers (unit/time)	C_{dS}	Deterioration cost for suppliers (unit/time)
O_s	Ordering cost for suppliers	W_m	Fixed waste disposal cost per cycle (rupees/cycle)
W_{mv}	Variable waste disposal cost (rupees/cycle)	d_1	Distance (Km)
p_2	Selling price for retailer	μ	Factor representing the percentage increase in $m(\mu)$

W	Average weight of solid waste production (kg/unit)	C_T	Fixed transportation cost per delivery
C_t	Variable transportation cost per delivery	Q_m	Manufacture's ordering quantity (unit/time)
Q_s	Supplier's ordering quantity (unit/time)	e_2	The carbon emission cost of the vehicle
D_{se}	Carbon emission (kg/unit)	W_{me}	Energy consumption from manufacturer's inventory holding (KWH/unit)
W_{se}	Energy consumption from retailer's inventory holding (KWH/unit)	d_{me}	Carbon emission from deterioration of product (kg CO_2 /unit)
E_{mV}	Solid waste standard emission from disposal (kg CO_2/KWH)	E_e	Emission from electricity consumption (Ton CO_2/KWH)
F_e	Vehicle standard emission from fuel consumption (kg CO_2/unit)	T_X	Carbon emission tax rate (Rupees/Kg Co_2)
e_p	Average energy consumption during production (KWH/unit)	I_e	Interest earns
I_c	Interest paid	M	The credit period offered by the manufacturer to the supplier
N	The credit period offered by the supplier to the retailer	n	Number of shipments
T	Total cycle time	V	Length of manufacturer's replenishment cycle per delivery
ξ	Preservation technology investment (decision variable)	t_1	Cycle time where production stops (decision variable)

5. Mathematical Modelling

5.1. Manufacturer's model

Figure 1.1 shows manufacturing starts at t=0, and the quantity is zero. Till t=t_1 production reaches its maximum level Q_m at a rate of production β. After that, inventory decreased due to demand and deterioration; the inventory level became zero at t=t_2. The differential equations of the manufacturer model are written below-

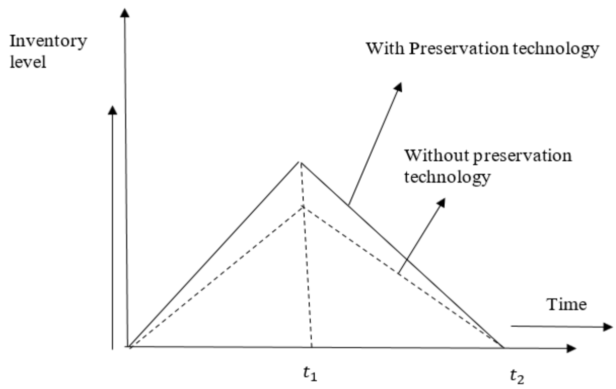

Fig. 1.1: Manufacturer Inventory Level Concerning Time.

Figure 1.1 illustrates the dynamics of inventory levels over time. Production commences at t=0, with inventory levels rising steadily as production ramps up to its maximum capacity Q_m at a rate β, reaching this peak at t=t_1. Beyond this point, inventory levels begin to decline due to the combined effects of demand and deterioration, ultimately dwindling to zero by t=t_2 marking the point at which production must be replenished to meet ongoing demand.

$$\frac{dI_1(t)}{dt} + \theta_1 I_1(t) = P - \left(a - bp + ct + dI_1(t)\right), 0 \le t \le t_1$$

$$\frac{dI_1(t)}{dt} + (\theta_1 - m(\mu) - \beta d + d)\, I_1(t) = (\beta - 1)(D + ct), 0 \le t \le t_1 \tag{1}$$

$$\frac{dI_2(t)}{dt} + (\theta_2 - m(\mu) + d) I_2(t) = -(D + ct), t_1 \le t \le T \tag{2}$$

With the boundary condition $I_1(0) = 0, I_2(T) = 0, I_1(t_1) = I_2(t_1) = Q_m$ (3)
The solution of these equations is-

$$I_1(t) = (\beta - 1)\left[\frac{D}{\theta_1 - m(\mu) - \beta d + d}\left(1 - e^{-(\theta_1 - m(\mu) - \beta d + d)t}\right) + \frac{ct}{\theta_1 - m(\mu) - \beta d + d} + \frac{c}{(\theta_1 - m(\mu) - \beta d + d)^2}\left(e^{-(\theta_1 - m(\mu) - \beta d + d)t} - 1\right)\right] \tag{4}$$

$$I_2(t) = \frac{D}{\theta_2 - m(\mu) + d}\left(e^{(\theta_2 - m(\mu) + d)(T - t)} - 1\right) + \frac{c}{\theta_2 - m(\mu) + d}\left(Te^{(\theta_2 - m(\mu) + d)(T - t)} - t\right) + \frac{c}{(\theta_2 - m(\mu) + d)^2}\left(1 - e^{(\theta_2 - m(\mu) + d)(T - t)}\right) \tag{5}$$

Calculate inventory cost for manufacturer's

Setup cost- $SC_m = O_m$ (6)

Production cost- $PC_m = (C_p + C_{Pe}) \int_0^{t_1} P\, dt = (C_p + C_{Pe})\left[\beta\left(Dt_1 + \frac{ct_1^2}{2}\right) + d\beta(\beta - 1)\left(\frac{D}{\theta_1 - m(\mu) - \beta d + d}\left(t_1 + \frac{e^{-(\theta_1 - m(\mu) - \beta d + d)t_1}}{\theta_1 - m(\mu) - \beta d + d} - \frac{1}{\theta_1 - m(\mu) - \beta d + d}\right)\right) + \frac{ct_1^2}{2(\theta_1 - m(\mu) - \beta d + d)} + \frac{c}{(\theta_1 - m(\mu) - \beta d + d)^2}\left(-\frac{e^{(\theta_1 - m(\mu) - \beta d + d)t_1}}{\theta_1 - m(\mu) - \beta d + d} - t_1 + \frac{1}{\theta_1 - m(\mu) - \beta d + d}\right)\right]$ (7)

Where $C_{Pe} = e_p E_e T_X$ is carbon emission cost produced by energy usages of the machine and handling operation in production
Where e_p= Average energy consumption during production (KWH/unit)
E_e = Emission from electricity consumption (ton Co_2/KWh)
T_X =Carbon emission tax rate (Rup/ Ton CO_2)

Holding cost- $$HC_m = (C_{hm} + C_{hme})\left[\int_0^{t_1} I_1(t)\,dt + \int_{t_1}^{T} I_2(t)\,dt\right] = (C_{hm} + C_{hme})\Bigg[(\beta - 1)\Bigg(\frac{D}{(\theta_1 - m(\mu) - \beta d + d)^2}\left(e^{-(\theta_1 - m(\mu) - \beta d + d)t_1} + t_1(\theta_1 - m(\mu) - \beta d + d) - 1\right) + \frac{ct_1^2}{2(\theta_1 - m(\mu) - \beta d + d)} + \frac{c}{((\theta_1 - m(\mu) - \beta d + d))^3}\left(\left(1 - e^{-(\theta_1 - m(\mu) - \beta d + d)}\right) - t_1(\theta_1 - m(\mu) - \beta d + d)\right)\Bigg) + \Bigg(\frac{D}{(\theta_2 - m(\mu) + d)^2}\left((t_1 - T)(\theta_2 - m(\mu) + d) + e^{(\theta_2 - m(\mu) + d)(T - t_1)} - 1\right) + \frac{c}{(\theta_2 - m(\mu) + d)^2}\left(\frac{t_1^2 - T^2}{2}(\theta_2 - m(\mu) + d) + e^{(\theta_2 - m(\mu) + d)(T - t_1)} - 1\right) + \frac{c}{(\theta_2 - m(\mu) + d)^3}\left((T - t_1)(\theta_2 - m(\mu) + d) + 1 - e^{(\theta_2 - m(\mu) + d)(T - t_1)}\right)\Bigg)\Bigg] \quad (8)$$

Deterioration cost- $$DC_m = (C_{dm} + C_{dme})\Bigg[(\beta - 1)(\theta_1 - (\mu))\Bigg(\frac{D}{(\theta_1 - m(\mu) - \beta d + d)^2}\left(e^{-(\theta_1 - m(\mu) - \beta d + d)t_1} - 1 + t_1(\theta_1 - m(\mu) - \beta d + d)\right) + \frac{ct_1^2}{2(\theta_1 - m(\mu) - \beta d + d)} + \frac{c}{((\theta_1 - m(\mu) - \beta d + d))^3}\left(\left(1 - e^{-(\theta_1 - m(\mu) - \beta d + d)}\right) - t_1(\theta_1 - m(\mu) - \beta d + d)\right)\Bigg) + (\theta_2 - m(\mu))\Bigg(\frac{D}{(\theta_2 - m(\mu) + d)^2}\left((t_1 - T)(\theta_2 - m(\mu) + d) + e^{(\theta_2 - m(\mu) + d)(T - t_1)} - 1\right) + \frac{c}{(\theta_2 - m(\mu) + d)^2}\left(\frac{t_1^2 - T^2}{2}(\theta_2 - m(\mu) + d) + e^{(\theta_2 - m(\mu) + d)(T - t_1)} - 1\right) + \frac{c}{(\theta_2 - m(\mu) + d)^3}\left((T - t_1)(\theta_2 - m(\mu) + d) + 1 - e^{(\theta_2 - m(\mu) + d)(T - t_1)}\right)\Bigg)\Bigg] \quad (9)$$

