

# Host-Parasitoid Model With Intraspecific Competitions

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## Abstract

In this paper a discrete-time host-parasitoid model with intraspecific competitions is proposed. Phase portraits are drawn for different types of intraspecific competitions to notice the effects of intraspecific competitions for biologically reasonable range of parameter values. Bifurcation analysis is done with respect to instantaneous search rate as well as handling time for different types of intraspecific competitions. Many forms of complex dynamics such as chaos, periodic windows etc. are observed. The stable coexistence as well as oscillatory coexistence of host and parasitoid is shown under different types of intraspecific competitions. The Hopf point and attractor crises exist for different intraspecific competitions.

**Keywords:** *Host-parasitoid; Intraspecific competitions; Chaos; Periodic windows; Oscillatory coexistence, Hopf.*

## 1 Introduction

Since 1974 the ecological research detected chaos and other forms of complex dynamics in simple population models which are continuously motivating Biologists, Physicists, Engineers and Applied Mathematicians to study ecological models. Parasitoids are insect species, usually wasps or flies that have at least one free-living and one parasitic stage in their life cycle. The adult female parasitoids usually lay eggs in the larval or pupal stage of its host, which hatch and develop, consuming the host from the inside and eventually killing it before they pupate. About ten percentages of all metazoan (multi-cellular) species on Earth are insect parasitoids. There is no universal Host-Parasitoid model in ecology. Usually, the host-parasitoid models use discrete time steps where time steps corresponding to generations (year). In 1929, Thompson [1] introduced a host-parasitoid model with a set of biological assumption. In 1935, Nicholson and Bailey [2] modified the Thompson's model [1] for better prediction of real life situations. After that, Rogers [3-4] applied the model of Holling to host-parasitoid system by assuming two kinds of limitation on Thompson's model and Nicholson-Bailey's model [2]. Many researchers [5-10] produced many discrete type host-parasitoid models with different ecological factors. Sanyi Tang and Lansun Chen [11] modified the Nicholson-Bailey's model [2, 12] by introducing Holling type-II and type-III interactions.

Competition is an interaction between organisms or species in ecology. Competition is one of many biotic and abiotic factors that affect community structure due to limited resource (food, space and water). Competition among members of same species for the same resources in an ecosystem is known as intraspecific competition. Competition is not straightforward, and can occur in both direct and indirect fashion [13].

In this paper, we focus our attention to observe the effects of intraspecific competition in the host-parasitoid model of Sanyi Tang and Lansun Chen [11]. Our model is a density dependent host-parasitoid model. We have drawn the phase portrait for different values of intraspecific competition parameter. Taking instantaneous search rate as parameter, bifurcation analysis is done for different types of intraspecific competitions. Bifurcation analysis is also done with respect to handling time.

## 2 Model

We consider discrete-time dynamics of Host-Parasitoid interactions. We define  $H_n$  to be the number of hosts at generation  $n$  and  $P_n$  to be the number of parasitoids at generation  $n$ .  $R_0$  to be the host's basic reproductive ratio, i.e. the per capita production of the host in the absence of parasitism. Let  $c$  be the average number of eggs laid by an adult parasitoid in a single host that will survive to breed in the next generation and  $f(H, P)$  is the fraction of host not parasitized.

The census takes place at the beginning of the season, before parasitism takes place, counting adult parasitoids and the stage of the host that is subject to parasitisation. After parasitism, the number of unparasitised host is  $H_n f(H_n, P_n)$ , and the number of parasitized hosts is  $H_n [1 - f(H_n, P_n)]$ . Therefore, the general Host-Parasitoid model is of the following form:

$$\begin{aligned} H_{n+1} &= R_0 H_n f(H_n, P_n) \\ P_{n+1} &= c H_n [1 - f(H_n, P_n)] \end{aligned} \quad (1)$$

Host-parasitoid model of Tang and Chen [11] with Holling type-II functional response is the following

