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Research paper

Modified Adomian Decomposition Method for Solving Singular Differential Equation of Lane-Emden Type

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Abstract

This paper presents a novel adaptation of the Adomian Decomposition Method specifically designed to solve Lane-Emden equations more effectively. Unlike traditional approaches, our method eliminates the need for integral operators, a common source of computational complexity in previous versions of the Adomian method. By relying exclusively on differential operators, we simplify the problem-solving process, allowing for direct differentiation without the approximation challenges associated with integral coefficients. This enhancement not only streamlines the computational workload but also significantly improves both the accuracy and efficiency of solutions to Lane-Emden equations. We provide rigorous demonstrations of our method through various examples, illustrating marked advantages over conventional techniques in terms of convergence rates and computational simplicity. The unique contribution of this work lies in its potential to inspire further research and applications across diverse fields that utilize higher-order differential equations.

Keywords: *Modified Adomian Method, Numerical Method, Singular Equations, Lane-Emden Equations, linear and Nonlinear Equations*

1. Introduction

The Adomian Decomposition Method (ADM), developed by George Adomian in the 1980s [1], has emerged as a powerful analytical tool for solving a wide range of linear and nonlinear differential equations. Its ability to generate both approximate and exact solutions without the need for linearization or discretization has led to extensive exploration and numerous refinements. Significant efforts have been made to enhance the method's accuracy and broaden its applicability, particularly through the development of different differential operators [2, 3]. In the mid-1990s, Adomian and Rach [4, 5] introduced updated versions of the Adomian polynomials, which are optimized for computational efficiency and demonstrate faster convergence than classical forms. Subsequently, Wazwaz [6] proposed a reliable modification by partitioning the initial term and restructuring the series to improve convergence rates. The versatility of ADM has been further demonstrated in recent studies; for instance, Zeidan et al. [7–9] provided comprehensive frameworks for solving Burgers' equations, the Riemann problem, and systems with non-prescribed data, highlighting the method's computational efficiency over traditional numerical techniques.
Recent literature from 2023 to 2026 reflects a continuous momentum in refining ADM variants for specialized physical models. Alsulami et al. [10] explored the application of ADM to special equations in physical sciences, addressing unique computational challenges. Furthermore, Dabwan and Hasan [11] introduced a modified ADM approach for higher-order ordinary differential equations, applying it to MHD flows and elastic beam problems. In early 2026, the framework was further extended to solve third-order Emden-Fowler equations [12], showcasing its adaptability to complex nonlinear structures.
While the majority of existing modifications rely heavily on integral operators, the present work introduces a fundamental methodological shift. We eliminate the dependence on integral operators, which often require complex approximation techniques for coefficients, and instead focus on a specifically designed differential operator. This approach significantly streamlines the calculation process, as differentiation is inherently more straightforward than integration in many singular contexts.
The primary objective of this research is to present a significant modification to the Adomian method centered on the exclusive use of differential operators to solve singular differential equations, particularly the Lane-Emden equations. Given the crucial role of these equations in astrophysics, fluid mechanics, and nonlinear dynamics, our proposed formulation provides a more robust and efficient computational framework for handling singularities.
In this study, we have significantly extended the scope of the paper by introducing a generalized framework of the modified differential operator. While the initial study focused on second-order equations, we now demonstrate its efficiency for third, fourth, and nonlinear

fifth-order singular problems. Additionally, a benchmarking analysis has been included to compare the results with the classical Adomian Decomposition Method (ADM).

2. Formulation of the technique

In this section, the new modification presented in this article will be explained, as the authors will study the following equation, and this modification is effective in solving this equation.
Consider the class of the form's singular value problem (Lane-Emden equation):

$$y'' + \frac{n}{x}y' + y = f(x,y) + g(x). \tag{1}$$

where ($f(x,y)$) and ($g(x)$) are real functions and n is integer number.
In an operator form, we write eq.(1) as:

$$Dy + y = f(x,y) + g(x), \tag{2}$$

where

$$Dy = x^{-n}\frac{d}{dx}x^n\frac{d}{dx}y,$$

we can write eq.(2) as

$$(D+1)y = f(x,y) + g(x),$$

$$y = \frac{1}{D+1}(f(x,y) + g(x)), \tag{3}$$

$$= (1 - D + D^2 - D^3 + \ldots)(f(x,y) + g(x)),$$

$$= \sum_{k=0}^{L}(-1)^k D^k (f(x,y) + g(x)),$$

The method by Adomian is given the solution $y(x)$

$$y(x) = \sum_{n=0}^{\infty} y_n(x), \tag{4}$$

and

$$f(x,y) = \sum_{n=0}^{\infty} A_n, \tag{5}$$

where the elements $y_n(x)$ of the solution $y(x)$ will be determined repeatable. Specific algorithms were seen in [1] to formulate Adomian polynomials. The following algorithm:

$$A_0 = F(y_0),$$

$$A_1 = y_1 F'(y_0),$$

$$A_2 = y_2 F'(y_0) + \frac{1}{2!}y_1^2 F''(y_0),$$

$$A_3 = y_3 F'(y_0) + y_1 y_2 F''(y_0) + \frac{1}{3!}y_1^3 F'''(y_0), \tag{6}$$

$$\ldots$$

The component $y(x)$ can be given by using Adomian decomposition method as follows

$$y_0 = \sum_{k=0}^{L}(-1)^k D^k g(x),$$

$$y_{(n+1)} = \sum_{k=0}^{L}(-1)^k D^k \sum_{n=0}^{\infty} A_n. \tag{7}$$

3. Theoretical Analysis

3.1. Convergence of the Method

To establish the mathematical rigor of the proposed modification, we provide a proof of convergence in the context of Banach spaces.

Theorem 1. *Let E be a Banach space of analytic functions. The series solution $y = \sum_{k=0}^{\infty}(-1)^k D^k G(x)$ produced by the modified method converges in E if there exists a constant $0 \leq \alpha < 1$ such that the differential operator D satisfies the contraction condition:*

$$\|D^{k+1}G\| \leq \alpha \|D^k G\|, \quad \forall k \geq 0. \tag{8}$$

Proof. Let $S_m = \sum_{k=0}^{m}(-1)^k D^k G$ be the sequence of partial sums. We aim to show that $\{S_m\}$ is a Cauchy sequence in E. The difference between two consecutive partial sums is:

$$\|S_{m+1} - S_m\| = \|(-1)^{m+1} D^{m+1} G\| = \|D^{m+1}G\|. \tag{9}$$

By applying the contraction property recursively, we have:

$$\|D^{m+1}G\| \leq \alpha \|D^m G\| \leq \alpha^2 \|D^{m-1}G\| \leq \cdots \leq \alpha^{m+1}\|G\|. \tag{10}$$

For any $p > q$, the distance between S_p and S_q is:

$$\|S_p - S_q\| = \left\| \sum_{k=q+1}^{p} (-1)^k D^k G \right\| \leq \sum_{k=q+1}^{p} \|D^k G\|. \tag{11}$$

Summing the geometric progression, we obtain:

$$\|S_p - S_q\| \leq \sum_{k=q+1}^{p} \alpha^k \|G\| = \|G\| \frac{\alpha^{q+1}(1-\alpha^{p-q})}{1-\alpha}. \tag{12}$$

Since $0 \leq \alpha < 1$, it follows that $\lim_{q\to\infty} \|S_p - S_q\| = 0$. Thus, $\{S_m\}$ is a Cauchy sequence, which implies the series converges to a limit $y \in E$. □

3.2. Error Estimation and Stability

Based on the convergence established in Theorem 1, we can determine the upper bound for the truncation error. If the series is truncated at the L-th term, the remainder error $R_L(x)$ is defined as:

$$R_L(x) = y(x) - \sum_{k=0}^{L}(-1)^k D^k G(x) = \sum_{k=L+1}^{\infty} (-1)^k D^k G(x). \tag{13}$$

The norm of the remainder error is bounded by:

$$\|R_L(x)\| \leq \frac{\alpha^{L+1}}{1-\alpha}\|G\|. \tag{14}$$

This inequality ensures that the error vanishes as $L \to \infty$. Notably, for the Lane-Emden equations presented in the following section, $G(x)$ consists of polynomials. Since D is a differential operator, there exists a finite k such that $D^k G = 0$, leading to $\|R_L(x)\| = 0$. This theoretically justifies why the proposed method achieves the exact solution in finite steps.

4. Applications

In this section, we presented three examples that demonstrate the success of the method used in this paper to solve second order singular value problem.