Preservation technology cost- $PTC_m = \int_0^T \xi\,dt = \xi T$ (10)

Waste disposal cost- $$WDC_m = W_m + W_{mv}\Bigg[\beta\left(Dt_1 + \frac{ct_1^2}{2}\right) + d\beta(\beta - 1)\Bigg(\frac{D}{\theta_1 - m(\mu) - \beta d + d}\left(t_1 + \frac{e^{(\theta_1 - m(\mu) - \beta d + d)t_1}}{\theta_1 - m(\mu) - \beta d + d} - \frac{1}{\theta_1 - m(\mu) - \beta d + d}\right)\Bigg) + \frac{ct_1^2}{2(\theta_1 - m(\mu) - \beta d + d)} + \frac{c}{(\theta_1 - m(\mu) - \beta d + d)^2}\left(-\frac{e^{(\theta_1 - m(\mu) - \beta d + d)t_1}}{\theta_1 - m(\mu) - \beta d + d} - t_1 + \frac{1}{\theta_1 - m(\mu) - \beta d + d}\right)\Bigg] \quad (11)$$

Transportation cost- $CT_m = C_T + C_t[(2d_1C_1 + d_1Q_MC_2)] + (2d_1e_1 + d_1Q_me_2)$

Where

$$Q_m = [I_1(t)]_{t = t_1} = (\beta - 1)\left[\frac{D}{\theta_1 - m(\mu) - \beta d + d}\left(1 - e^{-(\theta_1 - m(\mu) - \beta d + d)t_1}\right) + \frac{ct_1}{\theta_1 - m(\mu) - \beta d + d} + \frac{c}{(\theta_1 - m(\mu) - \beta d + d)^2}\left(e^{-(\theta_1 - m(\mu) - \beta d + d)t_1} - 1\right)\right]$$

The total average cost for the manufacturer is

$$TC_m = \frac{1}{T}[SC_m + PC_m + HC_m + DC_m + PTC_m + WDC_m + CT_m]$$

$$TC_m = \frac{1}{T}\Bigg[O_m + (C_p + C_{Pe})\Bigg[\beta\left(Dt_1 + \frac{ct_1^2}{2}\right) + d\beta(\beta - 1)\Bigg(\frac{D}{\theta_1 - m(\mu) - \beta d + d}\left(t_1 + \frac{e^{-(\theta_1 - m(\mu) - \beta d + d)t_1}}{\theta_1 - m(\mu) - \beta d + d} - \frac{1}{\theta_1 - m(\mu) - \beta d + d}\right)\Bigg) + \frac{ct_1^2}{2(\theta_1 - m(\mu) - \beta d + d)} + \frac{c}{(\theta_1 - m(\mu) - \beta d + d)^2}\left(-\frac{e^{(\theta_1 - m(\mu) - \beta d + d)t_1}}{\theta_1 - m(\mu) - \beta d + d} - t_1 + \frac{1}{\theta_1 - m(\mu) - \beta d + d}\right)\Bigg] + (C_{hm} + C_{hme})\Bigg[(\beta - 1)\Bigg(\frac{D}{(\theta_1 - m(\mu) - \beta d + d)^2}\left(e^{-(\theta_1 - m(\mu) - \beta d + d)t_1} + t_1(\theta_1 - m(\mu) - \beta d + d) - 1\right) + \frac{ct_1^2}{2(\theta_1 - m(\mu) - \beta d + d)} + \frac{c}{((\theta_1 - m(\mu) - \beta d + d))^3}\left(\left(1 - e^{-(\theta_1 - m(\mu) - \beta d + d)}\right) - t_1(\theta_1 - m(\mu) - \beta d + d)\right)\Bigg) + \Bigg(\frac{D}{(\theta_2 - m(\mu) + d)^2}\left((t_1 - T)(\theta_2 - m(\mu) + d) + e^{(\theta_2 - m(\mu) + d)(T - t_1)} - 1\right) + \frac{c}{(\theta_2 - m(\mu) + d)^2}\left(\frac{t_1^2 - T^2}{2}(\theta_2 - m(\mu) + d) + e^{(\theta_2 - m(\mu) + d)(T - t_1)} - 1\right) + \frac{c}{(\theta_2 - m(\mu) + d)^3}\left((T - t_1)(\theta_2 - m(\mu) + d) + 1 - e^{(\theta_2 - m(\mu) + d)(T - t_1)}\right)\Bigg)\Bigg] + (C_{dm} + C_{dme})\Bigg[(\beta - 1)(\theta_1 - m(\mu))\Bigg(\frac{D}{(\theta_1 - m(\mu) - \beta d + d)^2}\left(e^{-(\theta_1 - m(\mu) - \beta d + d)t_1} - 1 + t_1(\theta_1 - m(\mu) - \beta d + d)\right) + \frac{ct_1^2}{2(\theta_1 - m(\mu) - \beta d + d)} + \frac{c}{((\theta_1 - m(\mu) - \beta d + d))^3}\left(\left(1 - e^{-(\theta_1 - m(\mu) - \beta d + d)}\right) - t_1(\theta_1 - m(\mu) - \beta d + d)\right)\Bigg) + (\theta_2 - m(\mu))\Bigg(\frac{D}{(\theta_2 - m(\mu) + d)^2}\left((t_1 - T)(\theta_2 - m(\mu) + d) + e^{(\theta_2 - m(\mu) + d)(T - t_1)} - 1\right) + \frac{c}{(\theta_2 - m(\mu) + d)^2}\left(\frac{t_1^2 - T^2}{2}(\theta_2 - m(\mu) + d) + e^{(\theta_2 - m(\mu) + d)(T - t_1)} - 1\right) + \frac{c}{(\theta_2 - m(\mu) + d)^3}\left((T - t_1)(\theta_2 - m(\mu) + d) + 1 - e^{(\theta_2 - m(\mu) + d)(T - t_1)}\right)\Bigg)\Bigg] + \xi T + W_m + W_{mv}\Bigg[\beta\left(Dt_1 + \frac{ct_1^2}{2}\right) + d\beta(\beta - 1)\Bigg(\frac{D}{\theta_1 - m(\mu) - \beta d + d}\left(t_1 + \frac{e^{(\theta_1 - m(\mu) - \beta d + d)t_1}}{\theta_1 - m(\mu) - \beta d + d} - \frac{1}{\theta_1 - m(\mu) - \beta d + d}\right)\Bigg) + \frac{ct_1^2}{2(\theta_1 - m(\mu) - \beta d + d)} + \frac{c}{(\theta_1 - m(\mu) - \beta d + d)^2}\left(-\frac{e^{(\theta_1 - m(\mu) - \beta d + d)t_1}}{\theta_1 - m(\mu) - \beta d + d} - t_1 + \frac{1}{\theta_1 - m(\mu) - \beta d + d}\right)\Bigg] + C_T + C_t[(2d_1C_1 + d_1Q_MC_2)] + (2d_1e_1 + d_1Q_me_2)\Bigg]$$

6. Supplier's Model

The supplier's inventory behaviour with time is represented in Fig. Initially, the inventory level is Qs, and it starts declining due to the effect of the deterioration rate and demand rate. Here, the producer supplies the inventory to the supplier in n different shipments. The differential equations of the supplier's model are written below-

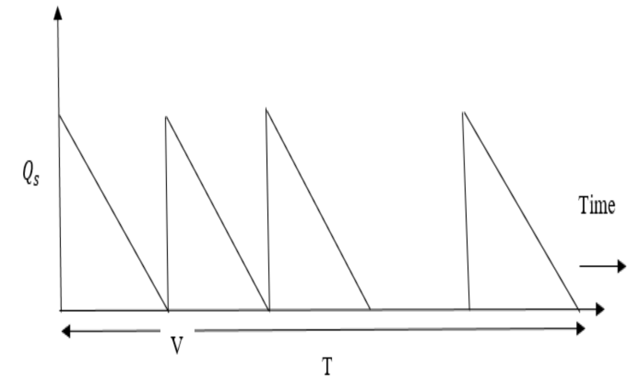

Fig. 1.2: Supplier's Inventory Model Concerning Time.

In Fig. 1.2 the supplier's inventory level begins at Qs and gradually depletes over time due to the dual impact of deterioration and demand. To maintain a steady supply chain, the producer replenishes the supplier's inventory in n discrete shipments.