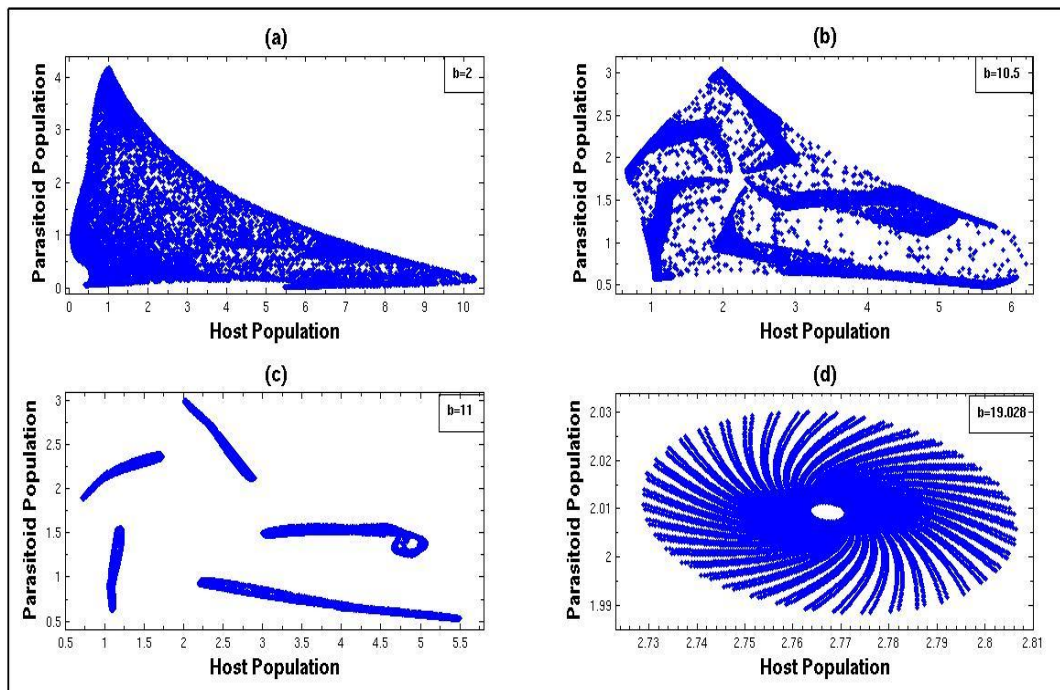
$$\begin{aligned} H_{t+1} &= H_t \exp \left[ r \left( 1 - \frac{H_t}{K} \right) - \frac{a T P_t}{1 + a T_h H_t} \right] \\ P_{t+1} &= H_t \left[ 1 - \exp \left( - \frac{a T P_t}{1 + a T_h H_t} \right) \right] \end{aligned} \quad (2)$$

Here,  $H_t$  and  $P_t$  are the hosts and parasitoids population size in generation  $t$  respectively.  $K$  is the carrying capacity of the environment,  $r$  is the intrinsic growth rate and  $a$  is the instantaneous search rate.  $T$  is searching time and  $T_h$  is the handling time. We formulate the model of host-parasitoid populations such that parasitoids are regulated by intraspecific competition. We shall assume the competition affects the fraction  $F(H_t, P_t)$ . There are various idealized forms of intraspecific competitions. In idealized contest competition exhibits exact compensation for large host density  $H_t$  and parasitoid density  $P_t$ , where increase mortality between time  $t$  and  $t + 1$  compensates exactly for any increase in number

at time  $t$ . Using the concept of Hassell [14], we modify the model (2) by introducing the following model:

$$\begin{aligned} H_{t+1} &= H_t \exp \left[ r \left( 1 - \frac{H_t}{K} \right) - \frac{aTP_t}{(1 + aT_h H_t)^b} \right] \\ P_{t+1} &= H_t \left[ 1 - \exp \left( - \frac{aTP_t}{(1 + aT_h H_t)^b} \right) \right] \end{aligned} \quad (3)$$

For  $b = 0$  there is no competition, for  $0 < b < 1$  under-compensation, for  $b = 1$  exact compensation, for  $b > 1$  over-compensation, and for  $b \rightarrow \infty$  scramble competition. In the next section we present the phase diagrams and bifurcation diagrams for different types of competition.