Example 1. *Consider the following linear Lane-Emden equation [2, 13–17]*

$$y'' + \frac{2}{x}y' + y = 6 + 12x + x^2 + x^3. \tag{15}$$

The exact solution is $y(x) = x^2 + x^3$.
We can write equation (15) in form

$$Dy + y = 6 + 12x + x^2 + x^3,$$

$$(D+1)y = 6 + 12x + x^2 + x^3,$$

$$y = \frac{1}{D+1}(6 + 12x + x^2 + x^3),$$

$$y = \sum_{k=0}^{L}(-1)^k D^k (6 + 12x + x^2 + x^3),$$

$$y = (1 - D + D^2 - D^3 + ...)(6 + 12x + x^2 + x^3) \tag{16}$$

where

$$Dy = x^{-2}\frac{d}{dx}x^2\frac{d}{dx}y. \tag{17}$$

$$y = 6 + 12x + x^2 + x^3 - D(6 + 12x + x^2 + x^3) + D^2(6 + 12x + x^2 + x^3).$$

We get:

$$y = 6 + 12x + x^2 + x^3 - (6 + \frac{24}{x} + 12x) + (\frac{24}{x}) + 0,$$

$$= x^2 + x^3.$$

As we noted, the exact solution is obtained directly, without resorting to complex calculations. This demonstrates the efficiency of the proposed method for solving this type of equation.

Example 2. *Consider the singular differential equation [18, 19]*

$$y'' + \frac{1}{x}y' + y = \frac{5}{4} + \frac{x^2}{16}, \tag{18}$$

the exact solution is $y(x) = 1 + \frac{x^2}{16}$.
Rewrite eq.(18) in the form

$$Dy + y = \frac{5}{4} + \frac{x^2}{16},$$

$$(D+1)y = \frac{5}{4} + \frac{x^2}{16},$$

$$y = \frac{1}{D+1}(\frac{5}{4} + \frac{x^2}{16}),$$

$$y = \sum_{k=0}^{L}(-1)^k D^k(\frac{5}{4} + \frac{x^2}{16}),$$

$$= (1 - D + D^2 - D^3 + ...)(\frac{5}{4} + \frac{x^2}{16}),$$

where

$$Dy = x^{-1}\frac{d}{dx}x^1\frac{d}{dx}y,$$

$$y = \frac{5}{4} + \frac{x^2}{16} - \frac{1}{4} + 0,$$

$$= 1 + \frac{x^2}{16}.$$

The exact solution is obtained.

Example 3. *Consider the following non-linear equation [3]*

$$y'' + \frac{2}{x}y' + y = 6 + x^2 - e^{x^2} + e^y. \tag{19}$$

The exact solution is $y(x) = x^2$.
We can write equation (19) in form

$$Dy + y = 6 + x^2 - e^{x^2} + e^y,$$

$$(D+1)y = 6 + x^2 - e^{x^2} + e^y,$$

$$y = \frac{1}{D+1}(6 + x^2 - e^{x^2} + e^y),$$

$$y = \sum_{k=0}^{L}(-1)^k D^k(6 + x^2 - e^{x^2} + e^y),$$

$$= (1 - D + D^2 - D^3 + ...)(6 + x^2 - e^{x^2} + e^y),$$

where

$$Dy = x^{-2}\frac{d}{dx}x^2\frac{d}{dx}y. \tag{20}$$

$$y = (1 - D + D^2 - D^3 + ...)(6 + x^2 - e^{x^2}) + (1 - D + D^2 - D^3 + ...)(e^y),$$

$$y = x^2 - e^{x^2}(55 + 76x^2 + 16x^4) + (1 - D + D^2 - D^3 + ...)(e^y).$$

Using the method in [6] we obtain

$$y_0 = x^2,$$

$$y_1 = -e^{x^2}(55 + 76x^2 + 16x^4) + (1 - D + D^2 - D^3 + ...)(A_0),$$

$$y_1 = -e^{x^2}(55 + 76x^2 + 16x^4) + (1 - D + D^2 - D^3 + ...)(e^{x^2}),$$

$$= -e^{x^2}(55 + 76x^2 + 16x^4) + e^{x^2}(55 + 76x^2 + 16x^4) = 0,$$

$$y_{m+1} = D(A_m) = 0, \qquad m \geq 0.$$

Then

$$y = x^2.$$

To evaluate the performance of the proposed differential operator, we provide a numerical comparison for the results obtained in Example 3. As shown in Table 2, the absolute error is calculated by comparing our results with the exact solution. The data demonstrates that our method achieves the exact solution with zero absolute error, which confirms the high precision and efficiency of the differential operator compared to traditional methods.

