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Research paper

Plane Symmetric Inflationary Universe with Hybrid Expansion Law and Time Varying Λ

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Abstract

Plane symmetric inflationary cosmological model using Hybrid Expansion Law (HEL) with flat potential and time varying Λ in General Theory of Relativity has been studied. The solution of the Einstein's field equations is obtained under the assumption of HEL which yields a time-dependent deceleration parameter presenting transition of the universe from the early decelerating phase to the recent accelerating phase. The physical and kinematical parameters of the models have been studied and discussed.

Keywords: *Plane Symmetric Universe; Higgs Field; Hybrid Expansion Law.*

1. Introduction

The idea of inflation that the universe experienced an exponential expansion at its very early stage seems to have been well established in the field of cosmology [1]. Among many versions of inflation, a class of them called the new inflation has turned out to be most successful [1]. Although there is no definite theory of new inflation can naturally explain the homogeneity, isotropy and flatness of the universe supports the basic validity of the idea .The essence of the new inflationary universe scenario is that assume a period of exponential expansion governed by the potential energy of a spatially homogenous classical scalar field. A detail investigation has been done by Guth and Pi [2] indicating that the scalar field can be regarded as a classical variable under certain circumstances and the slow-rollover picture is reliable.
Scalar field are the simplest classical fields and there exist an extensive literature containing numerous solutions of the Einstein's equations where the scalar field is minimally coupled to the gravitational field. In particular, self-interacting scalar fields play a central role in the study of inflationary cosmology. Several authors like [3-8] has studied different aspect of scalar field in the evolution of the universe and FRW models whereas [9-11] have studied the role of self-interacting scalar fields in inflationary cosmology.
The concept of Higgs field ϕ with potential $V(\phi)$ that has the flat region and the ϕ field evolves slowly but the universe expands in an exponential way due to vacuum field energy. In General Theory of Relativity, scalar fields facilitates in explaining the creation of matter in cosmological theories and may additionally describe the uncharged field. Reddy et al. [12-13] have discussed inflationary universes in general relativity in four and five dimensions. Also the same author [14] investigated a plane symmetric Bianchi type –I inflationary universe in general relativity. Katore and Rane [15] have studied the Kantowski-Sach inflationary universe in general relativity. Inflationary scenario in locally rotationally symmetric Bianchi Type-II space-time with masse less scalar field with flat potential have been discussed by Bali and Laxmi [16]. Recently, Bali and Swati [17] investigated LRS Bianchi Type-II inflationary scenario for a mass less scalar field and flat potential and time varying Λ. Kaluza-Klien inflationary universe in general relativity has been studied by Adhav [18]. A five dimensional Bianchi Type-I inflationary universe is investigated in the presence of mass less scalar field with a flat potential and plane symmetric inflationary universe with mass less scalar field and time varying Λ studied by Katore et al.[19- 20].
Motivated from the study outlined above in this paper, inflationary universe in a plane symmetric coordinate system with flat potential, time varying Λ and using the scale factor $a = a_0 t^{\alpha} e^{\beta t}$, where $a_0 > 0, \alpha \geq 0$ and $\beta \geq 0$ are constants is investigated. This generalized form (being the mixture of power-law and exponential-law cosmologies) of scale factor is called as the HEL. It is observed that the HEL leads to the power-law cosmology for $\beta = 0$ and the exponential-law for $\alpha = 0$. Thus, the power-law and exponential-law cosmology are the special cases of the HEL cosmology. Recently Akarsu et al. [21] studied the scalar fields with potential responsible for the evolution of the universe under the HEL within the framework of FRW space time in general relativity. Anisotropic Bianchi Type-V model with HEL has been studied by Kumar [22]. Raut et al. [23] have studied the Kantowski-Sachs cosmological model with anisotropic dark energy with HEL. Bianchi Type-VI$_0$ cosmological model of the universe with HEL has been studied by Adhav et al. [24].

2. Metric and field equations

Consider the plane symmetric metric in the form as

$$ds^2 = dt^2 - A^2(dx^2 + dy^2) - B^2 dz^2 , \tag{1}$$

where A and B are the scale factors (metric tensors) and functions of cosmic time t only.
Using the Stein-Schabes [25] approach with the natural unit $(8\pi G = 1$ and $c = 1)$, the Lagrangian is that of gravity minimally coupled to scalar field $V(\phi)$ given by

$$L=\int\sqrt{-g}\left(R-\frac{1}{2}g^{ij}\partial_i\phi\partial_j\phi-V(\phi)\right)d^4x\ , \tag{2}$$

Now from the variation of L with respect to the dynamical fields, the Einstein field equations for mass less scalar field $V(\phi)$ is given by

$$R_{ij}-\frac{1}{2}Rg_{ij}+\Lambda g_{ij}=-T_{ij}\ , \tag{3}$$

where ,

$$T_{ij}=\partial_i\phi\ \partial_j\phi-\left[\frac{1}{2}\partial_\rho\phi\ \partial^\rho\phi+V(\phi)\right]g_{ij}\ , \tag{4}$$

with

$$\frac{1}{\sqrt{-g}}\partial_i\left[\sqrt{-g}\ \partial^i\phi\right]=-\frac{dV}{d\phi}\ , \tag{5}$$

where g_{ij} is the metric tensor, v^i is the flow vector, ϕ is the Higgs field, $V(\phi)$ is the effective potential and Λ is the cosmological constant .