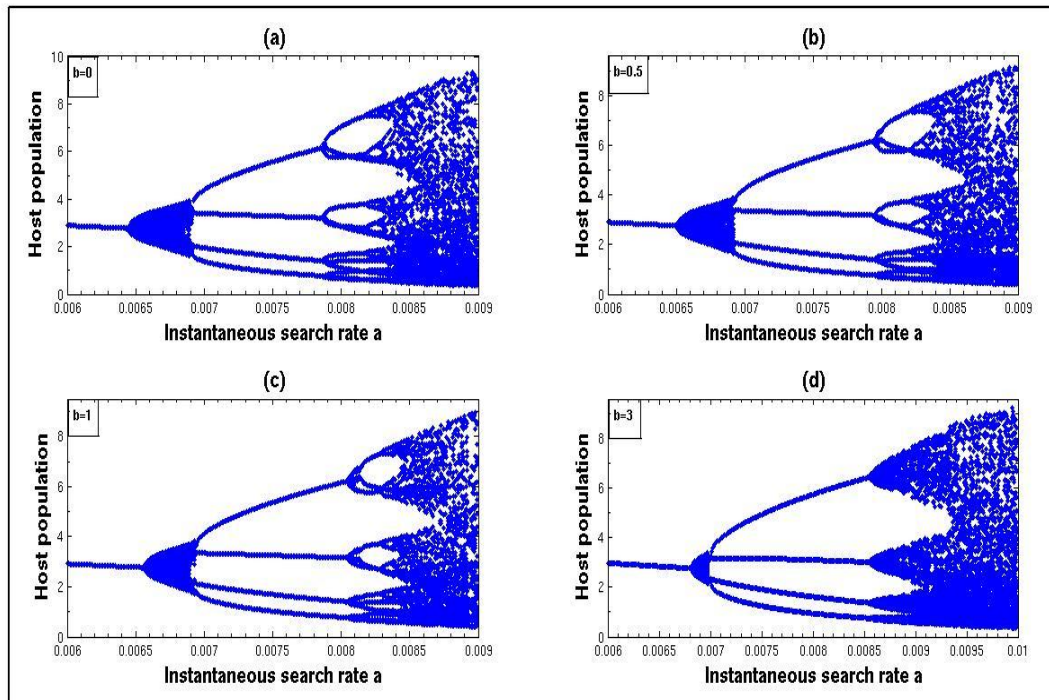
**Figure 1:** Phase portraits of the system (3) for different values of  $b$  taking  $a = 0.012$ .

### 3 Results

We choose  $r = 2.9$ ,  $T = 100$ ,  $T_h = 1$  and  $K = 5$  (Tang and Chen, [11]) for numerical simulation. We present phase diagram taking instantaneous search rate  $a = 0.012$  for different values of competition parameter  $b$ . We have done bifurcation analysis with respect to instantaneous search rate  $a$  as well as handling time  $T_h$  for different type of intraspecific competitions.

### 3.1 Phase Portrait

The phase portraits of the system (3) are displayed in figure 1, which shows The existence of quasi-periodic attractors, circular curves enclosing fixed point and chaotic attractors for different values of  $b$ .



**Figure 2:** Bifurcation diagrams of the system (3) with respect to instantaneous search rate  $a$  for different values of  $b$ .

### 3.2 Bifurcation analysis with respect to instantaneous search rate $a$

Bifurcation diagram with respect to  $a$  for no competition is presented in figure 2(a). For  $0.006 \leq a < 0.0064$ , a stable coexistence between host and parasitoid is observed. After  $a = 0.0064$ , the chaotic bands are observed. Therefore a Hopf bifurcation occurs at  $a = 0.0064$ . When the parameter  $a$  is further increased, chaotic behavior is observed. When the parameter  $a$  is increased beyond  $a = 0.00692$ , the chaotic attractors suddenly disappears. It is observed that the period-4 orbit bifurcates to period-12 orbit starting from  $a \approx 0.00786$ . As parameter  $a$  increases, the period-12 orbit bifurcates to period-24 orbits. In the period-doubling route system goes to chaotic region from  $a \geq 0.0084$ .