$\frac{dI_s(t)}{dt} + (\theta_2 - m(\mu) + d)I_s(t) = -(D + ct), 0 \leq t \leq k$

Ordering cost- $OC_s = O_s$

Holding cost- $HC_s = (C_{hs} + C_{hse})n\int_0^V I_2(t)dt = (C_{hs} + C_{hse})n\left(\frac{D}{(\theta_2 - m(\mu)+d)^2}\left(-V(\theta_2 - m(\mu) + d) + e^{(\theta_2 - m(\mu)+d)V} - 1\right) + \frac{c}{(\theta_2 - m(\mu)+d)^2}\left(\frac{-V^2}{2}(\theta_2 - m(\mu) + d) + Ve^{(\theta_2 - m(\mu)+d)(V)} - V\right) + \frac{c}{(\theta_2 - m(\mu)+d)^3}\left((V)(\theta_2 - m(\mu) + d) + 1 - e^{(\theta_2 - m(\mu)+d)(V)}\right)\right)$

Deterioration cost- $DC_s = n(C_{ds} + C_{dse})\int_0^V (\theta_2 - m(\mu))I_2(t)\,dt = n(C_{ds} + C_{dse})(\theta_2 - m(\mu))\left(\frac{D}{(\theta_2 - m(\mu)+d)^2}\left(-V(\theta_2 - m(\mu) + d) + e^{(\theta_2 - m(\mu)+d)V} - 1\right) + \frac{c}{(\theta_2 - m(\mu)+d)^2}\left(\frac{-V^2}{2}(\theta_2 - m(\mu) + d) + Ve^{(\theta_2 - m(\mu)+d)(V)} - V\right) + \frac{c}{(\theta_2 - m(\mu)+d)^3}\left((V)(\theta_2 - m(\mu) + d) + 1 - e^{(\theta_2 - m(\mu)+d)(V)}\right)\right)$

Preservation technology cost- $PCT_s = nV\xi$
Here, the total cost for the supplier is

$$TC_s = \frac{1}{T}\left[O_s + (C_{hs} + C_{hse})n\left(\frac{D}{(\theta_2 - m(\mu)+d)^2}\left(-V(\theta_2 - m(\mu) + d) + e^{(\theta_2 - m(\mu)+d)V} - 1\right) + \frac{c}{(\theta_2 - m(\mu)+d)^2}\left(\frac{-V^2}{2}(\theta_2 - m(\mu) + d) + Ve^{(\theta_2 - m(\mu)+d)(V)} - V\right) + \frac{c}{(\theta_2 - m(\mu)+d)^3}\left((V)(\theta_2 - m(\mu) + d) + 1 - e^{(\theta_2 - m(\mu)+d)(V)}\right)\right) + n(C_{ds} + C_{dse})(\theta_2 - m(\mu))\left(\frac{D}{(\theta_2 - m(\mu)+d)^2}\left(-V(\theta_2 - m(\mu) + d) + e^{(\theta_2 - m(\mu)+d)V} - 1\right) + \frac{c}{(\theta_2 - m(\mu)+d)^2}\left(\frac{-V^2}{2}(\theta_2 - m(\mu) + d) + Ve^{(\theta_2 - m(\mu)+d)(V)} - V\right) + \frac{c}{(\theta_2 - m(\mu)+d)^3}\left((V)(\theta_2 - m(\mu) + d) + 1 - e^{(\theta_2 - m(\mu)+d)(V)}\right)\right) + nV\xi\right]$$

Now, based on the permissible delay period allowed to the supplier, three cases arise:
Case 1: when $N_s \leq M_s \leq V + N_s$
Case 2: when $N_s \leq V + N_s \leq M_s$
Case 3: when $M_s \leq N_s \leq V + N_s$
For case 1

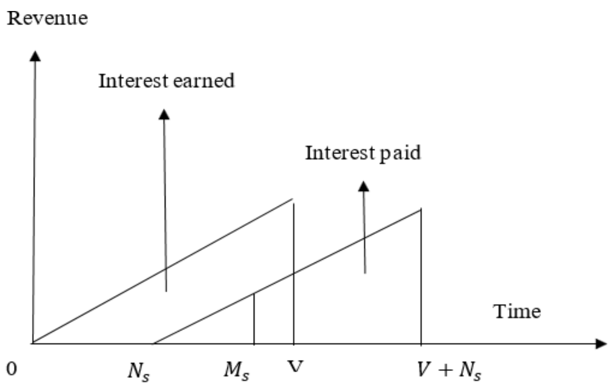

Fig. 1. 3: Revenue Level V/S Time.

In this scenario, the supplier offers a trade credit period (TCP) to retailer that is notably shorter than the TCP extended by the producer to the supplier. As a result, the supplier earns interest on average sales revenue over the time period (Ms−Ns) following receipt of payment from retailer. The supplier settles their account with the producer at time Ms, utilizing the earned interest to optimize their financial position. This financial arrangement is illustrated in Figure 1.3 The interest earned on items sold can be mathematically represented as.

$$IE_1 = nI_e p \int_{N_s}^{M_s} D dt = nI_e p \Big[D(M_s - N_s) + \tfrac{c}{2}(M_s^2 - N_s^2) + d\Big(\tfrac{D}{L^2}\big((M_s - N_s)L + e^{L(V-N_s)} - e^{L(V-M_s)}\big) + \tfrac{C}{L^2}\Big(V\big(e^{L(V-N_s)} - e^{L(V-M_s)}\big) + \tfrac{L}{2}(N_s^2 - M_s^2)\Big) + \tfrac{C}{L^3}\big(L(M_s - N_s) + e^{L(V-M_s)} - e^{L(V-N_s)}\big)\Big)\Big]$$

Where $L = \theta_2 - m(\mu) + d$

$$IP_1 = nI_p c\left[\tfrac{D}{L^2}\Big(e^{L(V-M_s)} - e^{L(-N_s)} + L(M_s - N_s - V)\Big) + \tfrac{c}{L^2}\Big(T\big(e^{L(V-M_s)} - e^{L(-N_s)}\big) + \tfrac{L}{2}\big(M_s^2 - (N_s + V)^2\big)\Big) + \tfrac{c}{L^3}\Big(e^{L(-N_s)} - e^{L(V-M_s)} + L(N_s + V - M_s)\Big)\right]$$

Total cost for the supplier

$$TC_{s1} = TC_s + IP_1 - IE_1$$

$$TC_{s1} = \frac{1}{T}\Bigg[O_s + (C_{hs} + C_{hse})n\Big(\frac{D}{(\theta_2 - m(\mu) + d)^2}\big(-V(\theta_2 - m(\mu) + d) + e^{(\theta_2 - m(\mu)+d)V} - 1\big) + \frac{c}{(\theta_2 - m(\mu) + d)^2}\Big(\frac{-V^2}{2}(\theta_2 - m(\mu) + d) + Ve^{(\theta_2 - m(\mu)+d)(V)} - V\Big) + \frac{c}{(\theta_2 - m(\mu) + d)^3}\Big((V)(\theta_2 - m(\mu) + d) + 1 - e^{(\theta_2 - m(\mu)+d)(V)}\Big)\Big) + n(C_{ds} + C_{dse})\big(\theta_2 - m(\mu)\big)\Big(\frac{D}{(\theta_2 - m(\mu) + d)^2}\big(-V(\theta_2 - m(\mu) + d) + e^{(\theta_2 - m(\mu)+d)V} - 1\big) + \frac{c}{(\theta_2 - m(\mu) + d)^2}\Big(\frac{-V^2}{2}(\theta_2 - m(\mu) + d) + Ve^{(\theta_2 - m(\mu)+d)(V)} - V\Big) + \frac{c}{(\theta_2 - m(\mu) + d)^3}\Big((V)(\theta_2 - m(\mu) + d) + 1 - e^{(\theta_2 - m(\mu)+d)(V)}\Big)\Big) + nV\xi + nI_p c\Big[\frac{D}{L^2}\Big(e^{L(V-M_s)} - e^{L(-N_s)} + L(M_s - N_s - V)\Big) + \frac{c}{L^2}\Big(T\big(e^{L(V-M_s)} - e^{L(-N_s)}\big) + \frac{L}{2}\big(M_s^2 - (N_s + V)^2\big)\Big) + \frac{c}{L^3}\Big(e^{L(-N_s)} - e^{L(V-M_s)} + L(N_s + V - M_s)\Big)\Big] - nI_e p\Big[D(M_s - N_s) + \frac{c}{2}(M_s^2 - N_s^2) + d\Big(\frac{D}{L^2}\big((M_s - N_s)L + e^{L(V-N_s)} - e^{L(V-M_s)}\big) + \frac{C}{L^2}\Big(V\big(e^{L(V-N_s)} - e^{L(V-M_s)}\big) + \frac{L}{2}(N_s^2 - M_s^2)\Big) + \frac{C}{L^3}\big(L(M_s - N_s) + e^{L(V-M_s)} - e^{L(V-N_s)}\big)\Big)\Big]\Bigg]$$

For case 2

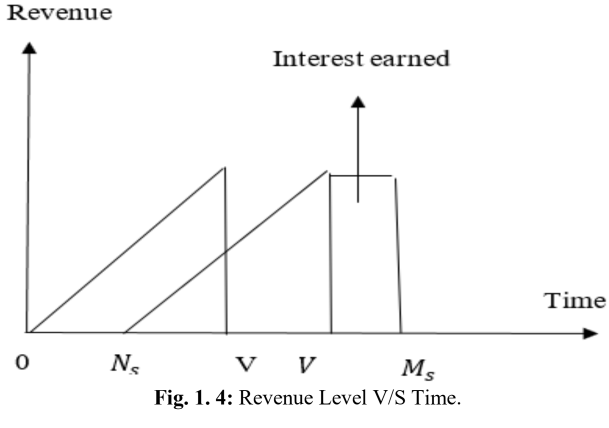

Fig. 1. 4: Revenue Level V/S Time.