x	Exact	Proposed Method	MADM in [3]	SADM [3]
0.	0.	0.	0.	0.
0.1	0.01	0.01	0.0100033	0.009919
0.2	0.04	0.04	0.040053	0.039732
0.3	0.09	0.09	0.090269	0.087944
0.4	0.16	0.16	0.160850	0.155278
0.5	0.25	0.25	0.252071	0.241076
0.6	0.36	0.36	0.364283	0.345113
0.7	0.49	0.49	0.497910	0.467239
0.8	0.64	0.64	0.653445	0.607400
0.9	0.81	0.81	0.831448	0.765645
1.	1.	1.	1.032540	0.942146

Table 1: Numerical comparison of the proposed MADM results with the exact solution and other existing methods.

x	Error (Proposed)	Error in [3] (MADM)	Error (Standard) in [3]
0.	0.	0.	0.
0.1	0.	3.33×10^{-6}	8.088×10^{-5}
0.2	0.	5.328×10^{-5}	0.00062799
0.3	0.	0.00026942	0.00205597
0.4	0.	0.00085008	0.00472246
0.5	0.	0.00207093	0.00892379
0.6	0.	0.00428297	0.01488729
0.7	0.	0.00790996	0.02276077
0.8	0.	0.01344528	0.03260004
0.9	0.	0.02144822	0.14435482
1.	0.	0.03253968	0.05785384

Table 2: Comparison of absolute errors between the proposed MADM and traditional approaches.

The comparison in the tables above shows that our method is much more accurate. While other methods produced extra terms, our approach directly reached the exact solution $y(x) = x^2$. This resulted in a **zero error** at all points, proving that the new modification is faster and avoids the mistakes found in previous studies.

The examples 1,2 and 3 illustrate that the modified method allows for straightforward attainment of complete solutions for both linear and nonlinear equations. This demonstrates the efficiency of the new modification, which exclusively utilizes the differential operator, in contrast to the traditional Adomian method and its various adaptations. We employed Mathematica to tackle the examples presented.

5. Generalization of the Modified ADM for n^{th}-Order Singular Equations

In this section, we present the general formulation of the modified Adomian Decomposition Method (ADM) for solving n^{th}-order nonlinear singular differential equations. Consider the following operator form:

$$(D+\lambda)y+f(x,y)=g(x), \quad n\geq 2 \tag{21}$$

where λ is a constant coefficient, $N(y)$ represents the nonlinear part, and D is the generalized n^{th}-order singular differential operator defined as:

$$D=x^{-c}\frac{d}{dx}\left(x^{c}\frac{d^{n-1}}{dx^{n-1}}\right) \tag{22}$$

To avoid the complexities of the multiple integral operator L^{-1}, we employ the inverse operator $(\mathscr{D}+\lambda)^{-1}$ through its geometric series expansion:

$$(D+\lambda)^{-1}=\frac{1}{\lambda}\sum_{k=0}^{\infty}\left(-\frac{1}{\lambda}D\right)^{k} \tag{23}$$

Substituting the decomposition series $y=\sum_{m=0}^{\infty}y_m$ and the Adomian polynomials A_m for $N(y)$, we obtain the generalized recurrence relation:

$$y_0=\frac{1}{\lambda}\sum_{k=0}^{L}\left(-\frac{1}{\lambda}D\right)^{k}g(x) \tag{24}$$

$$y_{m+1}=-\frac{1}{\lambda}\sum_{k=0}^{L}\left(-\frac{1}{\lambda}D\right)^{k}A_m, \quad m\geq 0 \tag{25}$$

This formulation demonstrates that the solution can be constructed using only differential operations, effectively bypassing the singular points at $x=0$ without the need for integration.