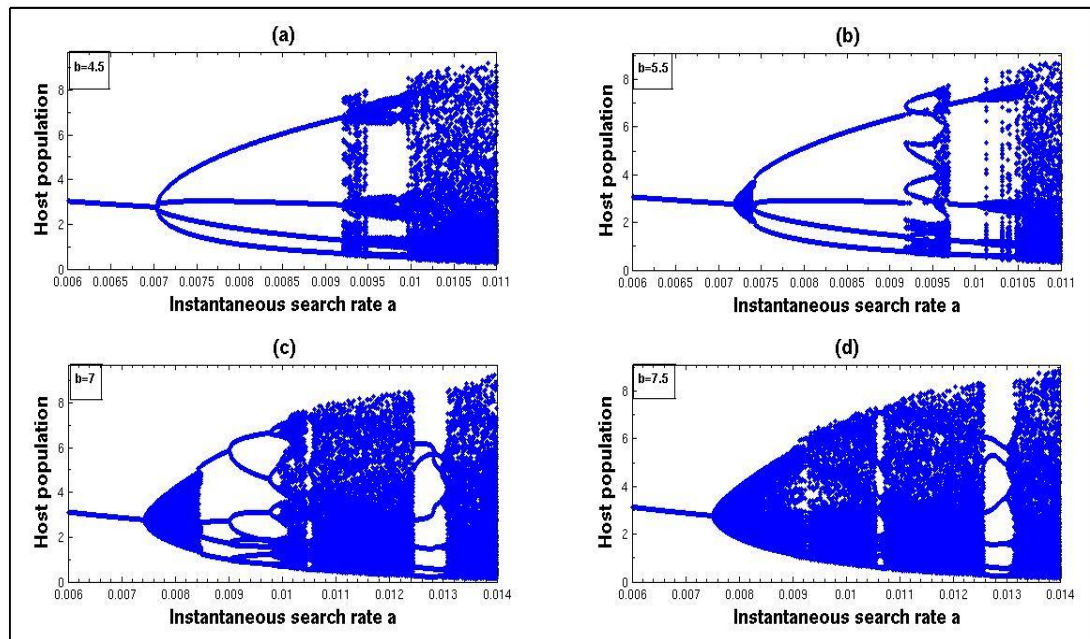
Figure 2(b) is a bifurcation diagram for model (3) for  $b = 0.5$ . A stable coexistence is observed for  $0.006 \leq a < 0.0065$  and chaos occurs after  $a \geq 0.0065$ . A Hopf bifurcation occurs at  $a = 0.0065$ . Beyond  $a = 0.00694$ , the chaotic attractors suddenly disappears, thus constituting a type of attractor crisis. The period-tripling bifurcation starts at  $a \approx 0.00795$ . As parameter  $a$  increases from 0.00795, the period-12 behavior bifurcates to period-24 cycles. We observe the period-doubling route to chaotic in the system for  $a \geq 0.00845$ .

Figure 2(c) is a bifurcation diagram for  $b = 1$ . The stable coexistence is observed when  $a$  belongs to  $(0.006, 0.00655)$ . When the parameter  $a$  further increases, a Hopf bifurcation occurs at  $a \approx 0.00655$ . When the parameter  $a$  is slightly increased beyond  $a = 0.00696$ , the chaotic attractors suddenly disappears. The period-tripling bifurcation starts from  $a \approx 0.00805$ . As parameter  $a$  increases from 0.0083, the period-12 behavior bifurcates to period-24 cycles through period-doubling. The period-doubling bifurcation rout to chaos is observed for in the system.

Figure 2(d) is a bifurcation diagram of the model (3) for  $b = 3$ . A stable coexistence is observed for  $0.006 \leq a < 0.0068$ . As the parameter  $a$  increases, a chaotic attractors appear from  $a \approx 0.0068$ . Thus a Hopf bifurcation occurs at  $a \approx 0.0068$ . When the parameter  $a$  is slightly increased beyond  $a \approx 0.00712$ , the chaotic attractors suddenly disappears. The system behaves chaotically for  $a \approx 0.0086$ .

Figure 3(a) is a bifurcation diagram for  $b = 4.5$ . The stable coexistence occurs when  $a$  belongs to  $(0.006, 0.00701)$ . When the parameter  $a$  increases, the period-4 orbits appear from  $a \approx 0.00701$ . As  $a$  is further increased, the chaos appears for  $a \approx 0.0092$ .