Fig:1.4- Revenue level v/s TimeWhen the producer offers a trade credit period (TCP) to the supplier that exceeds both the supplier's cycle length and the TCP offered to retailer, the supplier benefits from interest earnings on sales revenue. Specifically, the supplier earns interest on average sales revenue during the period (Ns,v+Ns) and on total sales revenue for the period (Ms−(v+Ns)). Notably, the supplier incurs no interest payable under this arrangement. This financial dynamic is illustrated in Figure 1.4. The interest earned on items can be mathematically represented as.

$$IE_2 = nI_e p\left[\int_{N_s}^{V+N_s} D\,dt + \int_{V+N_s}^{M_s} DV dt\right]$$

$$= nI_e p\Big[DV + \frac{c}{2}((V+N_s)^2 - N_s^2) + d\Big(\frac{D}{L^2}(e^{L(V-N_s)} - e^{L(-N_s)} - LV) + \frac{c}{L^2}\Big(T(e^{L(V-N_s)} - e^{L(-N_s)}) + \frac{L}{2}(N_s^2 - (V+N_s)^2)\Big) +$$

$$\frac{c}{L^3}(e^{L(T-V-N_s)} - e^{L(T-N_s)} + LV)\Big) + V\Big(D(M_s - V - N_s) + \frac{c}{2}(M_s^2 - (V+N_s)^2) + d\Big(\frac{D}{L^2}(e^{-LN_s} - e^{L(V-M_s)} + L(V+N_s-M_s)\Big) +$$

$$\frac{c}{L^2}\Big(T(e^{-LN_s} - e^{L(V-M_s)}) + \frac{L}{2}((V+N_s)^2 - M_s^2)\Big) + \frac{c}{L^3}\Big(e^{L(V-M_s)} - e^{-LN_s} + L(M_s - V - N_s)\Big)\Big)\Big)\Big]$$

Where $L = \theta_2 - m(\mu) + d$

$$IP_2 = 0$$

$$TC_{s2} = TC_s + IP_2 - IE_2$$

$$TC_{s2} = \frac{1}{T}\Big[O_s + (C_{hs} + C_{hse})n\Big(\frac{D}{(\theta_2 - m(\mu)+d)^2}(-V(\theta_2 - m(\mu) + d) + e^{(\theta_2 - m(\mu)+d)V} - 1) + \frac{c}{(\theta_2 - m(\mu)+d)^2}\Big(\frac{-V^2}{2}(\theta_2 - m(\mu) + d) +$$

$$Ve^{(\theta_2 - m(\mu)+d)(V)} - V\Big) + \frac{c}{(\theta_2 - m(\mu)+d)^3}\Big((V)(\theta_2 - m(\mu) + d) + 1 - e^{(\theta_2 - m(\mu)+d)(V)}\Big)\Big) + n(C_{ds} + C_{dse})(\theta_2 -$$

$$m(\mu))\Big(\frac{D}{(\theta_2 - m(\mu)+d)^2}(-V(\theta_2 - m(\mu) + d) + e^{(\theta_2 - m(\mu)+d)V} - 1) + \frac{c}{(\theta_2 - m(\mu)+d)^2}\Big(\frac{-V^2}{2}(\theta_2 - m(\mu) + d) + Ve^{(\theta_2 - m(\mu)+d)(V)} - V\Big) +$$

$$\frac{c}{(\theta_2 - m(\mu)+d)^3}\Big((V)(\theta_2 - m(\mu) + d) + 1 - e^{(\theta_2 - m(\mu)+d)(V)}\Big)\Big) + nV\xi - nI_e p\Big[DV + \frac{c}{2}((V+N_s)^2 - N_s^2) + d\Big(\frac{D}{L^2}(e^{L(V-N_s)} - e^{L(-N_s)} -$$

$$LV) + \frac{c}{L^2}\Big(T(e^{L(V-N_s)} - e^{L(-N_s)}) + \frac{L}{2}(N_s^2 - (V+N_s)^2)\Big) + \frac{c}{L^3}(e^{L(T-V-N_s)} - e^{L(T-N_s)} + LV)\Big) + V\Big(D(M_s - V - N_s) +$$

$$\frac{c}{2}(M_s^2 - (V+N_s)^2) + d\Big(\frac{D}{L^2}(e^{-LN_s} - e^{L(V-M_s)} + L(V+N_s-M_s)\Big) + \frac{c}{L^2}\Big(T(e^{-LN_s} - e^{L(V-M_s)}) + \frac{L}{2}((V+N_s)^2 - M_s^2)\Big) +$$

$$\frac{c}{L^3}\Big(e^{L(V-M_s)} - e^{-LN_s} + L(M_s - V - N_s)\Big)\Big)\Big)\Big]\Big]$$

For case-3

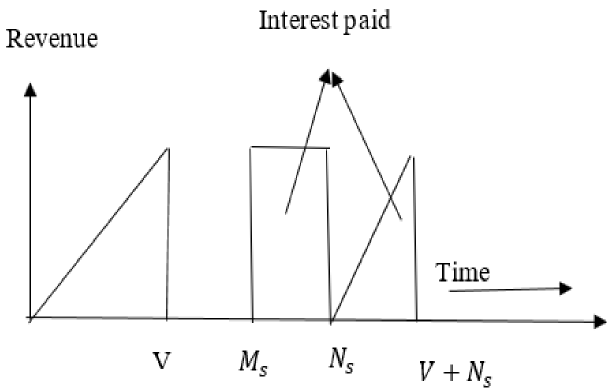

Fig. 1. 5: Revenue Level V/S Time.

In this case the supplier's trade credit period (TCP) offered to retailer exceeds the period granted by the producer. Consequently, the supplier does not earn interest but instead incurs interest charges on the entire order quantity for the duration (Ns–Ms). Additionally, the supplier pays interest on the average product held during the cycle time "v". This financial scenario is visually represented in Figure 1.5, highlighting the supplier's interest payment obligation.

$$IE_3 = 0$$

$$IP_3 = nI_p\Bigg[(N_s - M_s)\Big[\frac{D}{L^2}(e^{LV} - 1 - LV) + \frac{c}{L^2}\Big(e^{LV}V - V - \frac{LV^2}{2}\Big) + \frac{c}{L^3}(1 - e^{LV} + VL)\Big] + \Big[\frac{D}{L^2}\big(e^{L(V-N_s)} - e^{L(-N_s)} - VL\big) + \frac{c}{L^2}\Big(V\big(e^{L(V-N_s)} - e^{L(-N_s)}\big) + \frac{L}{2}(N_s^2 - (V+N_s)^2)\Big) + \frac{c}{L^3}\big(e^{L(-N_s)} - e^{L(V-N_s)} + VL\big)\Big]\Bigg]$$

Where $L = \theta_2 - m(\mu) + d$

$$TC_{s3} = TC_s + IP_3 - IE_3$$

$$TC_{s3} = \frac{1}{T}\Bigg[O_s + (C_{hs} + C_{hse})n\Bigg(\frac{D}{(\theta_2 - m(\mu)+d)^2}\big(-V(\theta_2 - m(\mu) + d) + e^{(\theta_2 - m(\mu)+d)V} - 1\big) + \frac{c}{(\theta_2 - m(\mu)+d)^2}\Big(\frac{-V^2}{2}(\theta_2 - m(\mu) + d) + Ve^{(\theta_2 - m(\mu)+d)(V)} - V\Big) + \frac{c}{(\theta_2 - m(\mu)+d)^3}\Big((V)(\theta_2 - m(\mu) + d) + 1 - e^{(\theta_2 - m(\mu)+d)(V)}\Big)\Bigg) + n(C_{ds} + C_{dse})\big(\theta_2 - m(\mu)\big)\Bigg(\frac{D}{(\theta_2 - m(\mu)+d)^2}\big(-V(\theta_2 - m(\mu) + d) + e^{(\theta_2 - m(\mu)+d)V} - 1\big) + \frac{c}{(\theta_2 - m(\mu)+d)^2}\Big(\frac{-V^2}{2}(\theta_2 - m(\mu) + d) + Ve^{(\theta_2 - m(\mu)+d)(V)} - V\Big) + \frac{c}{(\theta_2 - m(\mu)+d)^3}\Big((V)(\theta_2 - m(\mu) + d) + 1 - e^{(\theta_2 - m(\mu)+d)(V)}\Big)\Bigg) + nV\xi + nI_p\Bigg[(N_s - M_s)\Big[\frac{D}{L^2}(e^{LV} - 1 - LV) + \frac{c}{L^2}\Big(e^{LV}V - V - \frac{LV^2}{2}\Big) + \frac{c}{L^3}(1 - e^{LV} + VL)\Big] + \Big[\frac{D}{L^2}\big(e^{L(V-N_s)} - e^{L(-N_s)} - VL\big) + \frac{c}{L^2}\Big(V\big(e^{L(V-N_s)} - e^{L(-N_s)}\big) + \frac{L}{2}(N_s^2 - (V+N_s)^2)\Big) + \frac{c}{L^3}\big(e^{L(-N_s)} - e^{L(V-N_s)} + VL\big)\Big]\Bigg]\Bigg]$$