The corresponding field equations (3) for metric (1) with the help of equation (4) can be written as

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\Lambda=-\left[\frac{1}{2}\dot{\phi}^2+V(\phi)\right], \tag{6}$$

$$2\frac{\ddot{A}}{A}+\frac{\dot{A}^2}{A^2}\ -\Lambda=-\left[\frac{1}{2}\dot{\phi}^2+V(\phi)\right], \tag{7}$$

$$\frac{\dot{A}^2}{A^2}+2\ \frac{\dot{A}\dot{B}}{AB}-\Lambda=\left[\frac{1}{2}\dot{\phi}^2-V(\phi)\right]. \tag{8}$$

Using equation (5) for scalar field ϕ leads to

$$\ddot{\phi}+\left(2\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)\dot{\phi}=-\frac{dV}{d\phi}\ , \tag{9}$$

where the overhead dot (·) denote derivative with respect to the cosmic time *t*.

3. Solutions of the field equations

To solve the differential equations (6)-(8) in term of cosmic time t, assume the scale factor of the form

$$a=a_0\,t^\alpha\,e^{\beta t}\ , \tag{10}$$

where $\alpha_0>0$, $\alpha\geq 0$ and $\beta\geq 0$ are constants.

The spatial volume (V) and average scale factor (a) of the universe are define as

$$V=A^2B\quad ,\quad a=\left(A^2B\right)^{1/3}. \tag{11}$$

The directional Hubble's parameters in the directions of x, y and z axes respectively are defined as

$$H_x=H_y=\frac{\dot{A}}{A},\quad H_z=\frac{\dot{B}}{B}. \tag{12}$$

The mean Hubble parameter (H) is defined as

$$H=\frac{1}{3}\frac{\dot{V}}{V}=\frac{1}{3}\left(\frac{2\dot{A}}{A}+\frac{\dot{B}}{B}\right). \tag{13}$$

The deceleration parameter (q) is given by

$$q=-\frac{a\ddot{a}}{\dot{a}^2}. \tag{14}$$

By assuming that the region is flat, so for flat region the effective potential is constant.

$$V(\phi)=\text{constant}=k\ \text{(say)}. \tag{15}$$

Solving equation (9), we get

$$\dot{\phi}=\frac{c_1}{V}\ , \tag{16}$$

where c_1 is a constant of integration.

Using equation (10), (11) and (16), yields to

$$\dot{\phi}=c_1\,a_0^{-3}t^{-3\alpha}e^{-3\beta t}. \tag{17}$$

Integrating above equation (17), the scalar Higgs field is obtained as

$$\phi=-\ c_1\,a_0^{-3}\left(3\beta\right)^{3\alpha-1}\gamma\left[1-3\alpha,3\beta t\right]+c_2\ , \tag{18}$$

where c_2 is constant of integration.

The Higgs field decreases slowly and for large values of t, it tends to a constant value c_2 . However, if the constant c_2 is zero for large values of t, it tends to zero.

To get the deterministic solution, assume that $\Lambda\approx a^{-2}$ as considered by Chen and Wu [26]. Thus

$$\Lambda=\frac{1}{a^2}=a_0^{-2}t^{-2\alpha}e^{-2\beta t}\ , \tag{19}$$

where a is the average scale factor .

Subtracting equation (6) from equation (7), one may get

$$\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B}-\frac{\dot{A}^2}{A^2}-\frac{\dot{A}\dot{B}}{AB}=0\ , \tag{20}$$

Solving equation (20) and using equation (11), yields

$$\frac{d}{dt}\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)+\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)\frac{\dot{V}}{V}=0\,. \tag{21}$$

Integrating the above equation (21) gives

$$\frac{\dot{A}}{A}-\frac{\dot{B}}{B}=c_3\,a_0^{-2}\,t^{-2\alpha}\,e^{-2\beta t}\ , \tag{22}$$

where c_3 is constant of integration.

Solving equation (22), the values of scale factors are obtained as

$$A=\lambda^{1/3}\,a_0\,t^\alpha\,e^{\beta t}\exp\left(-\frac{c_3\,a_0^{-3}}{3}\left(3\beta\right)^{3\alpha-1}\gamma\left[1-3\alpha,3\beta t\right]\right), \tag{23}$$

$$B=\lambda^{-2/3}\,a_0\,t^\alpha\,e^{\beta t}\exp\left(\frac{2c_3\,a_0^{-3}}{3}\left(3\beta\right)^{3\alpha-1}\gamma\left[1-3\alpha,3\beta t\right]\right), \tag{24}$$

where c_3 and λ are the constants of integration and γ represents the lower incomplete gamma function . Here for the realistic value of the scale factors, $\alpha\leq 1/3$.