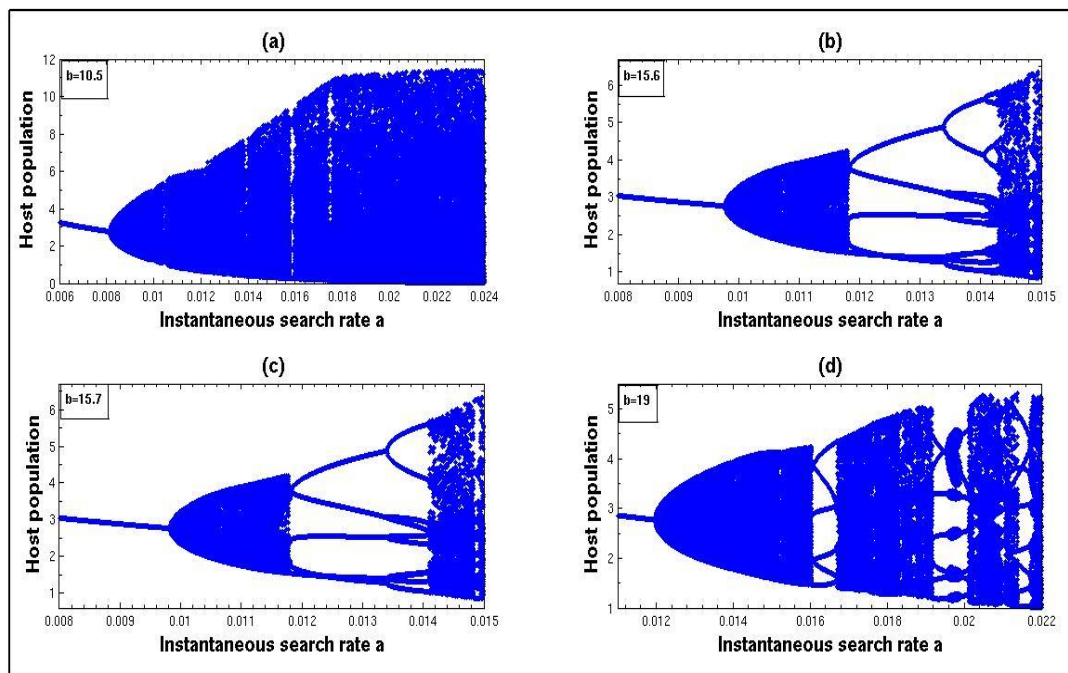
Bifurcation diagram for  $b = 5.5$  is shown in figure 3(b). For  $0.006 \leq a < 0.0072$ , a stable coexistence is observed. The chaotic attractor appear when  $a$  is increased from 0.0072. The Hopf bifurcation exists at  $a = 0.0072$ . As the parameter  $a$  further increases, the period-4 and period-16 behavior appears alternatively as we increased  $a$  from 0.00918 to 0.00955. When the parameter  $a$  is slightly increased beyond  $a \approx 0.00959$ , the system become chaotic. The chaotic bands disappear and form a period-4 cycle when  $a \approx 0.00969$  and the chaotic attractors again appear when the parameter  $a \geq 0.0102$ .



**Figure 3:** Bifurcation diagrams of the system (3) with respect to instantaneous search rate  $a$  for different values of  $b$ .

Figure 3(c) is a bifurcation diagram for  $b = 7$ . A stable coexistence is noticed when  $0.006 \leq a < 0.00747$ . The chaotic attractor reappears when  $a \geq 0.00747$  approximately. The Hopf point is observed at  $a = 0.00747$  and the crisis is observed at  $a \approx 0.00825$ . As the parameter  $a$  is increased further, the period-8 appears at  $a \approx 0.00901$  and period-16 appears at  $a \approx 0.00977$ . When  $a$  increased further, the period-doubling bifurcation is observed and the system becomes chaotic for  $a \approx 0.01$ . The periodic orbit appears in a narrow region in the neighborhood of  $a = 0.01044$ . Again chaotic behavior appears from 0.01054. The chaotic bands again disappear and form period-6 orbit from  $a \approx 0.0125$  and the chaotic attractors again appear when the parameter  $a$  is increased approximately starting from 0.0132.

Figure 3(d) is a bifurcation diagram for  $b = 7.5$ . A stable coexistence is observed when  $a$  is increased from 0.00377. The chaotic attractor appear when  $a$  is approximately increased from 0.0075. A periodic behavior is observed in the neighborhood of  $a = 0.00925$ . After that the chaotic bands reappear. Again the period-doubling bifurcation is observed near  $a = 0.0106$ . As the parameter  $a$  further increases, the period-doubling bifurcation goes to chaotic bands from  $a \approx 0.0107$ . The chaotic bands again disappear and form a period-6 cycle from  $a \approx 0.0126$  and the chaotic attractors again appear when the parameter  $a$  is increased approximately from 0.0132.