Total carbon emission of the manufacturer

$$TE_m = C_{pe}\Bigg[\beta\Big(Dt_1 + \frac{ct_1^2}{2}\Big) + d\beta(\beta - 1)\Bigg(\frac{D}{\theta_1 - m(\mu) - \beta d + d}\Big(t_1 + \frac{e^{-(\theta_1 - m(\mu) - \beta d + d)t_1}}{\theta_1 - m(\mu) - \beta d + d} - \frac{1}{\theta_1 - m(\mu) - \beta d + d}\Big)\Bigg) + \frac{ct_1^2}{2(\theta_1 - m(\mu) - \beta d + d)} + \frac{c}{(\theta_1 - m(\mu) - \beta d + d)^2}\Big(-\frac{e^{(\theta_1 - m(\mu) - \beta d + d)t_1}}{\theta_1 - m(\mu) - \beta d + d} - t_1 + \frac{1}{\theta_1 - m(\mu) - \beta d + d}\Big)\Bigg] + C_{hme}\Bigg[(\beta - 1)\Bigg(\frac{D}{(\theta_1 - m(\mu) - \beta d + d)^2}\big(e^{-(\theta_1 - m(\mu) - \beta d + d)t_1} + t_1(\theta_1 - m(\mu) - \beta d + d) - 1\big) + \frac{ct_1^2}{2(\theta_1 - m(\mu) - \beta d + d)} + \frac{c}{\big((\theta_1 - m(\mu) - \beta d + d)\big)^3}\Big(\big(1 - e^{-(\theta_1 - m(\mu) - \beta d + d)}\big) - t_1(\theta_1 - m(\mu) - \beta d + d)\Big)\Bigg) + \Bigg(\frac{D}{(\theta_2 - m(\mu)+d)^2}\Big((t_1 - T)(\theta_2 - m(\mu) + d) + e^{(\theta_2 - m(\mu)+d)(T - t_1)} - 1\Big) + \frac{c}{(\theta_2 - m(\mu)+d)^2}\Big(\frac{t_1^2 - T^2}{2}(\theta_2 - m(\mu) + d) + e^{(\theta_2 - m(\mu)+d)(T - t_1)} - 1\Big) + \frac{c}{(\theta_2 - m(\mu)+d)^3}\Big((T - t_1)(\theta_2 - m(\mu) + d) + 1 - e^{(\theta_2 - m(\mu)+d)(T - t_1)}\Big)\Bigg)\Bigg] + C_{dme}\Bigg[(\beta - 1)(\theta_1 - (\mu))\Bigg(\frac{D}{(\theta_1 - m(\mu) - \beta d + d)^2}\big(e^{-(\theta_1 - m(\mu) - \beta d + d)t_1} - 1 + t_1(\theta_1 - m(\mu) - \beta d + d)\big) + \frac{ct_1^2}{2(\theta_1 - m(\mu) - \beta d + d)} + \frac{c}{\big((\theta_1 - m(\mu) - \beta d + d)\big)^3}\Big(\big(1 - e^{-(\theta_1 - m(\mu) - \beta d + d)}\big) - t_1(\theta_1 - m(\mu) - \beta d + d)\Big)\Bigg) + (\theta_2 - m(\mu))\Bigg(\frac{D}{(\theta_2 - m(\mu)+d)^2}\Big((t_1 - T)(\theta_2 - m(\mu) + d) + e^{(\theta_2 - m(\mu)+d)(T - t_1)} - 1\Big) + \frac{c}{(\theta_2 - m(\mu)+d)^2}\Big(\frac{t_1^2 - T^2}{2}(\theta_2 - m(\mu) + d) + e^{(\theta_2 - m(\mu)+d)(T - t_1)} - 1\Big) + \frac{c}{(\theta_2 - m(\mu)+d)^3}\Big((T - t_1)(\theta_2 - m(\mu) + d) + 1 - e^{(\theta_2 - m(\mu)+d)(T - t_1)}\Big)\Bigg)\Bigg] + W_{mV}\Bigg[\Bigg[\beta\Big(Dt_1 + \frac{ct_1^2}{2}\Big) + d\beta(\beta - 1)\Bigg(\frac{D}{\theta_1 - m(\mu) - \beta d + d}\Big(t_1 + \frac{e^{(\theta_1 - m(\mu) - \beta d + d)t_1}}{\theta_1 - m(\mu) - \beta d + d} - \frac{1}{\theta_1 - m(\mu) - \beta d + d}\Big)\Bigg) + \frac{ct_1^2}{2(\theta_1 - m(\mu) - \beta d + d)} + \frac{c}{(\theta_1 - m(\mu) - \beta d + d)^2}\Big(-\frac{e^{(\theta_1 - m(\mu) - \beta d + d)t_1}}{\theta_1 - m(\mu) - \beta d + d} - t_1 + \frac{1}{\theta_1 - m(\mu) - \beta d + d}\Big)\Bigg] + (2de_1 + dQ_m e_2)$$

Total carbon emission for supplier

$$TE_s = nC_{hse}\Bigg(\Bigg(\frac{D}{(\theta_2 - m(\mu)+d)^2}\big(-V(\theta_2 - m(\mu) + d) + e^{(\theta_2 - m(\mu)+d)V} - 1\big) + \frac{c}{(\theta_2 - m(\mu)+d)^2}\Big(\frac{-V^2}{2}(\theta_2 - m(\mu) + d) + Ve^{(\theta_2 - m(\mu)+d)(V)} - V\Big) + \frac{c}{(\theta_2 - m(\mu)+d)^3}\Big((V)(\theta_2 - m(\mu) + d) + 1 - e^{(\theta_2 - m(\mu)+d)(V)}\Big)\Bigg)\Bigg) + nC_{dse}\Bigg(\big(\theta_2 - m(\mu)\big)\Bigg(\frac{D}{(\theta_2 - m(\mu)+d)^2}\big(-V(\theta_2 - m(\mu) + d) + e^{(\theta_2 - m(\mu)+d)V} - 1\big) + \frac{c}{(\theta_2 - m(\mu)+d)^2}\Big(\frac{-V^2}{2}(\theta_2 - m(\mu) + d) + Ve^{(\theta_2 - m(\mu)+d)(V)} - V\Big) + \frac{c}{(\theta_2 - m(\mu)+d)^3}\Big((V)(\theta_2 - m(\mu) + d) + 1 - e^{(\theta_2 - m(\mu)+d)(V)}\Big)\Bigg)\Bigg)$$

7. Numerical Analysis

Numerical analysis is done to bridge the gap between theoretical mathematics and practical problem-solving. Approximating solutions where analytical methods fail enables us to model, simulate, and understand complex systems across virtually every domain of human endeavour. For this model, the data values are-
a = 90, b = 0.04, c = 0.02, d =2, O_m = 100, C_{pm} =400 rupees/unit, β = 1.5, θ_1 = 0.015 %, θ_2 = 0.025 %, C_{hm} = 5 rupees/order, C_{dm}= 10 rupees/order, μ = 0.04, W_m = 80, W = 0.18, C_T = 90, C_t = 9, C_1 = 0.04, C_2 = 0.05, Q_m = 230, Q_M = 53, e_2 = 0.9, O_s = 150, C_{hs} = 3, C_{ds} =

8, D_{se} = 0.005 kg/unit, W_{me} = 0.55, W_{se} = 0.55, d_{me} = 0.06, E_{mv} = 0.65, E_e = 0.67, F_e = 2.64, T_x = 150, e_p = 1.65, I_1 = 0.14, I_2 = 0.12, p_1 = 1000 rupees, p_2 = 1700 rupees, M = 2.5 , N= 1, V = 3, C_{dm} = 1 rupees/order, n = 3, T = 9

Using the above numerical values of inventory parameters, Table 6.1 presents an optimum solution for the proposed model. From the table, the minimum cost of this model is considered in case 1. Therefore, the optimal values are $t1$= = 1.35529, ξ = 220.416, total cost = 223477, Carbon emission for supplier = 60204.6, and Carbon emission for retailer = 2081826.7. All these results are obtained by adopting the mathematical software Mathematica 12.0.

Table 1. 3: Comparison between Decision Variables for All Cases

Cases	ξ	t_1	Total Cost
Case-1	220.416	1.35529	223477 (min)
Case-2	220.596	1.35528	243456
Case-3	220.694	1.35528	247512

8. Graphical Representation

In this section, we represent convexity for all different cases of trade credit.

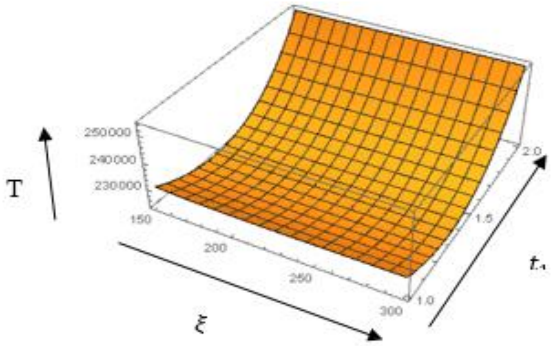

Fig. 1. 6: Convexity of the Total Cost When N_S≤M_S≤V+N_S.

Fig. 1 .6 represents the minimum total cost scenario, with a total cost of 223,477, ξ value of 220.416, and t_1 value of 1.35529. This optimal combination of ξ and t_1 results in the lowest total cost among the three cases, indicating that these values are the most desirable for minimizing costs. Notably, the graph for this case is convex, suggesting that small deviations from the optimal values would lead to increasing costs.

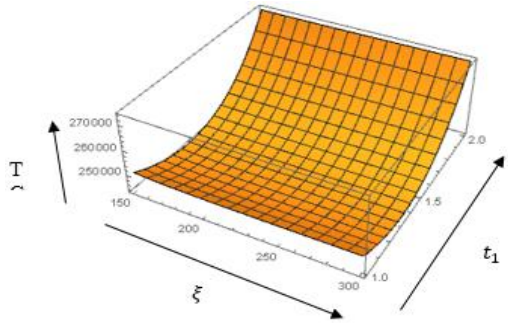

Fig. 1. 7: Convexity of the Total Cost When N_S≤V+N_S≤M_S.

In Fig. 1.7, the total cost increases to 243,456, with ξ and t_1 values of 220.596 and 1.35528, respectively. Compared to Fig. 1.7, the slightly higher ξ value and nearly identical t_1 value lead to a higher total cost, suggesting that small deviations from the optimal values can result in increased costs. The convex nature of the graph for this case reinforces the idea that the cost function is sensitive to changes in ξ and t_1.