Using equation (23) and (24) in equation (1) and after choosing suitable transformation coordinates and with choice of constants, the plane symmetric inflationary model is given by

$$ds^2=dt^2-t^\alpha\,e^{\beta t}\exp\left(-\frac{c_3\,a_0^{-3}}{3}\left(3\beta\right)^{3\alpha-1}\gamma\left[1-3\alpha,3\beta t\right]\right)\left(dx^2+dy^2\right)$$
$$-t^\alpha\,e^{\beta t}\exp\left(\frac{2c_3\,a_0^{-3}}{3}\left(3\beta\right)^{3\alpha-1}\gamma\left[1-3\alpha,3\beta t\right]\right)dz^2\ . \tag{25}$$

It is observed that the model in equation (25) initially has point type singularity at $t=0$ and it represents anisotropic space time in general.

4. Physical properties

The spatial volume (V), the mean Hubble parameter (H) and the expansion scalar (θ) are respectively given by

$$V=a_0^3\,t^{3\alpha}\,e^{3\beta t}\ . \tag{26}$$

$$H=\frac{\alpha}{t}+\beta\,. \tag{27}$$

$$\theta=3H=3\left(\frac{\alpha}{t}+\beta\right). \tag{28}$$

The average expansion anisotropy parameter (A_m) is defined as

$$A_m=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2\ ,$$

where $\Delta H_i=H_i-H$, which gives

$$A_m=\frac{5}{27}\frac{c_3^2\,a_0^{-6}\,t^{-6\alpha}\,e^{-6\beta t}}{\left(\alpha t^{-1}+\beta\right)^2}\ . \tag{29}$$

For isotropy we need $A_m = 0$. Therefore this considered plane symmetric universe isotropizes in two ways, first if the value of constant $c_3 = 0$ and second is for the large expansion of universe i.e. when $t \to \infty$.

The shear scalar (σ^2) is define and given by

$$\sigma^2 = \frac{3}{2} A_m H^2$$

$$\sigma^2 = \frac{5}{18} c_3^2 a_0^{-6} t^{-6\alpha} e^{-6\beta t}. \quad (30)$$

From equation (30) it is observed that for the value of constant $c_3 = 0$ and for the large expansion of the universe the shear scalar vanishes hence the universe approaches to isotropy.

The deceleration parameter (q) is given by

$$q = \frac{\alpha}{(\alpha + \beta t)^2} - 1. \quad (31)$$

Initially when the universe starts to expand, the sign of q becomes positive, which correspond to the standard decelerating behavior, which is consistent with the recent observations [28-30] as well as with the high red shifts of type Ia supernova, whereas with the expansion of the universe it is zero thus the model leads to de-sitter space time and the sign of q become negative, which correspond to the standard accelerating behavior of the universe, This scenario is also consistent with recent observations. This value is very near the observed value of deceleration parameter i.e. $-1 \le q \le 0$. This implies that the Bianchi type-I space-time shows flipping, from the decelerating to the accelerating phase. This behavior is depicted in Fig.1.

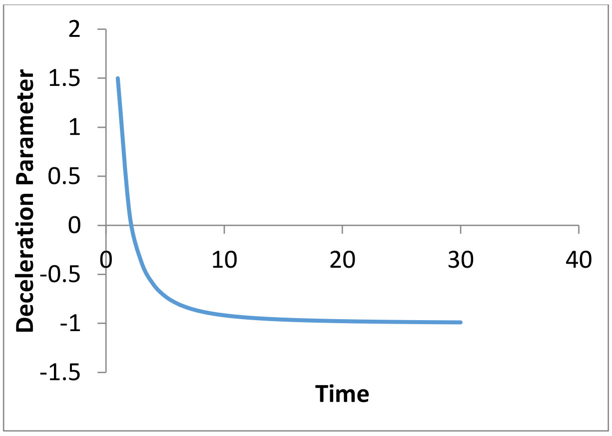

Fig. 1: Deceleration Parameter versus Cosmic Time with Appropriate Choice of Constants.

5. Conclusion

The plane symmetric inflationary cosmological model in the presence of scalar field and Hybrid Expansion Law with flat region of constant potential and time varying Λ is presented and discussed. It has been shown that the plane symmetric metric in the equation (25) is isotropized under the special condition as pointed out by Rothman and Ellis [27].

In this model with HEL, the time dependent deceleration parameter q has been obtained. A negative sign of q corresponds to accelerating model of the universe while positive q indicates the deceleration. Also in this model it is observed that at $t = \frac{\sqrt{\alpha} - \alpha}{\beta}$, the universe expand with constant exponent. The spatial volume increases as time increases. Thus with the expansion an inflationary scenario exists in plane symmetric universe. The expansion scalar θ is infinite at $t = 0$. As $t \to \infty$, it is observed that $\theta \to \beta$, $q \to -1$ and $\frac{dH}{dt} \to 0$ which implies the greatest value of the Hubble's parameter and the shear scalar $\sigma = 0$. Thus for large expansion it represents isotropic space-time in general. The model leads to de-sitter space time and represents accelerating universe. One should note that even though we consider Hybrid Expansion Law, the inflationary universe model obtain the considerable astrophysical significance.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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