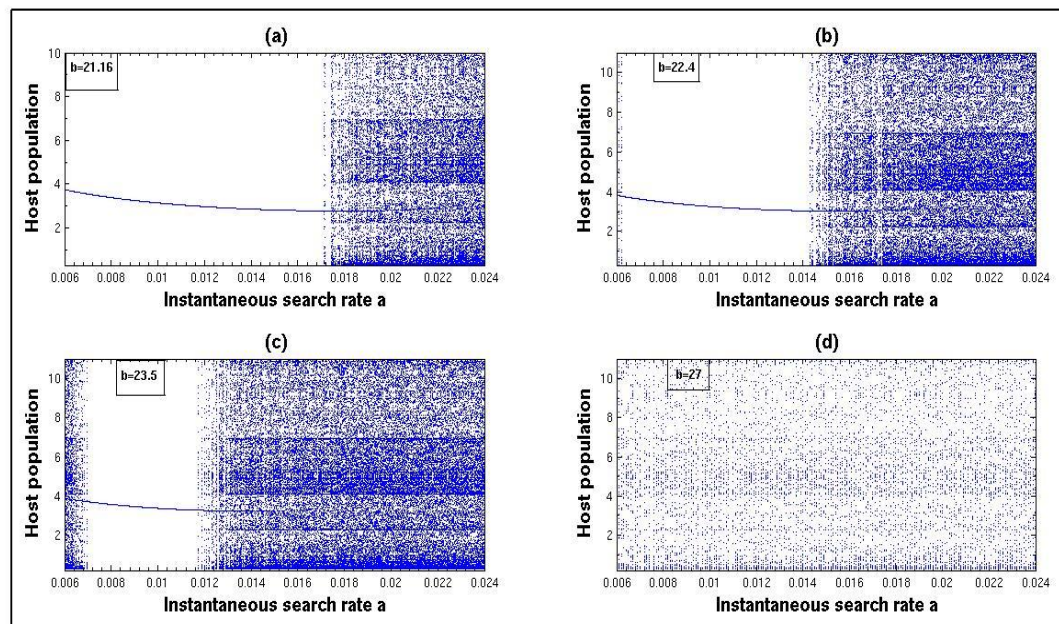
**Figure 4:** Bifurcation diagrams of the system (3) with respect to instantaneous search rate  $a$  for different values of  $b$ .

Figure 4(a) is a bifurcation diagram for  $b = 10.5$ . A stable coexistence is observed when  $a$  belongs to  $(0.00397, 0.0081)$ . The chaotic attractor appear when  $a$  is approximately equal to 0.0081. The regular and chaotic behaviors are alternately as we increased further up to 0.018. For  $a \geq 0.018$  the system behaves chaotically.

Bifurcation diagram for  $b = 15.6$  is presented in figure 4(b). A stable coexistence is observed for  $0.008 \leq a < 0.00973$ . The chaotic attractors appear when  $a$  is increased approximately from 0.00973. Therefore, a Hopf bifurcation occurs at  $a \approx 0.00973$  and the crisis period is observed at  $a \approx 0.01182$ . As the parameter  $a$  increases, the period-5 cycle bifurcates to period-10 at  $a = 0.01338$ . The period-10 goes to period-20 from  $a \approx 0.01046$ . The chaotic bands appear when  $a \approx 0.01435$ . A regular behavior appears in the neighborhood of  $a = 0.01484$  and the chaotic attractors again reappears for  $a \geq 0.01489$ .

Figure 4(c) is a bifurcation diagram for  $b = 15.7$ . A stable coexistence is observed when  $0.008 \leq a < 0.00972$ . The chaotic attractors appear when  $a$  is increased approximately from 0.00972. The Hopf and crisis is observed at  $a = 0.00972$  and  $a \approx 0.0118$  respectively. As the parameter  $a$  increases, the period-5 cycle bifurcates to period-10 from  $a \approx 0.0134$ . The chaotic bands reappear when  $a \approx 0.0141$ . Again, period-14 appears at  $a \approx 0.0148$  and then the chaotic band starts from  $a \geq 0.0149$ .