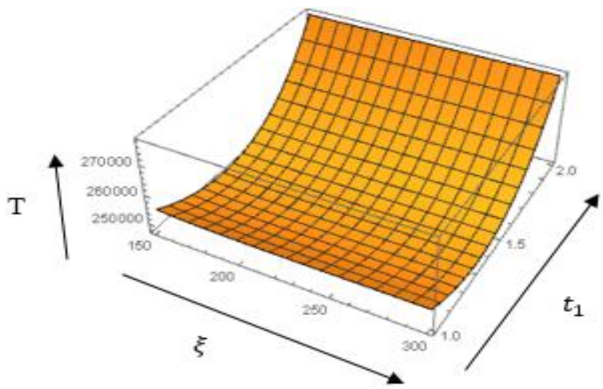

Fig. 1. 8: Convexity of the Total Cost When M_S≤N_S≤V+N_S

respectively. The higher ξ value and similar t_1 value compared to the other cases contribute to the increased total cost, highlighting the importance of carefully selecting ξ and t_1 values to minimize costs and optimize outcomes. The convex graph for this case further emphasizes the need for precise optimization to avoid higher costs.
Now, we check the effect of different parameters on the total cost for all three cases.

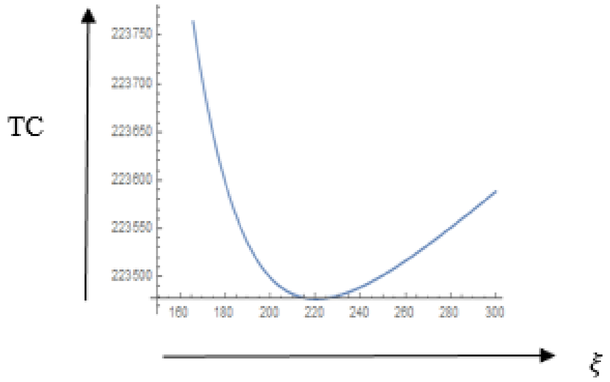

Fig. 1. 9: The Pictorial Graph between Total Cost and Ξ When N_S≤M_S≤V+N_S, Where the X-Axis represents Preservation Technology Investment and the Y-Axis Represents A Total Cost.

The Fig 1.9 shows a U-shaped curve, decreasing initially and then increasing. The minimum point is at ξ = 220.416, with a total cost of 223,477, indicating the optimal investment level that minimizes total costs.

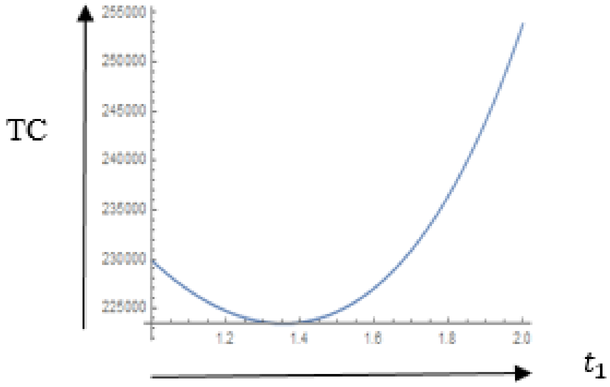

Fig. 1. 10: The Pictorial Graph between Total Cost and T_1 When N_S≤M_S≤V+N_S, Where the X-Axis Represents Time and the Y-Axis Represents A Total Cost.

The Fig 1.10 shows a U-shaped curve, decreasing initially and then increasing. The minimum point is at t1 = 1.35529, with a corresponding total cost, indicating the optimal time that minimizes total costs.

Table 1. 4:

Parameters	% change	ξ	t_1	Total Cost
a	+20%	228.268	1.35513	229410
	+10%	224.69	1.3552	226444
	-10%	221.421	1.35527	224136
	-20%	208.906	1.35564	217536
b	+20%	215.781	1.35541	225391
	+10%	218.206	1.35534	224434
	-10%	222.447	1.35524	222519
	-20%	224.326	1.3552	221562
c	+20%	220.42	1.3553	223470
	+10%	220.418	1.35529	223474
	-10%	220.415	1.35528	223480
	-20%	220.413	1.35528	223483
d	+20%	258.488	2.15814	546856
	+10%	239.95	1.80015	328507
	-10%	200.655	0.757242	168834
	-20%	181.363	-	134730
O_m	+20%	220.416	1.35529	223479
	+10%	220.416	1.35529	223478
	-10%	220.416	1.35529	223476
	-20%	220.416	1.35529	223475
C_{pm}	+20%	221.715	1.31037	226131
	+10%	221.086	1.33212	224833
	-10%	219.7	1.38006	222058
	-20%	218.93	1.4067	220568
β	+20%	220.518	0.878298	215956
	+10%	220.977	1.08349	220173
	-10%	217.991	1.73209	224456
	-20%	-	-	357975

θ_1	+20%	221.669	1.46716	228432
	+10%	221.455	1.44673	227534
	-10%	221.433	1.4446	227441
	-20%	221.218	1.42471	226563
θ_2	+20%	220.943	1.37096	225039
	+10%	220.68	1.36313	224254
	-10%	220.153	1.34742	222709
	-20%	219.888	1.33953	221949
C_{hm}	+20%	220.734	1.3633	224371
	+10%	220.576	1.35932	223935
	-10%	220.255	1.35121	223027
	-20%	220.092	1.34707	222576
C_{dm}	+20%	219.915	1.33908	221713
	+10%	220.168	1.34728	222598
	-10%	220.661	1.3631	224349
	-20%	220.902	1.37072	225214
μ	+20%	187.476	1.3552	223403
	+10%	202.543	1.35524	223437
	-10%	241.983	1.35525	223526
	-20%	268.553	1.35543	223586
W_m	+20%	220.416	1.35529	223479
	+10%	220.416	1.35529	223478
	-10%	220.416	1.35529	223476
	-20%	220.416	1.35529	223475
W	+20%	220.442	1.35406	223604
	+10%	220.418	1.35523	223483
	-10%	220.415	1.35535	223471
	-20%	220.414	1.35541	223464
C_T	+20%	220.416	1.35529	223469
	+10%	220.416	1.35529	223478
	-10%	220.416	1.35529	223467
	-20%	220.416	1.35529	223466
C_t	+20%	220.416	1.35529	260230
	+10%	220.416	1.35529	241854
	-10%	220.416	1.35529	205100
	-20%	220.416	1.35529	186724
C_{pm}	+20%	220.416	1.35529	223477
	+10%	220.416	1.35529	223477
	-10%	220.416	1.35529	223477
	-20%	220.416	1.35529	223477
C_{hm}	+20%	220.416	1.35529	223479
	+10%	220.416	1.35529	223482
	-10%	220.416	1.35529	223475
	-20%	220.416	1.35529	223472
Q_M	+20%	220.416	1.35529	259914
	+10%	220.416	1.35529	241695
	-10%	220.416	1.35529	205259
	-20%	220.416	1.35529	187040
Q_m	+20%	220.416	1.35529	223477
	+10%	220.416	1.35529	223477
	-10%	220.416	1.35529	223477
	-20%	220.416	1.35529	223477
E_{mV}	+20%	220.416	1.35529	223477
	+10%	220.416	1.35529	223477
	-10%	220.416	1.35529	223477
	-20%	220.416	1.35529	223477
O_s	+20%	220.416	1.35529	223480
	+10%	220.416	1.35529	223479
	-10%	220.416	1.35529	223475
	-20%	220.416	1.35529	223474
C_{hS}	+20%	220.425	1.35529	223551
	+10%	220.421	1.35529	223514
	-10%	220.412	1.35529	223440
	-20%	220.408	1.35529	223403
C_{dS}	+20%	220.409	1.35529	223286
	+10%	220.413	1.35529	223381
	-10%	220.42	1.35529	223573
	-20%	220.424	1.35529	223668
D_{se}	+20%	220.416	1.35529	223459
	+10%	220.416	1.35529	223468
	-10%	220.417	1.35529	223486
	-20%	220.417	1.35529	223495
W_{me}	+20%	226.345	1.42643	237432
	+10%	222.095	1.39725	228333
	-10%	218.89	1.31635	219339
	-20%	214.543	1.23213	210234
W_{se}	+20%	219.966	1.34073	221890
	+10%	220.193	1.34809	222687
	-10%	220.637	1.36233	224262

	-20%	220.854	1.36921	225041
d_{me}	+20%	220.419	1.35517	223490
	+10%	220.418	1.35523	223483
	-10%	220.415	1.35535	223471
	-20%	220.414	1.35541	223464
E_{mV}	+20%	220.419	1.35517	223490
	+10%	220.415	1.35535	223471
	-10%	220.418	1.35519	223487
	-20%	220.414	1.35541	223464
E_e	+20%	224.299	1.41526	235670
	+10%	222.45	1.38753	229628
	-10%	218.151	1.31717	217190
	-20%	215.588	1.27106	210728
F_e	+20%	220.416	1.35529	260226
	+10%	220.416	1.35529	241851
	-10%	220.416	1.35529	205103
	-20%	220.416	1.35529	186728
T_X	+20%	223.938	1.40381	270922
	+10%	222.252	1.38116	247235
	-10%	199633	218.4	1.32538
	-20%	216.159	1.29024	175683
e_p	+20%	220.975	1.33598	224605
	+10%	220.699	1.3455	224046
	-10%	220.126	1.36535	222897
	-20%	219.826	1.37571	222305
I_e	+20%	220.416	1.35529	223477
	+10%	220.416	1.35529	223477
	-10%	220.416	1.35529	223477
	-20%	220.416	1.35529	223477
I_c	+20%	220.361	1.35529	218672
	+10%	220.389	1.35529	221075
	-10%	220.444	1.35529	225879
	-20%	220.472	1.35529	228282
p_1	+20%	216.162	1.3554	214334
	+10%	218.38	1.35534	218905
	-10%	222.3	1.35525	228048
	-20%	224.051	1.35521	232619
p_2	+20%	220.074	1.3553	232696
	+10%	220.237	1.35529	227345
	-10%	220.611	1.35528	221092
	-20%	220.822	1.35528	220190
M	+20%	220.62	1.35528	219360
	+10%	220.516	1.35529	221297
	-10%	220.328	1.35529	225972
	-20%	220.263	1.35529	228873
N	+20%	220.656	1.35528	228468
	+10%	220.551	1.35529	226078
	-10%	220.246	1.35529	220643
	-20%	220.034	1.3553	217551
V	+20%	217.3	1.35536	214891
	+10%	218.936	1.35532	219827
	-10%	221.829	1.35526	226182
	-20%	223.232	1.35523	228192
C_{dm}	+20%	220.416	1.35529	223477
	+10%	220.416	1.35529	223477
	-10%	220.416	1.35529	223477
	-20%	220.416	1.35529	223477
n	+20%	218.138	1.35534	219936
	+10%	219.249	1.35532	221707
	-10%	221.646	1.35526	225247
	-20%	222.946	1.35523	227017
T	+20%	250.102	1.98286	296974
	+10%	235.155	1.67126	245948
	-10%	206.066	1.03116	219447
	-20%	192.43	0.691668	228746