6. Numerical Example

Example 4. *Consider the third-order singular equation:*

$$y'''+\frac{3}{x}y''+4y=24+4x^{3}, \tag{26}$$

The operator form is:

$$(D+4I)y=24+4x^{3}, \tag{27}$$

where $Dy=x^{-3}\frac{d}{dx}(x^{3}y'')$. Applying the inverse operator expansion:

$$y=\frac{1}{4}\left[1-\frac{D}{4}+\frac{D^{2}}{16}-\dots\right](24+4x^{3}). \tag{28}$$

Substituting these into the series solution:

$$y=\frac{1}{4}[(24+4x^{3})-24]=x^{3}. \tag{29}$$

The method converges precisely to the exact solution $y(x)=x^{3}$.

Example 5. *Consider the nonlinear singular equation of order $n=4$:*

$$y^{('''')}+\frac{5}{x}y^{('')}+4y=4\left(x^{3}+1\right)+\frac{24}{x}, \tag{30}$$

with $x^{3}+1$ as the exact solution.

$$(D+4)y=4\left(x^{3}+1\right)+\frac{24}{x}, \tag{31}$$

where $Dy=x^{-4}\frac{d}{dx}(x^{4}y''')$. Applying the inverse operator expansion:

$$y=\frac{1}{4}\left[I-\frac{D}{4}+\frac{D^{2}}{16}-\dots\right](4\left(x^{3}+1\right)+\frac{24}{x}). \tag{32}$$

Substituting these into the series solution:

$$y=\frac{1}{4}\left(4x^{3}+\frac{24}{x}+4\right)-\frac{6}{x}=1+x^{3}. \tag{33}$$

The method converges precisely to the exact solution $y(x)=1+x^{3}$.

Example 6. *Consider the nonlinear singular equation of order $n = 5$:*

$$y^{(v)} + \frac{5}{x} y^{(iv)} + y + y^2 = 720 + x^5 + x^{10}, \tag{34}$$

with $y(x) = x^5$ as the exact solution.
The operator for this case is:

$$D = x^{-5} \frac{d}{dx} \left(x^5 \frac{d^4}{dx^4} \right) \tag{35}$$

The recursive components are calculated as:

$$y = (1 - D + ...)(720 + x^5 + x^{10}) - (1 - D + ...)(A_n) \tag{36}$$

$$= x^5 + x^{10} - D(x^{10}) - (1 - D + ...)(A_n) \tag{37}$$

Let

$$y_0 = x^5, \tag{38}$$

and

$$y_1 = x^{10} - D(x^{10}) + (1 - D)(-A_0), \tag{39}$$

$$= x^{10} - D(x^{10}) + (1 - D)(-(x^5)^2), \tag{40}$$

It is clear that the noise terms x^{10} and $D(x^{10})$ cancel each other out, leading to:

$$y(x) = y_0 + y_1 = x^5 \tag{41}$$

These higher-order examples serve as a theoretical benchmark. It is observed that our differential approach recovers the exact analytical solution with zero residual error ($Res = 0$), avoiding the labor-intensive multiple integrations required by traditional methods for 5th-order systems.

7. Conclusion

This study introduces a substantial evolution in the resolution of second-order singular value problems through a **generalized modification** of the traditional Adomian Decomposition Method (ADM). A primary contribution of this research is the development of a unified and robust differential operator that successfully extends the method's applicability beyond second-order models to **higher-order singular equations**, including linear and nonlinear cases of the **third, fourth, and fifth orders**.
By leveraging these specifically designed differential operators, we have effectively eliminated the complications and computational burdens typically associated with traditional integral operators. The numerical results across these higher-order models—benchmarked in this revised version—demonstrate that our generalized framework ensures **exact analytical solutions** with a residual error of zero ($Res = 0$).
The implications of these findings are profound, particularly for the **Lane-Emden** equations and other multi-singular systems vital in astrophysics and fluid mechanics. Notably, our approach maintains mathematical elegance and significantly reduces the algebraic steps required for 5th-order systems, thereby outperforming classical and existing modified versions in terms of both speed and reliability. In conclusion, this work not only enriches the existing body of knowledge within the realm of differential equations but also sets a solid foundation for future research aimed at exploring even more complex nonlinear physical phenomena.
In conclusion, the proposed modified differential operator has demonstrated high efficiency and accuracy in solving various classes of singular nonlinear differential equations. As a direction for future research, we intend to generalize this framework to include non-singular differential equations. This expansion will aim to establish a unified computational tool capable of handling a broader spectrum of nonlinear problems, regardless of the presence of singularities, thereby enhancing its applicability across diverse physical and engineering models.

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