A stable coexistence appears for  $0.011 < a < 0.0119$  is shown in figure 4(d) for  $b = 19$ . The chaotic attractor appears when  $a \geq 0.0119$ . Therefore, a Hopf bifurcation occurs at  $a = 0.0119$ . As the parameter  $a$  increases, the period-6 bifurcates to chaos for  $a \geq 0.01677$ . Within  $0.0192 < a < 0.0201$ , the period-7 and chaotic bands appear alternately. The chaotic attractors again reappear from  $a \approx 0.0201$  and limit cycle periods appear in the neighborhood of  $a \approx 0.0215$ . Finally, the chaotic attractors appear from  $a \geq 0.0217$ .



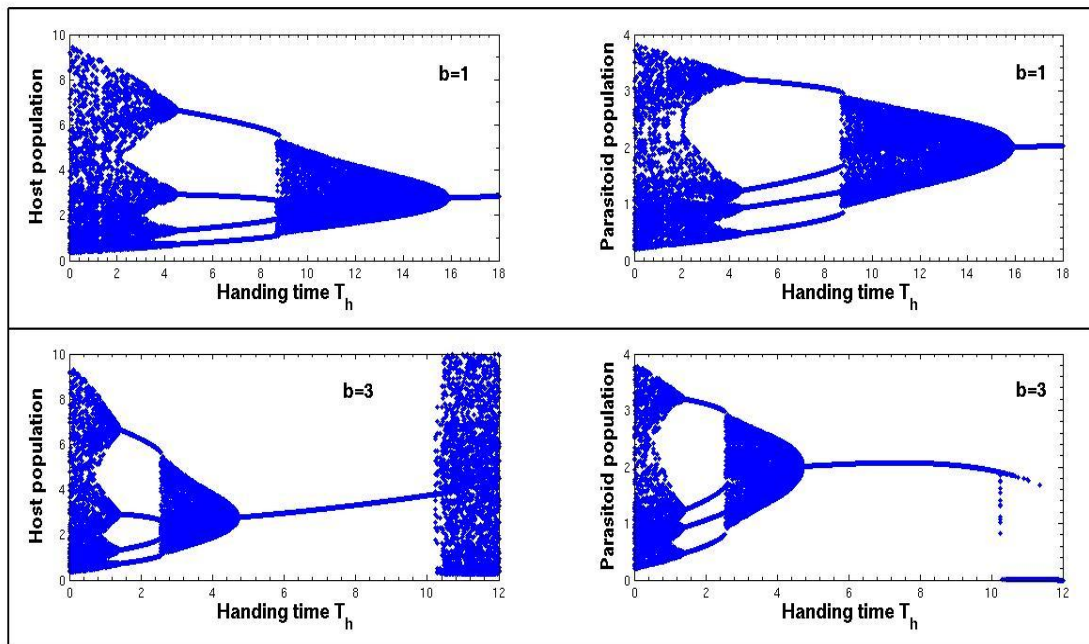
**Figure 5:** Bifurcation diagrams of the system (3) with respect to instantaneous search rate  $a$  for different values of  $b$ .

For  $0 \leq a < 0.00564$ , figure 5(a) shows the chaotic attractors for  $b = 21.16$ . A stable coexistence is observed when  $a \geq 0.00564$ . The chaotic attractors are starting when  $a > 0.0171$ . Figure 5(b) is a bifurcation diagram for  $b = 22.4$ . The chaotic attractors