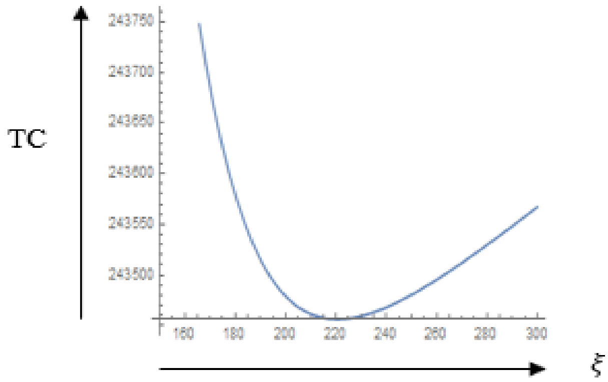

Fig. 1. 11: The Pictorial Graph between Total Cost and Ξ When N_S≤V+N_S≤M_S Where the X-Axis Represents Preservation Technology Investment and the Y-Axis Represents Total Cost.

The Fig 1.11 shows a curve with a minimum point at ξ = 220.596, corresponding to a total cost of 243,456. The curve likely decreases initially and then increases, indicating an optimal preservation technology investment level that minimizes total costs.

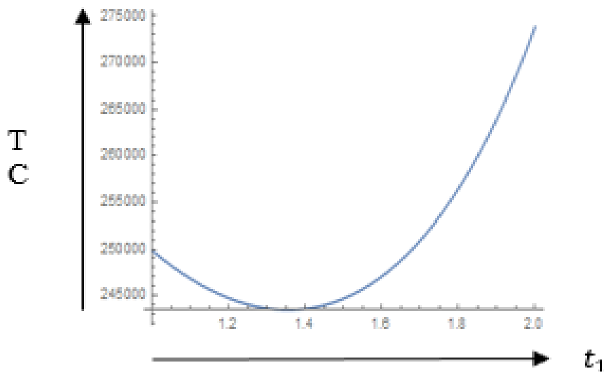

Fig. 1. 12: The Pictorial Graph between Total Cost and T_1 When N_S≤V+N_S≤M_S Where the X-Axis Represents Time and the Y-Axis Represents A Total Cost.

The Fig-1.12 shows a curve with a minimum point at t1 = 1.35528, corresponding to a total cost of 243,456. The curve likely decreases initially and then increases, indicating an optimal time that minimizes total costs.

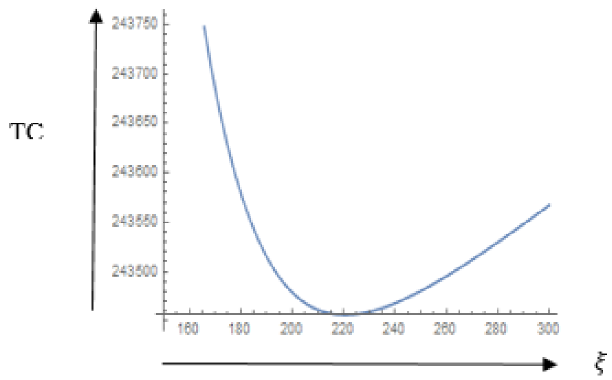

Fig. 1. 13: The Pictorial Graph between Total Cost and Ξ When M_S≤N_S≤V+N_S Where the X-Axis Represents Preservation Technology Investment and the Y-Axis Represents A Total Cost.

The Fig-1.13 shows a curve with a minimum point at ξ = 220.694, corresponding to a total cost of 247,512. The curve likely decreases initially and then increases, indicating an optimal preservation technology investment level that minimizes total costs.

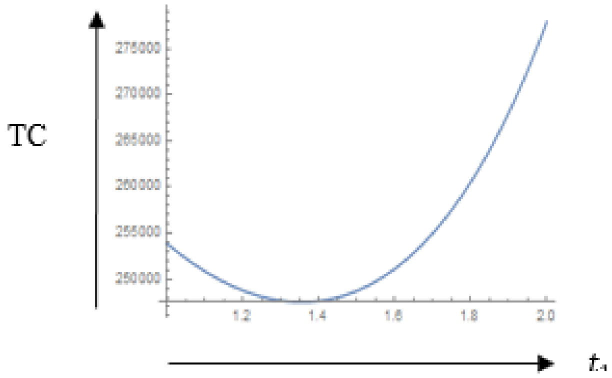

Fig. 1. 14: The Pictorial Graph between Total Cost and T_1 When M_S≤N_S≤V+N_S Where the X-Axis Represents Time and the Y-Axis Represents A Total Cost.

The Fig 1.14 shows a curve with a minimum point at t1 = 1.35528, corresponding to a total cost of 247,512. The curve likely decreases initially and then increases, indicating an optimal time that minimizes total costs.

1.4. Sensitivity analysis table

9. Observations

- When parameter an increase in parameter a (scaling constant) increases, preservation technology investment and total cost increase while time $t1$ is decreases.
- When there is an increase in parameter b (scaling factor), then preservation technology investment is decreasing while total cost and t_1 are decreasing.
- When there is an increase in parameter c (scaling factor), then preservation technology investment and t_1 are increasing while total cost is decreasing.
- When there is an increase in parameter d (scaling factor), then preservation technology investment, $t1$ and total cost increase.
- When there is an increase in parameter Om (Ordering cost for the manufacturer), preservation technology investment and t_1 are constant while total cost is slightly increasing.
- When there is an increase in parameter Cpm (Production cost for the manufacturer), then preservation technology investment and total cost increase while t_1 is decreasing.
- When there is an increase in parameter β (Production rate), then preservation technology investment fluctuates while t_1 and total cost are decreasing.
- When increase in parameter θ_1 (Deterioration rate for 0≤t≤t_1), then preservation technology investment, t_1 and total cost are increasing.
- When increase in parameter θ_2 (Deterioration rate for t_1≤t≤T), then preservation technology investment, t_1 and total cost are increasing.
- When there is an increase in parameter Chm (Holding cost for the manufacturer), then preservation technology investment, $t1$ and total cost increase.
- When there is an increase in parameter Cdm (Deterioration cost for the manufacturer), then preservation technology investment, $t1$ and total cost decrease.
- When there is an increase in parameter μ (Factor representing the percentage increase in m(ξ)), then preservation technology investment, t_1 and total cost decrease.
- When there is an increase in parameter W_m (Fixed waste disposal cost per cycle), preservation technology investment and t_1 are constant while total cost is slightly increasing.
- When there is an increase in parameter W (Average weight of solid waste production), preservation technology investment and total cost are increasing while t_1 is decreasing.
- When the parameter increases parameter CT (Fixed transportation cost per delivery), preservation technology investment and $t1$ are constant while total cost fluctuates.
- When there is an increase in parameter C_t (Variable transportation cost per delivery), then preservation technology investment and t_1 are constant while total cost is increasing.
- When an increase in parameter Chm (Holding cost for the manufacturer), then preservation technology investment and t_1 are constant while total cost is slightly increasing.
- When there is an increase in parameter Q_m (Manufacture's ordering quantity), then preservation technology investment, t_1 and total cost are constant.
- When there is an increase in parameter E_mV (Solid waste standard emission from disposal), then preservation technology investment, t_1 and total cost are constant.
- When there is an increase in parameter $d1$ (distance), then preservation technology investment, $t1$ and total cost are increased.
- When parameter Os (Ordering cost for suppliers) increases, preservation technology investment and $t1$ remain constant, and total cost increases.
- When there is an increase in parameter ChS (Holding cost for suppliers), then preservation technology investment and total cost increase while t_1 is constant.
- When there is an increase in parameter CdS (Deterioration cost for suppliers), then preservation technology investment and total cost decrease while t_1 is constant.