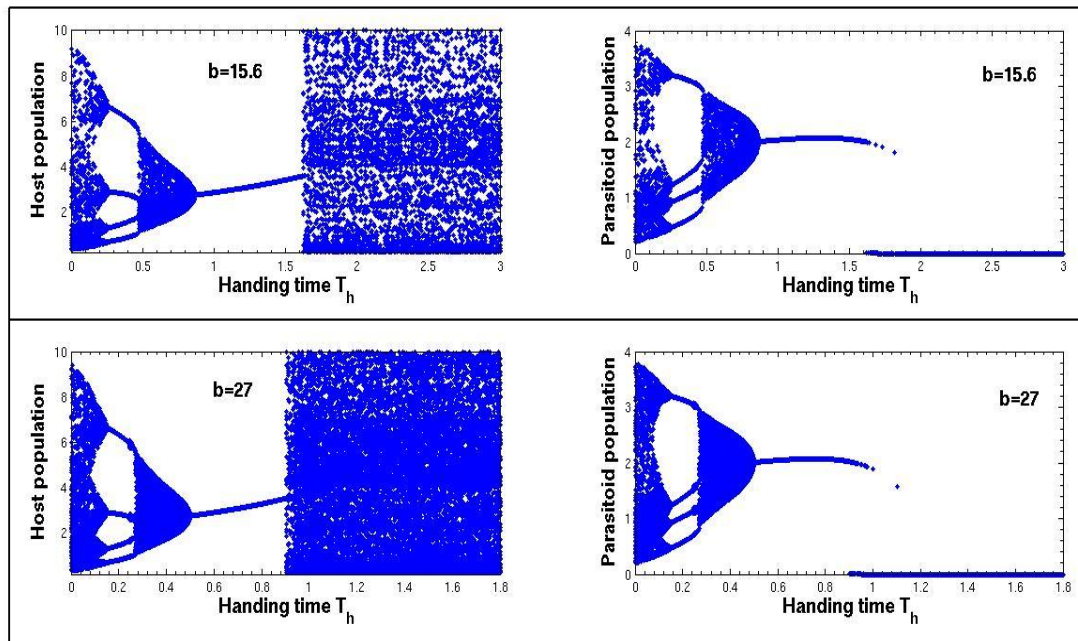
between host and parasitoid are observed for  $a \approx 0.00614$ . The stable coexistence is observed when  $a$  belongs to  $(0.00614, 0.0142)$ . Again the chaotic attractor is starting when  $a > 0.0142$ . Figure 5(c) is a bifurcation diagram for model (3) with  $b = 23.5$ . The chaotic attractors are observed within  $0 \leq a < 0.00681$ . The stable coexistence is observed when  $a$  is belongs to  $(0.00681, 0.0121)$ . Again the chaotic attractor reappears when  $a \geq 0.0121$ . In figure 5(d) bifurcation analysis is done taking  $b = 27$  and chaotic attractors are observed for all values of  $a$ .

### 3.3 Bifurcation analysis with respect to $T_h$

Bifurcation analysis with respect to handing time for different types of intraspecific competitions is done. From figure 6, we have observed that both host and parasitoid population have period-4 orbits and chaotic bands for exact competition ( $b = 1$ ). For Over-compensation ( $b = 3$ ), the host population have period-4, chaotic bands and settle downs to stable fixed point.



**Figure 6:** Bifurcation diagrams of the system (3) with respect to handing time  $T_h$  for different values of  $b$ .



**Figure 7:** Bifurcation diagrams of the system (3) with respect to handling time  $T_h$  for different values of  $b$ .

The parasitoid population also have period-4, chaotic bands but, it extinct after certain handling time. The stable coexistence is observed for both exact and over compensation. If handling time increases, the parasitoid population extinct due to over compensation. From figure 7 we observe that for higher values of intraspecific competitions parasitoid should have small handling time otherwise parasitoid species will extinct. Therefore handling time is very important biological parameter for stable or oscillatory coexistence of host and parasitoid species.

## 4 Conclusion

A Host-parasitoid model with intraspecific competition is proposed and analyzed. We have done the bifurcation analysis with respect to instantaneous search rate as well as handling time for different types of intraspecific competitions taking as parameter separately. We observe various types of non-unique dynamics having stable fixed point, chaotic bands, periodic windows, quasi-periodic orbits. We also observe Hopf bifurcation and attractor crises. Stable and oscillatory coexistence of host and parasitoid are observed in different types of competitions for some values of instantaneous search rate. We observe stable coexistence as well as oscillatory coexistence of host and parasitoid species for all types of competitions for specific values of handling time in presence of very high intraspecific competitions. For survival of the parasitoid species at high intraspecific competition, parasitoid should have small handling time otherwise parasitoid species will extinct. Complex dynamic patterns have been observed in Host-parasitoid model with intraspecific competition which can produce high-period oscillations and chaos. Identifying complicated, possibly chaotic, dynamics in population data has remained a major challenge in ecological studies [15] and it is not easier to predict this complexity by the theory. The present research helps to

understand the dynamical behavior of host-parasitoid interactions with intraspecific competition which can be used to improve the classical biological control of pests.

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