- When there is an increase in parameter Dse (Carbon emission), then preservation technology investment and total cost decrease while t_1 is constant.
- When there is an increase in parameter Wme (Energy consumption from manufacturer's inventory holding), then preservation technology investment, $t1$ and total cost are increased.
- When there is an increase in parameter Wse (Energy consumption from retailer's inventory holding), then preservation technology investment, $t1$ and total cost decrease.
- When there is an increase in parameter d_me (Carbon emission from deterioration of product), then preservation technology investment and total cost are increasing while t_1 is decreasing.
- When there is an increase in parameter E_mV (Solid waste standard emission from disposal), then preservation technology investment and total cost are increasing while t_1 is decreasing.
- When there is an increase in parameter Ee (Emission from electricity consumption), then preservation technology investment, $t1$ and total cost increase.
- When there is an increase in parameter F_e (Vehicle standard emission from fuel consumption), then preservation technology investment and t_1 are constant while total cost is increasing.
- When parameter an increase in parameter TX (Carbon emission tax rate) increases, preservation technology investment increases while $t1$ and total cost fluctuate.
- When there is an increase in parameter e_p (Average energy consumption during production), then preservation technology investment and total cost are increasing while t_1 is decreasing.
- When there is an increase in parameter Ie (Interest earned), then preservation technology investment, t_1 and total cost are constant.
- When there is an increase in parameter Ic (interest charge), preservation technology investment and total cost decrease while t_1 is constant.
- When parameter an increase in parameter $p1$ (Selling price for the manufacturer) increases, preservation technology investment and total cost decrease while t_1 is rising.
- When there is an increase in parameter $p2$ (Selling price for the retailer), then preservation technology investment and total cost decrease while t_1 is increasing.
- When there is an increase in parameter M (Credit period offered by the manufacturer to retailer), then preservation technology investment is increasing, t_1 is almost constant while total cost is decreasing.
- When there is an increase in parameter N (Credit period offered by the retailer to customer), then preservation technology investment and total cost are increasing while t_1 is decreasing.
- When there is an increase in parameter V (Length of manufacturer's replenishment cycle per delivery), then preservation technology investment and total cost decrease while $t1$ increases.
- When an increase in parameter Cdm (Deterioration cost for the manufacturer), then preservation technology investment, t_1 and total cost are constant.
- When the parameter n (Number of shipments) increases, preservation technology investment and total cost decrease while t_1 is rising.
- When parameter T (Total cycle time) increases, preservation technology investment and $t1$ are increase while total cost fluctuates.

10. Limitations

While our integrated supply chain model for bakery items offers a robust framework for optimizing production, inventory, and logistics decisions, it is not without its limitations. The model's computational complexity may pose challenges for larger problem instances, and its scalability may be limited by the size and intricacy of the supply chain. Furthermore, the assumptions and simplifications inherent in the model, such as the demand function and no shortages, may not always reflect real-world complexities. Additionally, the model's reliance on accurate data and parameters may be difficult to fulfill in practice. Nevertheless, our model provides a valuable foundation for bakery supply chain management, and future research can build upon this work to develop more sophisticated models that address these limitations and provide even greater insights for industry practitioners.

11. Conclusion

The two-echelon supply chain inventory management literature highlights the increasing importance of integrating environmental concerns, particularly carbon emissions, into decision-making processes. Research has shown that factors such as product deterioration, trade-credit terms, and demand dependencies on time, price, and stock significantly influence inventory policies. While individual studies have addressed these elements in isolation or limited combinations, there is a growing need for comprehensive models that integrate all these factors in a unified framework. Incorporating controllable deterioration and multi-level trade-credit periods offers opportunities to optimise financial and environmental outcomes. Similarly, accounting for dynamic demand patterns driven by temporal, pricing, and stock-level factors can enhance supply chain responsiveness and efficiency. However, existing models often lack scalability and practical applicability, particularly in multi-echelon and complex systems. Future research should focus on developing holistic, computationally efficient models that balance cost, sustainability, and operational complexity. By addressing these gaps, supply chains can achieve greater resilience, sustainability, and profitability, meeting the demands of businesses and environmental stakeholders in the modern era. Our integrated supply chain model is designed for the bakery industry, balancing freshness and profitability by optimizing production, inventory, and logistics while reducing waste and environmental impact. It incorporates a price, stock, and time-dependent demand function, assumes no shortages, and utilizes preservation technology to mitigate deterioration. The model also accounts for carbon emissions from production, transportation, and inventory holding, providing a comprehensive and sustainable framework for bakeries to thrive in a competitive market while minimizing their environmental footprint.

References

[1] Chaudhary, R., Singh, S. R., & Padiyar, S. V. S. (2025). Integrated supply chain imperfect production inventory model using green product with carbon emissions. RAIRO-Operations Research, 59, 2031-2050. https://doi.org/10.1051/ro/2025069.

[2] Chen, X., Benjaafar, S., & Elomri, A. (2013). The carbon-constrained EOQ. Operations Research Letters, 41(2), 172-179. https://doi.org/10.1016/j.orl.2012.12.003.

[3] Das, S., Choudhury, M., Mahato, C., & Mahata, G. C. (2023). Dual-channel supply chain inventory model under two-warehouse setting with order volume-linked trade credit, all-units discount and partial backordering. Soft computing, 27(21), 15817-15852. https://doi.org/10.1007/s00500-023-08824-3.

[4] Dong, G., Liang, L., Wei, L., Xie, J., & Yang, G. (2023). Optimization model of trade credit and asset-based securitization financing in carbon emission reduction supply chain. Annals of Operations Research, 1-50.

[5] D. S. Chaudhary, S. R. Singh, D. Yadav, and R. Yadav, "Sustainable supply chain inventory model with carbon emissions and investment in preservation technology under inflation," International Conference on Advances in Pure & Applied Mathematics (ICAPAM), January 2025, https://doi.org/10.1063/5.0265215.

[6] Govindan, K., Azevedo, S. G., Carvalho, H., & Cruz-Machado, V. (2014). Impact of supply chain management practices on sustainability. Journal of Cleaner Production, 85, 212-225. https://doi.org/10.1016/j.jclepro.2014.05.068.

[7] Ghosh, S. K. (2017). A Two Echelon Supply Chain Inventory Model for a Deteriorating Item When Demand is Time and Credit Sensitive. Journal of New Theory (17), 82-97.

[8] Handa, N., Singh, S. R., & Punetha, N. (2022). Impact of carbon emission in two-echelon supply chain inventory decision with controllable deterioration and two-level trade-credit period. Opsearch, 59(4), 1555-1586. https://doi.org/10.1007/s12597-022-00599-9

[9] Jaggi, C. K., Goyal, S. K., & Goel, S. K. (2008). Retailer's optimal replenishment decisions with credit-linked demand under permissible delay in payments. European Journal of Operational Research, 190(1), 130-135. https://doi.org/10.1016/j.ejor.2007.05.042.

[10] Jaggi, C. K., Tripathy, M., Sharma, A. K., & Sharma, G. (2020). An inventory model for non-instantaneous deteriorating item under progressive trade credit policy. Rev. Investigicion Oper., 41(6), 804-825.

[11] Jauhari, W. A., Wangsa, I. D., Sofiana, A., & Utama, D. M. (2024). An integrated inventory model for a supply chain system with two competing retailers, carbon emissions, and price-and-service dependent demand. Sustainability Analytics and Modeling, 4, 100029. https://doi.org/10.1016/j.samod.2024.100029.

[12] Lu, C. J., Gu, M., Yang, C. T., Wang, Y. W., & Chen, D. R. (2024). Imperfect production–inventory models for deteriorating items with carbon cap-and-trade policy and advance-cash-credit payment. IEEE Access. https://doi.org/10.1109/ACCESS.2024.3393149.

[13] Mahata, P., & Mahata, G. C. (2021). Two-echelon trade credit with default risk in an EOQ model for deteriorating items under dynamic demand. J Ind Manag Optim, 17(6), 3659-3684. https://doi.org/10.3934/jimo.2020138.

[14] Mahato, F., Mahato, C., & Mahata, G. C. (2024). Sustainable inventory models with controllable deterioration under two-level trade credit policy and various carbon emissions regulations. Modeling Earth Systems and Environment, 10(3), 4449-4474. https://doi.org/10.1007/s40808-024-02022-2.

[15] Mahato, F., & Mahata, G. C. (2024). Two-echelon supply chain with production disruption and controllable deterioration considering carbon emission under Stackelberg game approach. RAIRO-Operations Research, 58(3), 2339-2365. https://doi.org/10.1051/ro/2024076.

[16] Mishra, N. K. (2022). A supply chain inventory model for deteriorating products with carbon emission-dependent demand, advanced payment, carbon tax and cap policy. Mathematical Modelling of Engineering Problems, 9(3). https://doi.org/10.18280/mmep.090308.

[17] Priyan, S., Mala, P., & Gurusamy, R. (2020). Optimal inventory strategies for two-echelon supply chain system involving carbon emissions and fuzzy deterioration. International Journal of Logistics Systems and Management, 37(3), 324-351. https://doi.org/10.1504/IJLSM.2020.111386.

[18] Shah, N. H., Patel, D. G., Shah, D. B., & Prajapati, N. M. (2023). A sustainable production inventory model with green technology investment for perishable products. Decision Analytics Journal, 8, 100309. https://doi.org/10.1016/j.dajour.2023.100309.

[19] Sebatjane, M. (2024). A sustainable inventory model for a two-echelon cold chain with green technology investments and stock-dependent demand under carbon emissions tax regulation. Cleaner Logistics and Supply Chain, 13, 100173. https://doi.org/10.1016/j.clscn.2024.100173.

[20] Taleizadeh, A. A., Haghighi, F., & Niaki, S. T. A. (2019). Modelling and solving a sustainable closed-loop supply chain problem with pricing decisions and discounts on returned products. Journal of Cleaner Production, 207, 163-181. https://doi.org/10.1016/j.jclepro.2018.09.198.

[21] Vandana, Singh, S. R., Yadav, D., Sarkar, B., & Sarkar, M. (2021). Impact of energy and carbon emission of a supply chain management with two-level trade-credit policy. Energies, 14(6), 1569-1588. https://doi.org/10.3390/en14061569.